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A Modified PSO Based Solution Approach for Economic Ordered Quantity Problem with Deteriorating Inventory, Time Dependent Demand Considering Order Size Limits, Stock Limits and Prohibited Ordering Segments

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Abstract:- *This paper presents formulation of Economic Ordered Quantity (EOQ) problem considering Order Size Limits, Stock Limits and Prohibited Ordering Segments, after that a modified PSO algorithm that utilizes the PSO with double chaotic maps is presented to solve this problem. In proposed approach, the logistic map and lozi map are applied alternatively to the velocity updating function of the particles. Using PSO with irregular velocity updates which is performed by these maps forces the particles to search greater space for best global solution. However the random function itself derived from a well-defined mathematical expression which limits its redundancy hence in the paper we are utilizing the two different chaotic maps which are used alternatively this mathematically increased the randomness of the function. The simulation of the algorithm for the formulated EOQ problem verifies the effectiveness and superiority of the algorithm over standard algorithms for such a complex problem which are difficult to solve by analytical approaches.*

Keywords:- Economic Ordered Quantity (EOQ) problem, PSO, Chaotic Maps, Logistic Map, Lozi Map.

I. INTRODUCTION

The Economic Ordered Quantity (EOQ) in inventory framework is the key part of inventory systems and considered as a significant important part of inventory systems. The EOQ issue is considered as optimization issue in which minimization of the aggregate inventory holding expenses and requesting expenses is situated as principle goal which ought to be found inside the equality and inequality constraints (operational compels) limitations. The operational requirements are alluded as maximum inventory level restrictions, change in every unit expense relying on request size, accumulating rate points of confinement, and deterioration losses are considered for reasonable operation. Additionally the base requested amount impacts might likewise be considered. These contemplations make the EOQ issue a vast scale very non-direct compelled streamlining issue. An alternate viewpoint other than expense which compels to utilize the EOQ is the new stockpiling approaches and regulations which governs the inventory managers to consider the environmental effects of the operation. Under these circumstances, requested inventory is not just governed by the unit's capacity of minimizing the total inventory holding costs and ordering cost, but also their capability of satisfying the governing policies requirements. In this paper the EOQ problem under the constrains for order size limits, stock limits and prohibited ordering segments is discussed and then after applied to the EOQ inventory mathematical model for deteriorating items with exponentially decreasing demand. Finally the objective function for the combined model is derived to use with PSO algorithm. The rest of the paper is arranged as second segment shows a concise audit of the related works, the third and fourth section talks about the issue definition and mathematical modeling, while fifth section clarifies the PSO and the variations utilized followed by and sixth sections which presents a brief review of chaotic maps, at last in section seventh and eight separately exhibits the simulated results and conclusion.

II. LITERATURE REVIEW

This section discusses some of the recent literatures related to the EOQ problem, inventory modeling and particle swarm optimization techniques. Liang Yuh Ouyang et al. [1] presented an EOQ inventory mathematical model for deteriorating items with exponentially decreasing demand. Their model also handles the shortages and variable rate partial backordering which dependents on the waiting time for the next replenishment. Kuo-Lung Hou et al. [10] presents an inventory model for deteriorating items considering the stock-dependent selling rate under inflation and time value of money over a finite planning horizon. The model allows shortages and partially backlogging at exponential rate. Lianxia Zhao [7] studied an inventory model with trapezoidal type demand rate and partially backlogging for Weibull-distributed deterioration items and derived an optimal inventory replenishment policy. Kai-Wayne Chuang et al. [2] studied pricing strategies in marketing, with objective to find the optimal inventory and pricing strategies for maximizing the net present value of total profit over the infinite horizon. The studied two variants of models: one without considering shortage, and the other with shortage. Jonas C.P. Yu [4] developed a deteriorating inventory system with only one supplier and one buyer. The system considers the collaboration and trade credit between supplier and buyer. The objective is to maximize the total profit of the whole system when shortage is completely backordered. The literature also discuss the negotiation mechanism between supplier and buyer in case of shortages and payment delay. Michal Pluhacek et al [15] compared the performance of two popular evolutionary computational techniques (particle swarm optimization and differential evolution) is compared in the task of batch reactor geometry optimization. Both algorithms are enhanced with chaotic pseudo-random number generator (CPRNG) based on Lozi chaotic map. The application of Chaos Embedded Particle Swarm Optimization for PID Parameter Tuning is presented in [16]. Magnus Erik et al [17] gives a list of good choices of parameters for various optimization scenarios which should help the practitioner achieve better results with little effort.

