**International Refereed Journal of Engineering and Science (IRJES)**
*ISSN (Online) 2319-183X, (Print) 2319-1821*
*Volume 5, Issue 5 (May 2016), PP.34-39*

# Numerical-Based Radar Cross Section Estimation of a Dielectric Cylinder

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**Abstract:-**In this paper two dimensional integral equation based numerical approach is used to analyze the scattered field from a homogeneous dielectric circular cylinder. The surface integral equation method replaces the dielectric scatterer surface into equivalent surface electric and magnetic currents. The cylinder contour is discretized into N sections to obtain 2N linear equations, represented in the matrix form. The equation is approximated as a sum of weighted terms using known expansion functions. The resultant matrix equation is solved to obtain electric and magnetic currents and hence cylinder RCS.

**Index-Terms:-**Dielectric homogeneous cylinder, method of moments, polarization, radar cross section.

## I. INTRODUCTION

The surface of many practical scatterers, such as fuselage of aircraft, missiles, etc. can be conveniently represented by cylindrical shapes [2]. In particular, circular cylinder is simplest and its scattered field solution can be expressed in terms of Bessel and Hankel functions. The EM scattering analysis of different conducting and dielectric cylindrical structures has been carried out using different high-frequency and numerical-based techniques. Among the low-frequency numerical-methods, Method of Moments (MoM) is one of the efficient methods. This method essentially converts surface or volume integral equation into matrix form towards the solution. The estimation of the scattered field and hence radar cross section (RCS) depends on the size of moment matrix.

The scattering of arbitrary inhomogeneous and anisotropic dielectric body is done using such approach [1]. The scattered field formulation is obtained in form of an integral equation and is solved using pulse function expansion and point matching method. The conjugate gradient method and fast Fourier transform has been employed to make computations efficient and faster. The drawback of this method is that for large value of relative permittivity, rate of convergence becomes slow. Thus pulse basis block model was proposed [2] for solving electric field integral equations for dielectric scatterer with large permittivity. The main approach was to use face-centered node points and unique way to choose the unknown field at this node points. Conventionally the fields are sampled at the centre of each cell requiring accurate prediction of induced polarization charges in the cell especially for higher permittivities. Further the fields at node points are chosen in such a way that the off-diagonal elements of moment matrix are of small magnitude. This maintains the condition number of matrix and facilitates accurate solution even at higher permittivities. Moreover this method preserves convolution property of electric field integral equations.

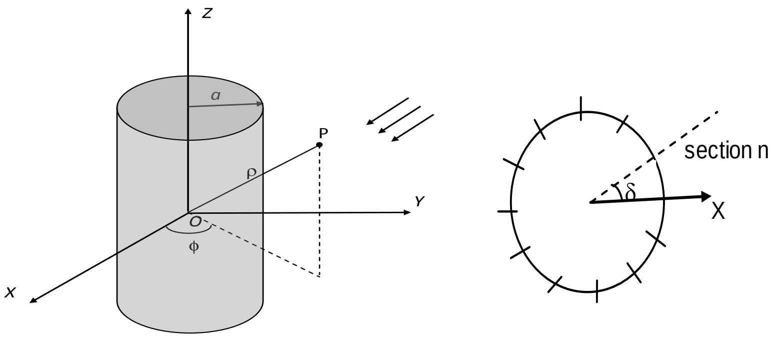
Trivedi and Khankhoje [3] proposed MOM technique for computing EM scattering from homogeneous dielectric rough cylinder and conducting cylinder at TM polarization. The system of linear equations is solved by discretizing EFIE to arrive at cylinder RCS. The proposed method is applicable to both 2-D and 3-D structures. For an infinitely long cylinder, in MoM the problem is solved by truncating the cylinder. Zoughi and Qaddoumi [4] treated scattering analysis of finite-length electrically large defected cylinder for various polarizations. The dependence of RCS on the frequency was analyzed. It was shown that the number of lobes in RCS pattern and scattering level increases with frequency. The results are discussed for both solid and hollow cylinders using MoM. The method can be extended for higher frequencies and electrically large cylinders by making the impedance matrix symmetric. Rao *et al.* [5] used MoM to solve coupled integral equations for finite conducting and dielectric structures. The planar triangular patches were used for modeling of disk and cylinders. The technique used employs same basis functions for expanding both electric and magnetic currents.

