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A Firefly Algorithm for Optimizing Spur Gear Parameters Under Non-Lubricated Condition

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Abstract:- Firefly algorithm is one of the emerging evolutionary approaches for complex and non-linear optimization problems. It is inspired by natural firefly's behavior such as movement of fireflies based on brightness and by overcoming the constraints such as light absorption, obstacles, distance, etc. In this research, firefly's movement had been simulated computationally to identify the best parameters for spur gear pair by considering the design and manufacturing constraints. The proposed algorithm was tested with the traditional design parameters and found the results are at par in less computational time by satisfying the constraints.

Keywords: - Constraints, Firefly Algorithm, Optimization, Spur gear

I. INTRODUCTION

Optimization is the process of identifying the better solution from the predefined solution space by considering the constraints. i.e. all the suitable parameter values influencing the problem are possible solutions, which used to form the solution space and the best value in the solution space is the optimum solution [1]. Often optimization problems are solved through deterministic algorithms and stochastic algorithms. As the name implies, the former algorithm follows a rigorous procedure to identify the optimal solution, but it might not be best suitable for all kinds of optimization problems. Later algorithm is further sub-divided into heuristic and meta-heuristic algorithms. Heuristic algorithms are the more efficient and effective algorithm for solving a particular category of the optimization problems and cannot be the best suitable for all category of the problems. On the other hand, nature-inspired meta-heuristic algorithms can be used to solve global optimization problems. Because, these algorithms use random values to identify the best suitable path and to avoid the stagnation at local optimal points, similar to the nature. In contradict, the random values can also deviated the convergence path away from the solution. Thus there exists a tradeoff between randomization and local search [1, 2, 3]. So the meta-heuristic optimization algorithms are having a deterministic component and random component for randomly sampling the search space and for random walk along the path. One of the best meta-heuristic algorithm commonly used by the researchers is the Genetic Algorithm (GA) [4, 5]. In recent years, many modern meta-heuristic algorithms were developed based on swarm intelligence like PSO and AFSA [6, 7].

Firefly algorithm is one of the emerging meta-heuristic algorithms developed by Xin-She Yang, which shows its superiority over some traditional algorithms [8, 9]. Firefly algorithm is inspired by the natural behavior of fireflies in identifying their mating pair. Fireflies are the flying insects capable of producing light through its special photogenic organs in their body surface [10]. The intensity of the light generated from them is attracted by the other fireflies. Also the firefly does not memorize any history of better situation and they move regardless of it, thereby it searches in the global space instead of local search. So the similar concept was simulated in firefly algorithm for solving optimization problems.

This paper aims to identify the optimal parameters for the spur gear pair by considering the design and manufacturing constraints in an non-lubricated conditions. The rest of this paper is organized as follows: Section II and Section III outlines the Firefly algorithm and Spur gear design respectively. Experimental procedure and results are presented in section IV. Section V concludes the research.

II. FIREFLY ALGORITHM

Firefly algorithm was first introduced by Xin-She Yang [2] and it is mimicking the behavior of identifying the mating pair by its flashing characteristics. Some of the assumptions to be considered for simulating the firefly algorithms as=re as follows.

1. All fireflies are unisex.
   So that, any firefly can be attracted towards other fireflies regardless of their sex.
2. Attractiveness is proportional to their light intensity.
   The fireflies are attracted and move towards the brighter firefly. Also the brightness decreases with increase in distance, so no one is permanently brighter.
3. The light intensity of a firefly is affected by the environmental condition.

The brightness of a firefly can be influenced by the environmental conditions like clear sky, mist, foggy surrounding, rain, etc.

As the light intensity decides the firefly movement, in this research, the objective function value used to decide the light intensity. The combined objective function for optimizing the spur gear parameter is given in the Equation 1.

$$COF = \{ [(F_1 / \text{max. } F_1) + (F_2 / \text{min. } F_2) + (F_3 / \text{max. } F_3) + (F_4 / \text{min. } F_4)] / 4\} \quad (1)$$

Whereas,

F_1 is power transmitted by the gear given in the Equation 2.

$$F_1 = P \quad \text{where, } P^{(L)} \leq P \leq P^{(U)} \quad (2)$$

P is Power transmitting capacity and between lower and upper limit.