III. PROBLEM FORMULATION

The objective of an EOQ problem is to minimize the total inventory holding costs and ordering costs which should be found within the equality and inequality constraints (operational constrains) limitations. The simplified cost function of each inventory item can be represented as described in (2)

$$C_T = \sum_{i=1}^{n} c_i(S_i) \dots\dots\dots\dots\dots\dots\dots..(3.1)$$

$$c_i(S_i) = \alpha_i * S_i \dots\dots...(3.2)$$

where

$$C_T = Total\ Inventory\ Cost$$
$$c_i = Cost\ Function\ of\ Invertory\ i$$
$$\alpha_i = per\ unit\ cost\ of\ Invertory\ i$$
$$S_i = Ordered\ size\ of\ Invertory\ i$$

3.2. Equality and Inequality Constraints

3.2.1 Demand and Stock Balance Equation: For Demand and Stock balance, an equality constraint should be satisfied. The total stock should be equal or greater than the total demand plus the total Deterioration loss

$$\sum_{i=1}^{n} S_{i,demand} + S_{i,loss} \dots\dots\dots\dots..(3.14)$$

where $S_{i,demand}$ $S_{i,loss}$ *represents the total demand and Deterioration loss of* i^{th} *inventory is a function of the units ordered that can be represented using Demand* (D_i) *and Deterioration* (L_i) *coefficients* [2] *as follows*:

$$\sum_{i=1}^{n} S_i D_i + \sum_{i=1}^{n} S_i L_i \dots\dots\dots.(3.15)$$

3.3.1 Minimum and Maximum Order Size Limits: the order size of each inventory should be within its minimum and maximum orderable size limits. Corresponding inequality constraint for each inventory is

$$S_{i,min} \le S_i \le S_{i,max} \dots\dots\dots\dots..(9)$$

where $S_{i,min}$ *and* $S_{i,max}$ *are the minimum and maximum orderable size of* i^{th} *invertory, respectively.*

3.3.2 Stock Limits: The actual storing quantities of all the inventories are restricted by their corresponding stock size limits. The Stock Limits constraints can be written as follows:

$$S_{i,ordered} + S_{i,onstock}^{0} \le US_i\ and\ S_{i,ordered} + S_{i,onstock}^{0} \ge LS_i \dots\dots\dots\dots(10)$$

where S_i^0 *is the onsotck quantity of the* i^{th} *inventory*
US_i *and* LS_i *are the* max *and minstock limits of* i^{th} *inventory item, respectively.*

To consider the stock limits and Order limits constraints at the same time, (10) and (9) can be rewritten as an inequality constraint as follows:

$$\max\{S_{i,min}, S_{i,instock}^{0} + US_i\} \le S_{i,ordered} \le \min\{S_{i,max}, S_{i,instock}^{0} + LS_i\}.(11)$$

3.3.3 EOQ Problem Considering Prohibited Ordering Segments: In some cases, the entire ordering range of an inventory is not always available due to physical operation limitations. Items may have prohibited ordering segments due to nature of items themselves or associated auxiliaries. Such situation may lead to improper ordering in certain ranges of inventory [6]. Therefore, for items with prohibited ordering segments, there are additional constraints on the items ordering segments as follows:

$$P_i \in \begin{cases} S_{i,min} \leq S_i \leq S_{i,1}^{l} \\ S_{i,k-1}^{u} \leq S_i \leq S_{i,k}^{l} \quad , \quad k = 2,3, \ldots pz_i \\ S_{i,pz_i}^{u} \leq S_i \leq S_{i,max} \end{cases}$$

$$i = 1,2, \ldots n_{PZ} \ldots \ldots \ldots (12)$$

where $S_{i,k}^{l}$ *and* $S_{i,k}^{u}$ *are, respectively, the lower and upper bounds of prohibited ordering segments of inventory* i. *Here* pz_i, *is the number of prohibited zones of inventory* i *and* n_{PZ} *is the number of inventories which have prohibited ordering segments.*

IV. MATHEMATICAL MODELING

The mathematical model in this paper is rendered from reference [1] with following notation and assumptions. However the modification according to different models are performed and marked during the explanation.

Notation:

c_1 : Holding cost, (\$/per unit)/per unit time.
c_2 : Cost of the inventory item, \$/per unit.
c_3 : Ordering cost of inventory, \$/per order.
c_4 : Shortage cost, (\$/per unit)/per unit time.
c_5 : Opportunity cost due to lost sales, \$/per unit.
t_1 : Time at which shortages start.
T : Length of each ordering cycle.
W : The maximum inventory level for each ordering cycle.
S : The maximum amount of demand backlogged for each ordering cycle.
Q : The order quantity for each ordering cycle.
$Inv(t)$: The inventory level at time t.