The EM scattering from homogeneous chiral cylinder of arbitrary cross section has been dealt by Majeed*et al.* [6]. The equivalence principle has been used to compute equivalent surface currents by solving coupled integral equations using MoM. The point matching technique is used for the expansion of function. The electric field produced by 2-D surface current is expressed in terms of Green's dyadic function. It is shown that the presence of chirality changes the dielectric characteristics. The technique can efficiently used for large cylinders. The moment matrix elements for surface formulation are much complicated than that for volume formulation due to the use of Green's function. This is reason why surface formulation is not able to handle inhomogeneous cylinder like volume formulation do.

Further Mautz and Harrington [7] presented the scattering analysis of homogeneous dielectric body of revolution such as dielectric sphere using MoM. The computations involve PMCHW formulation (formed by the initials of the investigators, *viz.*Poggio, Miller, Chang, Harrington, and Wu) [8]-[10] and Muller formulation [11]. The solutions are shown to be unique at all frequencies.

The scattering analysis of electrically large conducting and dielectric objects are discussed in [12]. The technique used is based on spatial decomposition (SDT) of large body into sub-zones, reducing the size of MoM matrix. The proposed method is reported to reduce the dimensionality and ill-conditioning of MoM matrix computation burden. The sub-divided sub-zones are separated by virtual surfaces having tangential electric and magnetic currents. The appropriate boundary conditions are imposed to solve these currents. The results are shown for electrically large perfectly conducting thin strip, rectangular and wedge shaped structure, and dielectric rectangular cylinder.

In this paper the scattering analysis of dielectric cylinder based on surface integral equation (SIE) method has been presented. The SIE method replaces the dielectric scatterer surface into equivalent surface electric and magnetic currents. When boundary conditions are imposed on the field components, produced by the equivalent currents, coupled integral equations can be obtained. These integral equations are solved for unknown equivalent current towards the scattered field. The numerical approach, Method of Moments (MoM) has beenused for solving integral equations.In this method, the integral equations are reduced to a system of linear equations, expressed in matrix form. The equation is first approximated as a sum of weighted terms using known expansion functions (basis function), which is solved to arrive at cylinder RCS. The effect of parameters such as dimension of cylinder and material characteristics on RCS value is analyzed.




**(a)** **(b)**

**Figure 1** (a) Schematic of homogeneous dielectric cylinder. (b) Cylinder contour divided into sections.

## II. ELECTRIC FIELD INTEGRAL EQUATION FOR HOMOGENEOUS DIELECTRIC CYLINDER

Surface discretization is used for homogenous dielectric cylinder rather than volume discretization since fewer unknowns result from surface integral equation. A homogeneous dielectric cylinder of circular cross section is depicted in Fig. 1. Cylinder of radius, *a* and dielectric constant $\varepsilon_d$ is considered. *P* is the observation point at a distance $\rho$ from the origin. A TM polarized plane electromagnetic wave is assumed to be incident on the cylinder, expressed as

$$E_i = E_o\hat{z} = e^{-jk_o x}\hat{z} \qquad (1)$$

The scattering analysis startswith the electric field integral equations,given by [13]

$$E_i\big|_s = K(\phi) + j\eta_o k_o A_{zo}\big|_s + \left[\frac{\partial \Phi_{yo}}{\partial x} - \frac{\partial \Phi_{xo}}{\partial y}\right]\Bigg|_{s^+} \qquad (2)$$

$$0 = -K(\phi) + j\eta_d k_d A_{zd}\big|_s + \left[\frac{\partial \Phi_{yd}}{\partial x} - \frac{\partial \Phi_{xd}}{\partial y}\right]\Bigg|_{s-} \qquad (3)$$

$A_z$ , $\Phi_y$ indicates magneticand electric vector potential respectively.*k*is the wave number and $\eta$ represents the intrinsic impedance.The subscript '*o*'denotes air and '*d*'is for dielectric.The electric and magnetic vector potential can be expressed in terms of equivalent electric, $J(\phi)$ and magnetic currents, $K(\phi)$. The expressions (2) and (3) corresponds to the case with observation point just outside ($s^+$) and inside ($s^-$) the surface respectively.

The contour of the cylinder is discretized into $N$ sections, each section is of an angular width, $\delta = \frac{2\pi}{N}$.