F_2 is the weight of the gear and is given in the Equation 3.

$$F_2 = \{[\frac{\pi}{4} \times d_1^2 \times b \times \rho] + [\frac{\pi}{4} \times d_2^2 \times b \times \rho]\} \quad (3)$$

d_1, d_2 = Pitch circle diameter of pinion and gear in mm
b = Thickness of pinion and gear in mm
ρ = Density of the material in kg/mm^3

F_3 is the efficiency of the gear given in the Equation 4.

$$F_3 = 100 - P_L \quad (4)$$

P_L = Power loss which is calculated by the Equation 5.

$$P_L = \frac{50 f}{\text{Cos}\Phi} \times \frac{(H_s^2 + H_t^2)}{(H_s + H_t)} \quad (5)$$

H_s = Specific sliding velocity at start of approach action
H_t = Specific sliding velocity at end of recess action
f = Coefficient of friction
Φ = Pressure angle in degrees

H_s and H_t are calculated by the Equations 6 & 7.

$$H_t = \frac{(i+1)}{i} \times \sqrt{\left(\left[\frac{r_0}{r}\right]^2 - cos^2\Phi\right)} - \sin\Phi \quad (6)$$

$$H_s = (i + 1) \times \sqrt{\left(\left[\frac{R_0}{R}\right]^2 - cos^2\Phi\right)} - \sin\Phi \quad (7)$$

Where, R & R_o = Pitch and Outside circle radius of gear in mm.
R & r_o = Pitch and Outside circle radius of pinion in mm
R_0 = R + one addendum

One addendum for 20° full depth involute system = one module = m.

$$r_o = r + m = \frac{d_1}{2} + m$$

$$Ro = R + m = \frac{d_2}{2} + m$$

F_4 is minimization of center distance and is given in Equation 8.

$$F_4 = \frac{(d_1 + d_2)}{2} = \frac{m}{2} (Z_1 + Z_2) \quad (8)$$

Where, z_1, z_2 = Number of teeth in pinion and gear respectively.

For solving an optimization problem, Firefly algorithm has to be applied iteratively. Initially the solution space 'S' should be created and 'n' fireflies need to be allowed for searching the optimal path in 'S'. For the first iteration, fireflies are dislocated randomly in S and for forthcoming iterations, firefly movements can employed with some deterministic strategies. Attractiveness of a firefly by the other fireflies is determined by its light intensity which in turn is proportional to the objective function. i.e. the light intensity 'I' of a firefly at a particular distance 'x' can be chosen from the Equation 9.

$$I(x) \ \alpha \ COF \tag{9}$$

However, the attractiveness 'β' is relative with respect to distance 'r_{ij}'. 'r_{ij}' is the distance between i^{th} firefly to j^{th} firefly in the 'S'. As the distance 'r_{ij}' increases, the brightness I(x) decrease and inturn the attractiveness 'β' decreases proportionally. Also the light is absorbed in the surrounding media and leads to decrease in the light intensity. So the attractiveness depends on the coefficient of absorption [11]. The Equation 10 is used for calculating the light intensity I(r).

$$I(r) = I_0 \ e^{-\gamma r} \tag{10}$$

where 'I_0' is the initial light intensity and 'γ' is the light absorption coefficient. The firefly attractiveness is calculated using the Equation 11.

$$\beta = \beta_0 \ e^{-\gamma r2} \tag{11}$$

where β_0 is the initial attractiveness of the fireflies which is set a unit value. $\gamma = 0$ if the fireflies are at same point. In general, the range of attractiveness should be between 0 and 1. The distance between the i^{th} firefly and the j^{th} firefly are determined by the Equation 12.

$$r_{ij} = \{\Sigma \ (X_{ik}-X_{jk})^2\}^{\wedge \ (1/2)} \quad k = \{ 1, .. d\} \tag{12}$$

The fireflies are attracted towards the more brighter firefly and is determined using the Equation 13.

$$X_i = X_i + \beta_0 \ e^{-\gamma r2} \ (X_j-X_i) + \alpha\varepsilon^i \tag{13}$$

The second term in the equation is depend on the attractiveness and the third term is the random number generation based on the Gaussian distribution which decides the convergence rate. Thus the firefly are allowed to move towards the brighter firefly and finally to terminate. In each and every iteration, the light intensity should be updated using exploration and exploitation concept. The same procedure has to be repeated for more number of iteration and the final best value will be the optimal value for the spur gear optimization problem

III. SPUR GEAR DESIGN OPTIMIZATION PROBLEM

Gears are defined as the friction wheels, which transmits rotational power among the shafts by increasing or decreasing the speed and torque. To increase the friction and transmission effectiveness, teethes are formed over the surface of the gear wheel. The straight teethed gears are normally called as spur gears. Generally one gear receives power and transmitted to the other. The former is called driver and the later is called driven. Spur gears are easy to manufacture and commonly used gears in all type of machine tools and automobiles to transmit power between the parallel shafts. So the design of spur gear pair is taken in this research for optimization of the parameters.

3.1 Constraints

Generally all the mechanical component design should be within the ultimate strength and the allowable stress values called as design constraints. Also the gear parameter should align with the standard design parameters for ease of manufacturing and are generally called as manufacturing constraints. major constraints considered in this research are as follows.