Assumptions:

1. The inventory system involves only one item and the planning horizon is infinite.
2. The replenishment occurs instantaneously at an infinite rate.
3. The deteriorating rate, θ $(0 < \theta < 1)$, is constant and there is no replacement or repair of deteriorated units during the period under consideration.
4. The demand rate $R(t)$, is known and decreases exponentially.

$$R(t) = \begin{cases} Ae^{-\lambda t}, I(t) > 0 \\ D, I(t) \leq 0 \end{cases} \ldots \ldots \ldots \ldots \ldots \ldots .(4.1)$$

Where A (> 0) is initial demand and λ $(0 < \lambda < \theta)$ is a constant governing the decreasing rate of the demand.
5. During the shortage period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be. Hence, the proportion of customers who would like to accept backlogging at time t is decreasing with the waiting time $(T - t)$ waiting for the next replenishment. To take care of this situation we have defined the backlogging rate to be $\frac{1}{1+\delta(T-t)}$ when inventory is negative. The backlogging parameter δ is a positive constant $t_1 < t < T$.

4.1 MODEL FORMULATION

Here, the replenishment policy of a deteriorating item with partial backlogging is considered. The objective of the inventory problem is to determine the optimal order quantity and the length of ordering cycle so as to keep the total relevant cost as low as possible. The behavior of inventory system at any time is depicted in Figure 1.

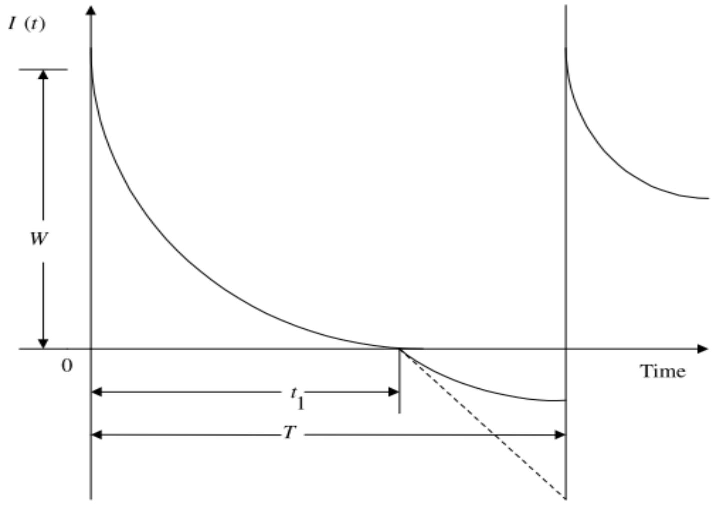

Figure 4: Inventory level $I(t)$ $vs.t$ $(time)$.

Replenishment is made at time $t = 0$ and the inventory level is at its maximum W. Due to both the market demand and deterioration of the item, the inventory level decreases during the period $[0, t_1]$, and ultimately falls to zero at $t = t_1$. Thereafter, shortages are allowed to occur during the time interval $[t_1, T]$ and all of the demand during the period $[t_1, T]$ is partially backlogged.

As described above, the inventory level decreases owing to demand rate as well as deterioration during inventory interval $[0, t_1]$. Hence, the differential equation representing the inventory status is given by

$$\frac{dInv(t)}{dt} + \theta Inv(t) = -Ae^{-\lambda t}, 0 \leq t \leq t_1 \dots\dots\dots.(4.2)$$

with the boundary condition $Inv(0) = W$. The solution of equation (1) is

$$Inv(t) = \left(\frac{A\mathrm{e}^{-t(\lambda-\theta)}}{\lambda-\theta} + W - \frac{A}{\lambda-\theta}\right)\mathrm{e}^{-\theta t} \dots\dots\dots (4.3)$$

Since the inventory falls to zero at time t_1, applying the condition to equation (2) gives

$$Inv(t_1) = \left(\frac{A\mathrm{e}^{-t_1(\lambda-\theta)}}{\lambda-\theta} + W - \frac{A}{\lambda-\theta}\right)\mathrm{e}^{-\theta t_1} = 0 \dots\dots (4.4)$$

From the above equation we can get the value of W (maximum inventory level)

$$W = -\frac{A\left(e^{-t_1(\lambda-\theta)} - 1\right)}{\lambda-\theta} \dots\dots\dots..(4.5)$$

Where W must satisfy $LB \leq W \leq UB$.