The triangular basis function is used to express $J(\phi)$ and $K(\phi)$ which are unknown quantities.

$$J(\phi) = \sum_{n=0}^{N-1} j_n \Delta_n(\phi) \tag{4}$$

$$K(\phi) = \sum_{n=0}^{N-1} k_n \Delta_n(\phi)\,, \tag{5}$$

where triangular function is given by [3]

$$\Delta_n(\phi) = \begin{cases} 1 - \frac{1}{\delta}\left|\phi - n\delta\right| & ;\ for (n-1)\delta < \phi < (n+1)\delta \\ 0 & ; otherwise \end{cases} \tag{6}$$

The integral equationsareconverted into 2N linearly independent equations,

$$e_m = \frac{3\delta}{4} k_m + \frac{\delta}{8}(k_{m-1} + k_{m+1}) + j\eta_o k_o \sum_{n=0}^{N-1} \alpha_{n,m}^{o}\ j_m + \sum_{n=0}^{N-1} \beta_{n,m}^{o} k_m \tag{7}$$

$$0 = -\frac{3\delta}{4} k_m + \frac{\delta}{8}(k_{m-1} + k_{m+1}) + j\eta_d k_d \sum_{n=0}^{N-1} \alpha_{n,m}^{d}\ j_m + \sum_{n=0}^{N-1} \beta_{n,m}^{o} k_m \text{ where, } e_m = \delta e^{-jk_o a\cos(m\delta)} \tag{8}$$

$$\alpha_{m,n}^{(p)} = \frac{1}{4j} a\alpha_{m,n,1}^{(p)} + \frac{1}{4j} a\alpha_{m,n,2}^{(p)} + \frac{1}{4j} a\alpha_{m,n,3}^{(p)} \tag{9}$$

$$\beta_{m,n}^{(p)} = \frac{k_p}{4j}\left(\beta_{m,n,1}^{(p)} + \beta_{m,n,2}^{(p)} + \beta_{m,n,3}^{(p)}\right) \tag{10}$$

There are four conditions, given by

(i) For $m \neq n$, $n-1$, $n+1$

$$\alpha_{m,n,1}^{(p)} = \frac{\delta^2}{8} H_o^{(2)}(k_p R_{m,n}) \tag{11a}$$

$$\alpha_{m,n,2}^{(p)} = \frac{3\delta^2}{4} H_o^{(2)}(k_p R_{m,n}) \tag{11b}$$

$$\alpha_{m,n,3}^{(p)} = \frac{\delta^2}{8} H_o^{(2)}(k_p R_{m,n}) \tag{11c}$$

$$\beta_{m,n,1}^{(p)} = \frac{\delta^2}{8} \frac{H_o^{(2)'}(k_p R_{m,n})}{R_{m,n}} F_{m,n} \tag{11d}$$

$$\beta_{m,n,2}^{(p)} = \frac{3\delta^2}{4} \frac{H_o^{(2)'}(k_p R_{m,n})}{R_{m,n}} F_{m,n} \tag{11e}$$

$$\beta_{m,n,3}^{(p)} = \frac{\delta^2}{8} \frac{H_o^{(2)'}(k_p R_{m,n})}{R_{m,n}} F_{m,n} \tag{11f}$$

where,

$R_{m,n} = \sqrt{a^2 - 2a^2\cos(m-n)\delta}$ and

$F_{m,n} = a^2\cos((n-m)\delta) - a^2$

(ii) For $m=n$

$$\alpha_{m,m,1}^{(p)} = \frac{\delta^2}{8}\left[1 - \frac{2j}{\pi}\ln(\frac{\gamma k_p}{2} a)\right] - \frac{2j\delta^2}{\pi}\left[\frac{1}{8}\ln(\delta) - 0.0699\right] \tag{12a}$$

$$\alpha^{(p)}_{m,m,2} = \frac{3\delta^2}{4}\left[1 - \frac{2j}{\pi}\ln(\frac{\gamma k_p}{2}a)\right] - \frac{2j\delta^2}{\pi}\left[\frac{3}{4}\ln(\delta) - 1.15057\right] \quad (12b)$$

$$\alpha^{(p)}_{m,m,3} = \frac{\delta^2}{8}\left[1 - \frac{2j}{\pi}\ln(\frac{\gamma k_p}{2}a)\right] - \frac{2j\delta^2}{\pi}\left[\frac{1}{8}\ln(\delta) - 0.0699\right] \quad (12c)$$