(i) Bending Stress Constraint

While transmitting the power between the shafts, the top portion of the gear teeth will subject to bending stress and the condition for the bending stress constraint is given in the Equation 14.

$$\sigma_b \leq [\sigma_b]_{al} \tag{14}$$

Whereas, $[\sigma_b]_{al}$ = Allowable bending stress in N/mm^2.

σ_b is the actual bending stress in N/mm^2 and is given in Equation 15.

$$\sigma_b = \frac{(i+1)}{(a\,m\,b\,y)} \times [M_t] \tag{15}$$

Where, i = gear ratio
a = Center distance between gear and pinion
y = Form factor
$[M_t]$ = Design twisting moment in Nmm, and is given in Equation 16.

$$[Mt] = M_t \times k \times k_d \quad (16)$$

M_t = Normal twisting moment transmitted by the pinion in Nmm
K and k_d are the Load Concentration factor and Dynamic load factor

(ii) Compressive Stress Constraint

Compressive stresses are created at the bottom surface of teethes of the mating gears. The compressive stress constraint is given in the Equation 17.

$$\sigma_c \leq [\sigma_c]_{al} \quad (17)$$

Where, $[\sigma_c]_{al}$ = Allowable crushing stress in N/mm^2.
σ_c = Actual crushing stress in N/mm^2 and is given in Equation 18.

$$\sigma_c = 0.74\left(\frac{i+1}{a}\right) \times \sqrt{\left[\left(\frac{i+1}{ib}\right) \times E \times [M_t]\right]} \quad (18)$$

Where, E = Young's Modulus of the gear material in N/mm^2

(iii) Module Constraint

The condition for module is given in the Equation 19.

$$m \geq m_{min} \quad (19)$$

m_{min} is the minimum module and is given in the Equation 20.

$$m_{min} = 1.26 \times \sqrt[3]{\frac{[M_t]}{(y\,\sigma_b \Psi_m Z_1)}} \quad (20)$$

Ψ_m = ratio between the gear pair thickness and module.
The obtained module value should be standardized to the 'R' series values.

3.2 Spur Gear Parameters Calculation

The design of spur gear is given in the following equations.

(i) Gear Ratio

The gear ratio is the ratio of speed of the pinion and the gear wheel. The formula for calculating gear ratio is given in the Equation 21.

$$i = \frac{Z_2}{Z_1} \text{ (or) } \frac{d_2}{d_1} = \frac{N_1}{N_2} \quad (21)$$

(ii) Center Distance between pinion and gear

The size of the gear wheels decides the centre distance [12] and the formula for calculating the centre distance is given in the Equation 22 and the condition to be satisfied is given in the Equation 23.

$$a = \frac{(d_1 + d_2)}{2} = \frac{m}{2}\,[Z_1 + Z_2] \quad (22)$$

$$a \geq a_{min} \quad (23)$$

The minimum center distance can be calculated from the Equation 24.

$$a_{min} = (i+1)\sqrt[3]{\left[\left(\frac{0.74}{[\sigma_c]}\right)^2 \times \left(\frac{E\,[M_t]}{i\Psi}\right)\right]} \quad (24)$$

Ψ = ratio between the gear pair thickness and center distance.

3.3 Gear Surface Temperature

The condition considered for the spur gear design is in a non-lubricated condition and the surface temperature of the gear wheels should be kept within the allowable limits while optimizing the design, because the gear life and lubrication depend mainly on the amount of heat generated [13]. The maximum contact temperature is obtained by Equation 25.

$$\theta_{B\,max} = \theta_M + \theta_{fl\,max} \qquad (25)$$

Whereas, θ_M is tooth temperature,
$\theta_{fl\,max}$ is maximum flash temperature along the line-of-action, which is calculated by Blok's relation given in the Equation 26.

$$\theta_{fl} = 31.62\ K\ \mu_m\ (X_r\ W_n / \sqrt{(b_H)})\ x\ \{(|V_{r1} - V_{r2}|) / [(B_{M1}\sqrt{(V_{r1})}) - (B_{M2}\sqrt{(V_{r2})})]\} \qquad (26)$$

Whereas, K = Hertzian distribution of frictional heat = 0.8;
μ_m = Mean coefficient of friction; X_r = Load sharing factor;
W_n = Normal unit load;
B_{M1} & B_{M2} = Thermal contact coefficients of the pinion and wheel;
V_{r1} & V_{r2} = Rolling tangential velocities in m/s of the pinion and wheel;

3.4 Gear Noise Calculation

Noise in the gearing system is due to vibration and transmission error, i.e. the irregularities of the gear motion caused by change in tooth topology, shaft deflections and mesh stiffness variation along the line of action [14]. Transmission error along the line of action is given in the Equation 27.

$$TE = R_b\ \{\delta 2 - (i\ \delta 1)\} \qquad (27)$$

Whereas, R_b is gear base radius;
$\delta 1$, $\delta 2$ are the angular rotation of pinion and gear;

3.5 Primary Gear Parameters

For experimentation purpose, the primary gear parameters are set with the limits and the limit can be changed by the user depend upon the application. The primary gear parameters used to form the solution space are given in the Table 1.