Now putting the value of equation (4.5) into equation (4.3)

$$Inv(t) = \left(\frac{Ae^{-t(\lambda-\theta)}}{\lambda-\theta} - \frac{A\left(e^{-t_1(\lambda-\theta)} - 1\right)}{\lambda-\theta} - \frac{A}{\lambda-\theta}\right)e^{-\theta t}..(4.6)$$

By simplifying the equation (4), the inventory level at time t can be given as

$$Inv(t) = -\frac{A\left(-e^{-t(\lambda-\theta)} + e^{-t_1(\lambda-\theta)}\right)e^{-\theta t}}{\lambda-\theta} \dots\dots..(4.7)$$

During the shortage interval $[t_1, T]$, the demand at time t is partly backlogged at the fraction $\frac{1}{1+\delta(T-t)}$ Thus, the differential equation governing the amount of demand backlogged is as below.

$$\frac{dInv(t)}{dt} = \frac{D}{1+\delta(T-t)}, t_1 < t \leq T \dots\dots\dots\dots\dots.(4.8)$$

with the boundary condition $I(t_1) = 0$. The solution of equation (6) can be given by

$$Inv(t) = \frac{D}{\delta}\{\ln[1+\delta(T-t)] - \ln[1+\delta(T-t_1)]\}, t_1 \leq t \leq T \dots\dots\dots.(4.9)$$

Let $t = T$ in (7), we obtain the maximum amount of demand backlogged per cycle as follows:

$$S = -Inv(T) = \frac{D}{\delta}\ln[1+\delta(T-t_1)] \dots\dots\dots\dots\dots (4.10)$$

Hence, the ordered quantity per cycle is given by

$$Q = W + S = \frac{A}{\theta - \lambda}\left[e^{(\theta-\lambda)t_1} - 1\right] + \frac{D}{\lambda}\ln[1 + \delta(T - t_1)] \;\ldots\ldots\ldots\ldots..(4.11)$$

Where Q must satisfy $S_{min} \leq Q \leq S_{max}$ *and* $Q \notin S_{\text{Prohibited}}$.

The inventory holding cost per cycle is

$$HC = \int_0^{t_1} c_1 Inv(t)dt = \frac{c_1 A}{\theta(\theta - \lambda)} e^{-\lambda t_1}\left[e^{\theta t_1} - 1 - \frac{\theta}{\lambda}\left(e^{\lambda t_1} - 1\right)\right] \ldots\ldots\ldots..(4.12)$$

The deterioration cost per cycle is

$$DC = c_2[W - \int_0^{t_1} R(t)dt]$$

$$= c_2[W - \int_0^{t_1} Ae^{-\lambda t}]$$

$$= c_2 A\left\{\frac{1}{\theta - \lambda}\left(e^{(\theta-\lambda)t_1} - 1\right) - \frac{1}{\lambda}\left(1 - e_1^{-\lambda t}\right)\right\} \ldots\ldots..(4.13)$$

The shortage cost per cycle is

$$SC = c_4\left[-\int_{t_1}^{T} I(t)dt\right] = c_4 D\left\{\frac{T - t_1}{\delta} - \frac{1}{\delta^2}\ln[1 + \delta(T - t_1)]\right\} \;\ldots.(4.14)$$

The opportunity cost due to lost sales per cycle is

$$BC = c_5 \int_{t_1}^{T}\left[1 - \frac{1}{1 + \delta(T - t)}\right] D\, dt = c_5 D\left\{(T - t_1) - \frac{1}{\delta}\ln[1 + \delta(T - t_1)]\right\} \ldots(4.15)$$

Therefore, the average total cost per unit time per cycle is

$$TVC \equiv TVC(t_1, T)$$

= (holding cost + deterioration cost + ordering cost + shortage cost + opportunity cost due to lost sales)/ length of ordering cycle

$$TVC = \frac{1}{T}\left\{\frac{c_1 A}{\theta(\theta - \lambda)} e^{-\lambda t_1}\left[e^{\theta t_1} - 1 - \frac{\theta}{\lambda}\left(e^{\lambda t_1} - 1\right)\right] + c_2 A\left[\frac{e^{(\theta-\lambda)t_1} - 1}{\theta - \lambda} - \frac{1 - e^{-\lambda t_1}}{\lambda}\right] + c_3 D\left(\frac{c_4}{\lambda} + c_5\right)\left[T - t_1 - \left(\frac{\ln[1 + \delta(T - t_1)]}{\delta}\right)\right]\right\} \ldots 4.16$$