$$\beta^{p}_{m,m,1} = \beta^{p}_{m,m,3} = \frac{j\delta^2}{8\pi k_p} \quad (12d)$$

$$\beta^{o}_{m,m,2} = \frac{3j}{4\pi k_o}\delta(\delta - 2\pi) \quad (12e)$$

$$\beta^{d}_{m,m,2} = \frac{3j}{4\pi k_o}\delta(\delta + 2\pi) \quad (12f)$$

(iii) For $m=n+1$

$$\alpha^{(p)}_{m,m-1,1} = \frac{\delta^2}{8}H^{(2)}_o(k_p R_{m,n}) \quad (13a)$$

$$\alpha^{(p)}_{m,m-1,2} = \frac{3\delta^2}{4}\left[1 - \frac{2j}{\pi}\ln(\frac{\gamma k_p}{2}a)\right] - \frac{2j\delta^2}{\pi}\left[0.749\ln(\delta) - 0.33\right] \quad (13b)$$

$$\alpha^{(p)}_{m,m-1,3} = \frac{\delta^2}{8}\left[1 - \frac{2j}{\pi}\ln(\frac{\gamma k_p}{2}a)\right] - \frac{2j\delta^2}{\pi}\left[0.3124\ln(\delta) - 0.29\right] \quad (13c)$$

$$\beta^{(p)}_{m,n,1} = \frac{\delta^2}{8}\frac{H^{(2)'}_o(k_p R_{m,n})}{R_{m,n}}F_{m,n} \quad (13d)$$

$$\beta^{p}_{m,m+1,2} = \frac{3j\delta^2}{4\pi k_d} \quad (13e)$$

$$\beta^{o}_{m,m+1,3} = \frac{j\delta}{8\pi k_o}(\delta - 2\pi) \quad (13f)$$

$$\beta^{d}_{m,m+1,3} = \frac{j\delta}{8\pi k_d}(\delta + 2\pi) \quad (13g)$$

(iv) For$m=n$-1

$$\alpha^{(p)}_{m,m+1,1} = \frac{\delta^2}{8}\left[1 - \frac{2j}{\pi}\ln(\frac{\gamma k_p}{2}a)\right] - \frac{2j\delta^2}{\pi}\left[0.3124\ln(\delta) - 0.29\right] \quad (14a)$$

$$\alpha^{(p)}_{m,m+1,2} = \frac{3\delta^2}{4}\left[1 - \frac{2j}{\pi}\ln(\frac{\gamma k_p}{2}a)\right] - \frac{2j\delta^2}{\pi}\left[0.749\ln(\delta) - 0.33\right] \quad (14b)$$

$$\alpha^{(p)}_{m,m+1,3} = \frac{\delta^2}{8}H^{(2)}_o(k_p R_{m,n}) \quad (14c)$$

$$\beta^{o}_{m,m-1,1} = \frac{j\delta}{8\pi k_o}(\delta - 2\pi) \quad (14d)$$

$$\beta^{d}_{m,m-1,1} = \frac{j\delta}{8\pi k_d}(\delta + 2\pi) \quad (14e)$$

$$\beta^{p}_{m,m-1,2} = \frac{3j\delta^2}{4\pi k_p} \tag{14f}$$

$$\beta^{(p)}_{m,m-1,3} = \frac{\delta^2}{8}\frac{H_o^{(2)'}(k_p R_{m,n})}{R_{m,n}} F_{m,n} \tag{14g}$$

The equation (7) can be expressed in matrix form as shown below

$$\begin{bmatrix} e_o \\ e_1 \\ . \\ e_N \\ 0 \\ 0 \\ . \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{1,1} & Z_{1,2} & Z_{1,3} & . & . & . & Z_{1,2N-1} & Z_{1,2N} \\ Z_{2,1} & Z_{2,2} & Z_{2,3} & . & . & . & Z_{2,2N-1} & Z_{2,2N} \\ & & & & & & & \\ . & & & & & & & \\ . & & & & & & & \\ . & & & & & & & \\ . & & & & & & & \\ Z_{2N,1} & Z_{2N,2} & Z_{2N,3} & . & . & . & Z_{2N,2N-1} & Z_{2N,2N} \end{bmatrix} \cdot \begin{bmatrix} j_1 \\ . \\ . \\ j_N \\ k_1 \\ . \\ . \\ k_N \end{bmatrix} \tag{15}$$