Table 1: Primary Spur Gear Parameters

Power	: 20 to 40 Kw
Module	: 1 to 24 mm
Tooth Thickness	: 10 to 100 mm
Number of Teeth	: 12 to 60

3.6 Secondary Gear Parameters

In order to reduce the computational time and search space, secondary parameters are fixed with the standard values. The values are taken from the PSG design data book and are given in the Table 2.

Table 2: Secondary Spur gear parameters

Coefficient of friction	: 0.05
Thermal conductivity	: 48 W/ (m K)
Density	: 8.836×10^{-6} kg/mm^3
Specific heat	: 544 J/ (kg K)
Material of gear and pinion	: 40 Ni 2 Cr 1 Mo 28 (Cr-Mo series)
Gear ratio (i)	: 2
Young's modulus	: 2.15×10^5 N/mm^2
Pressure Angle (Φ)	: 20°
Tool dedendum coefficient	: 1.2
Backlash coefficient	: 0.048
Minimum topland coefficient	: 0.20
Minimum root clearance	: 0.15
Allowable bending stress	: 400N/mm^2
Allowable compressive stress	: 1100N/mm^2

Thus the objective function values are calculated based on the above equations and the parameter values.

IV. EXPERIMENTAL IMPLEMENTATION

Performance of firefly algorithm is tested on a number of spur gear design problems. In proposed firefly algorithm, sensitivity analysis had done to initialize the firefly parameters. The population size is set to 30 numbers of fireflies and all the fireflies can move randomly in the search space initially. Later 20 fireflies have to follow the firefly procedure to select its path and the remaining 10 fireflies can move at random order in the search space thus the stagnation and trapping of the fireflies at the variable local optimal peaks can be avoided [15]. It also helps in exploring the entire search space. The search space is formed with the power, tooth thickness, module and number of teeth. These parameters are normalized between 0 and 1 in the search space with the interval of 0.01. The condition for a firefly to terminate its fly inside the search space is that a firefly should move to all the parameters only once. β_0, γ and α are set to1, 1 and 0.05 respectively with the iteration size of 500. In every iteration, the best firefly should be saved in the local best database for updating the firefly's intensity for next iterations. Once all the iteration has been completed, the average of the local best database will be updated to generate final best solution space and again fireflies are allowed to search the best solution. By this methodology, away fireflies can shrinks to global best and locate the best optimal solution in a better place. The Figure 1 shows the convergence rate of the best iteration obtained during computational simulation. Form the figure, it is clear that the convergence rate of the firefly algorithm is faster and steady throughout the iterations.

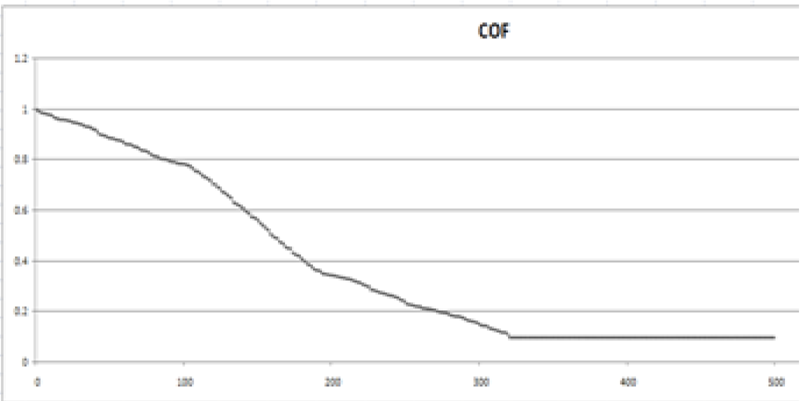

Figure 1: Convergence of the firefly algorithm

V. CONCLUSION

In this paper, firefly algorithm has been used to identify the better optimal spur gear parameters by considering the design and manufacturing constraints in a non-lubricated environment. Initial values of firefly parameters are decided by conducting sensitivity analysis and thereby prevent the trapping into local optimum. After some iteration, this parameter shrinks that causes focus on global optimum. Also by this approach, search space explored a large and in final iteration converged with best solutions. Simulation results show a better performance than standard design procedure. Thus the algorithm can be extended further to various applications.

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