Further simplification gives

$$TVC = \frac{1}{T}\left\{\frac{A(c_1 + \theta c_2)}{\theta(\theta - \lambda)}\left[e^{(\theta-\lambda)t_1} - (\theta - \lambda)t_1 - 1\right] - \frac{A(c_1 + \theta c_2)}{\theta\lambda}\left[1 - \lambda t_1 - e^{-\lambda t_1}\right] + c_3 + \frac{D(c_4 + \delta c_5)}{\delta}\left[T - t_1 - \frac{\ln[1 + \delta(T - t_1)]}{\delta}\right]\right\} \ldots\ldots.. \ldots (4.17)$$

Under the following constrains

$$S_{min} \leq \frac{A}{\theta - \lambda}\left[e^{(\theta-\lambda)t_1} - 1\right] + \frac{D}{\lambda}\ln[1 + \delta(T - t_1)] \leq S_{max} \;\ldots (4.18)$$

$$\frac{A}{\theta - \lambda}\left[e^{(\theta-\lambda)t_1} - 1\right] + \frac{D}{\lambda}\ln[1 + \delta(T - t_1)] \notin S_{\text{Prohibited}} \;\ldots (4.19)$$

$$LB \leq -\frac{A\left(e^{-t_1(\lambda-\theta)} - 1\right)}{\lambda - \theta} \leq UB \ldots\ldots\ldots\ldots.(4.20)$$

The objective of the model is to determine the optimal values of t_1 and T in order to minimize the average total cost per unit time (TVC) within the given constrains.

V. PARTICLE SWARM OPTIMIZATION (PSO)

The PSO algorithm is inspired by the natural swarm behavior of birds and fish. It was introduced by Eberhart and Kennedy in 1995 as an alternative to other ECTs, such as Ant Colony Optimization, Genetic Algorithms (GA) or Differential Evolution (DE). Each particle in the population represents a possible solution of the optimization problem, which is defined by its cost function. In each iteration, a new location (combination of cost function parameters) of the particle is calculated based on its previous location and velocity vector (velocity vector contains particle velocity for each dimension of the problem).The PSO algorithm works by simultaneously maintaining several candidate solutions in the search space. During each iteration of the algorithm, each candidate solution is evaluated by the objective function being optimized, determining the fitness of that solution. Each candidate solution can be thought of as a particle "flying" through the fitness landscape finding the maximum or minimum of the objective function. Initially, the PSO algorithm chooses candidate solutions randomly within the search space. It should be noted that the PSO algorithm has no knowledge of the underlying objective function, and thus has no way of knowing if any of the candidate solutions are near to or far away from a local or global maximum. The PSO algorithm simply uses the objective function to evaluate its candidate solutions, and operates upon the resultant fitness values.

Each particle maintains its position, composed of the candidate solution and its evaluated fitness, and its velocity. Additionally, it remembers the best fitness value it has achieved thus far during the operation of the algorithm, referred to as the individual best fitness, and the candidate solution that achieved this fitness, referred to as the individual best position or individual best candidate solution. Finally, the PSO algorithm maintains the best fitness value achieved among all particles in the swarm, called the global best fitness, and the candidate solution that achieved this fitness, called the global best position or global best candidate solution.

The PSO algorithm consists of just three steps, which are repeated until some stopping condition is met:
1. Evaluate the fitness of each particle
2. Update individual and global best fitness's and positions
3. Update velocity and position of each particle
4. Repeat the whole process till the

The first two steps are fairly trivial. Fitness evaluation is conducted by supplying the candidate solution to the objective function. Individual and global best fitness's and positions are updated by comparing the newly evaluated finesses against the previous individual and global best fitness's, and replacing the best fitness's and positions as necessary.
The velocity and position update step is responsible for the optimization ability of the PSO algorithm. The velocity of each particle in the swarm is updated using the following equation:

$$v(i+1) = w * v(i) + c_1 * \big(pBest - x(i)\big) + c_2 * \big(gBest - x(i)\big) \dots\dots\dots (5.1)$$

Modified PSO with chaos driven pseudorandom number perturbation

$$v(i+1) = w * v(i) + c_1 * Rand * \big(pBest - x(i)\big) + c_2 * Rand * \big(gBest - x(i)\big) \dots\dots\dots (5.2)$$