By solving (15), the electric and magnetic currentsare obtained, which are used to determine the bistatic scattering cross section $\sigma_{par}(\phi)$ for parallel polarization.

$$\sigma_{par}(\phi) = \frac{k_o}{4}\left|\sum_{n=1}^{N}\left[\eta_o j_n - k_n \cos(\phi - n\delta)\right] a\delta\, e^{jk_o a\cos(\phi - n\delta)}\right|^2 \tag{16}$$

## III. RESULTS AND DISCUSSION

The RCS of homogeneous dielectric cylinder is computed based on the analytical expressions discussed above. Fig. 2 presents the scattering cross section of a homogeneous dielectric infinitely long circular cylinder ($a$=2$\lambda$; $\varepsilon$=2$\varepsilon_o$). The cylinder contour is discretized into $N$=100 sections ($\delta$=0.063 rad).

Next the effect of permittivity on the cylinder RCS is analyzed towards the parametric analysis. Fig. 3 represents the contour plot of the cylinder RCS for different permittivity and azimuthal angle.

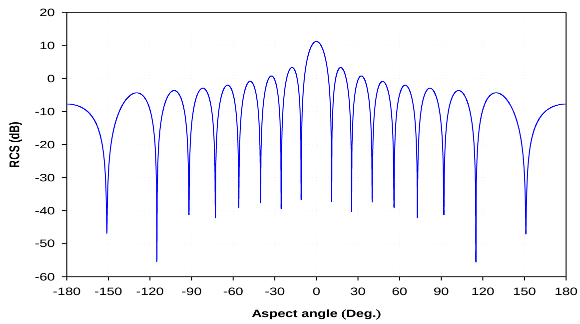



**Figure 2:** Radar cross section of homogeneous dielectric cylinder ( $\varepsilon$=2$\varepsilon_o$ and $a$=2$\lambda$).

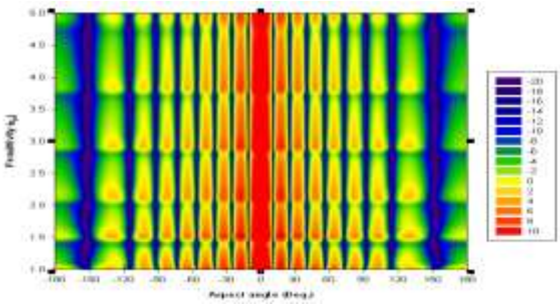


**Figure 3** Radar cross section of dielectric circular cylinder ($a$=2$\lambda$) for different permittivity

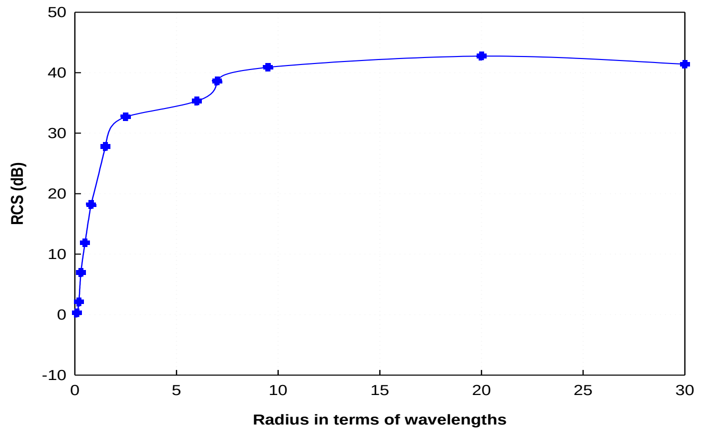



**Figure 4:** Dependence of radar cross section on the radius of dielectric cylinder

Fig. 4 represents the RCS variation with radius of cylinder. As the electrical radius increases, the specular RCS of dielectric cylinder ($\varepsilon$=2$\varepsilon_o$) increases but saturates beyond $a$=10λ .

## IV. CONCLUSION

The cylindrical geometry being one of the important components in the geometry of aerospace structures has been considered for RCS analysis. A numerical-based approach has been employed for obtaining scattered field of an infinitely-long homogeneous dielectric circular cylinder. The effect of parameters such as electrical radius, constitutive parameters of dielectric material on cylinder RCS has been analyzed. This parametric analysis is towards the RCS optimization of cylindrical surfaces.

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