A chaos driven pseudorandom number perturbation ($Rand$) is used in the main PSO formula (Eq. (13)) that determines new ''velocity'' and thus the position of each particle in the next iterations (or migration cycle). The perturbation facilities the better search in the available search space hence provides much better results.
$Where$:
$v(i+1) - New\ velocity\ of\ a\ particle.$
$v(i) - Current\ velocity\ of\ a\ particle.$
$c_1, c_2 - Priority\ factors.$
$pBest - Best\ solution\ found\ by\ a\ particle.$
$gBest - Best\ solution\ found\ in\ a\ population.$
$Rand - Random\ number, interval\ (0,1). Chaos\ number\ generator\ is\ applied\ only\ here.$
$x(i) - Current\ position\ of\ a\ particle.$
The new position of a particle is then given by (5.3), where $x(i+1)$ is the new position:

$$x(i+1) = x(i) + v(i+1) \dots\dots\dots\dots\dots (5.3)$$

Inertia weight modification PSO strategy has two control parameters w_{start} and w_{end} . A new w for each iteration is given by (5.4), where i stand for current iteration number and n for the total number of iterations.

$$w = w_{start} - \frac{\big((w_{start} - w_{end}) * i\big)}{n} \dots\dots\dots\dots\dots (5.4)$$

Each of the three terms $(w * v(i), c_1 * Rand * \big(pBest - x(i)\big) and\ c_2 * Rand * \big(gBest - x(i)\big)$ of the velocity update equation have different roles in the PSO algorithm.

The first term w is the inertia component, responsible for keeping the particle moving in the same direction it was originally heading. The value of the inertial coefficient w is typically between 0.8 and 1.2, which can either dampen the particle's inertia or accelerate the particle in its original direction. Generally, lower values of the inertial coefficient speed up the convergence of the swarm to optima, and higher values of the inertial coefficient encourage exploration of the entire search space.

The second term $c_1 * Rand * \big(pBest - x(i)\big)$ called the cognitive component, acts as the particle's memory, causing it to tend to return to the regions of the search space in which it has experienced high individual fitness.
The cognitive coefficient c_1 is usually close to 2, and affects the size of the step the particle takes toward its individual best candidate solution $pBest$.

The third term $c_2 * Rand * (gBest - x(i))$, called the social component, causes the particle to move to the best region the swarm has found so far. The social coefficient c_2 is typically close to 2, and represents the size of the step thfe particle takes toward the global best candidate solution $gBest$ the swarm has found up until that point.

VI. CHAOTIC MAPS

This section contains the description of discrete chaotic maps used as the chaotic pseudorandom inventory for PSO. In this research, direct output iterations of the chaotic map were used for the generation of real numbers for the main PSO formula that determines new velocity, thus the position of each particle in the next iteration (See (2) in section 2). The procedure of embedding chaotic dynamics into evolutionary algorithms is given in [15][16] while the techniques for selecting proper parameter values in discussed in [17].

6.1 LOGISTIC MAP

The logistic map is a polynomial mapping (equivalently, recurrence relation) of degree 2, shows the complex, chaotic behavior from very simple non-linear dynamical equations. Mathematically, the logistic map is written

$$X_{n+1} = \mu X_n(1 - X_n) \dots \dots \dots \dots \dots (6.1)$$

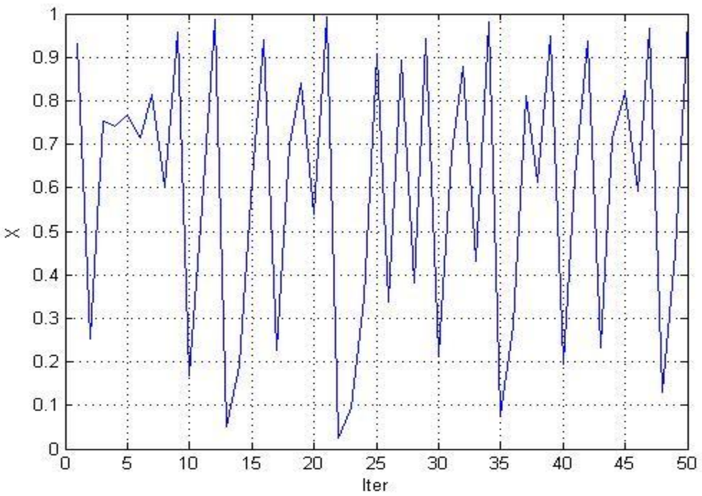

Figure 6:1: Plot of logistic map $\mu = 4$ and $X_0 = 0.63$ after 100 iterations.

6.2. LOZI MAP

The Lozi map is a simple discrete two-dimensional chaotic map. The map equations are given in (17).

$$X_{n+1} = 1 - x|X_n| + bY_n \dots \dots \dots \dots . (6.2a)$$
$$Y_{n+1} = X_n \dots \dots \dots \dots \dots \dots . (6.2b)$$

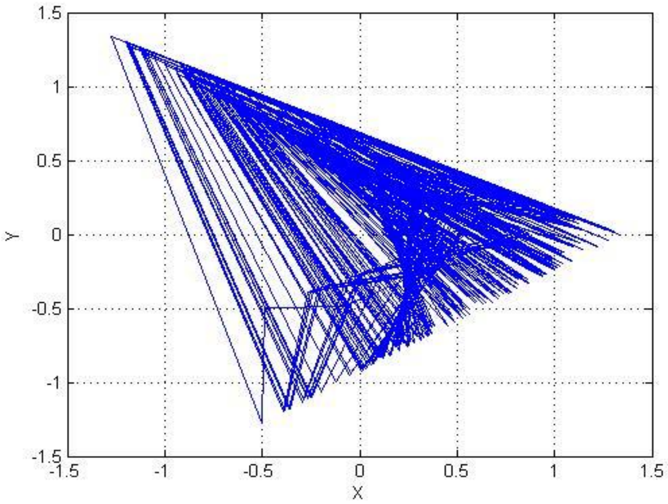

Figure 6:2: The 2D Plot of Lozi Map for $a = 1.7, b = 0.5$ after 1000 iterations

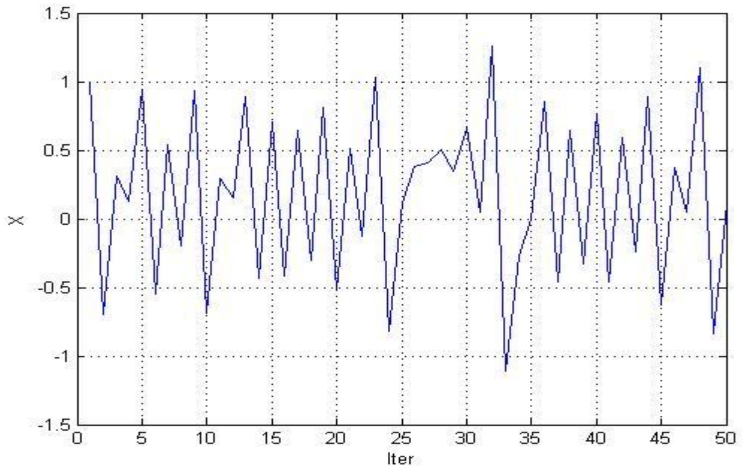

Figure 6:3: Plot of Lozi map for $a = 1.7, b = 0.5$ after 100 iterations

VII. IMPLEMENTATION OF IMPROVED PSO ALGORITHM FOR ECONOMIC ORDERED QUANTITY (EOQ) PROBLEMS

Since the decision variables in EOQ problems are t_1 and T with $S = \{S_1, S_2, \ldots., S_n\}$ where S_i ordering quantity of i^{th} inventory, the structure of a particle is composed of a set of elements corresponding to the $[t_1, T, S]$. Therefore, particle's position at iteration k can be represented as the vector $X_i^k = (P_{i1}^k, P_{i2}^k \ldots \ldots.., P_{im}^k)$ where $m = n + 2$ and n is the number of inventories. The velocity of particle i corresponds to the generation updates for all inventories. The process of the proposed PSO algorithm can be summarized as in the following steps.

1. Initialize the position and velocity of a population at random while satisfying the constraints.
2. Update the velocity of particles.
3. Modify the position of particles to satisfy the constraints, if necessary.
4. Generate the trial vector through operations presented in section 4.
5. Update and Go to Step 2 until the stopping criteria is satisfied.

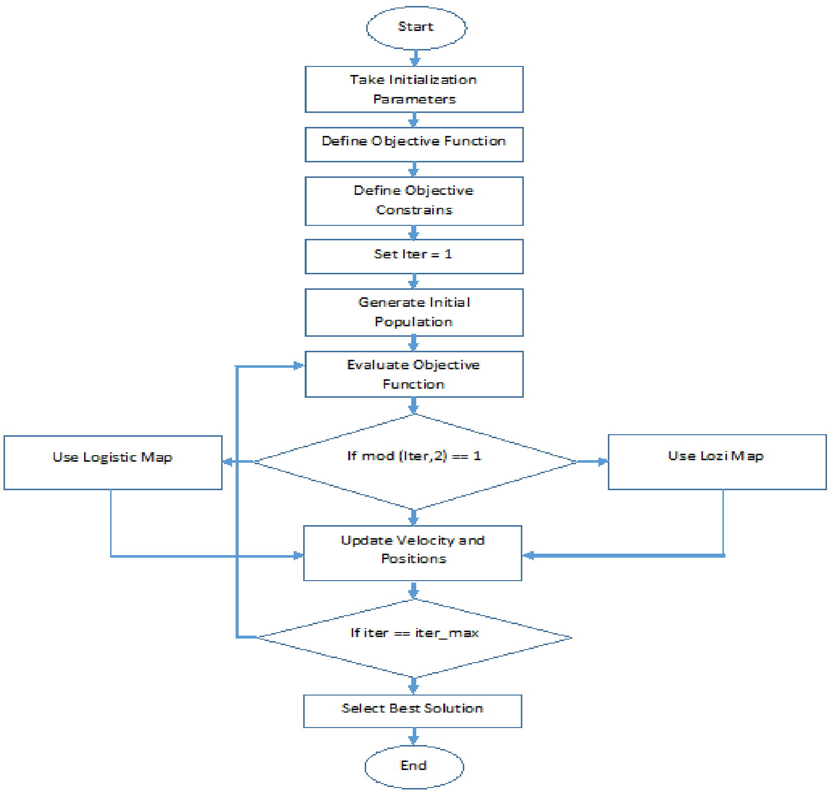

Figure 7: Flow Chart of the Proposed Algorithm.

VIII. SIMULATION RESULTS

The proposed IPSO approach is applied to three different inventory systems explained in section 3 and evaluated by all three PSO models as follows:
• The conventional PSO
• The PSO with chaotic sequences
• The PSO with alternative chaotic operation

The simulation of all algorithms is performed using MATLAB. The population size N_P and maximum iteration number $iter_{max}$ are set as 100 and 100, respectively. w_{max} and w_{min} are set to 0.9 and 0.1 respectively because these values are widely accepted and verified in solving various optimization problems. The list of all values used for the system are shown in the table below

Table 1: parameter values used for different PSO algorithms

Name of Variable	Value Assigned
c_1	2
c_2	1
w_{max}	0.9
w_{min}	0.1
μ (logistic map)	4.0
k (logistic map)	0.63
a (lozi map)	1.7
b (lozi map)	0.5
Total Particles	100
Maximum Iterations	100

Table 2: values of system variables:

	Variable Value	
Variable Name	Scenario 1	Scenario 2
A	12	12
θ	0.08	0.08
δ	2	2
λ	0.03	0.03
c_1	0.5	0.5
c_2	1.5	1.5
c_3	10	10
c_4	2.5	2.5
c_5	2	2
D	8	8
α	5	5
β	10	10
γ	0.04	N/A
S_{min}	1	1
S_{max}	100	95
N	N/A	15
S_{seg}	N/A	6
β_1	N/A	10

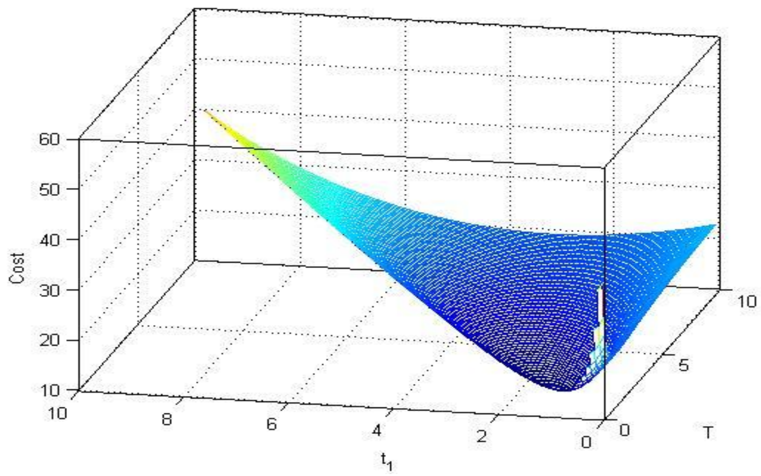

Figure 8.1: surface plot for the normal inventory system. With respect to t_1 (the time at which shortage starts) and T(ordering cycle time) the figure shows a smooth and continuous curve and hence can be solved by analytical technique also.

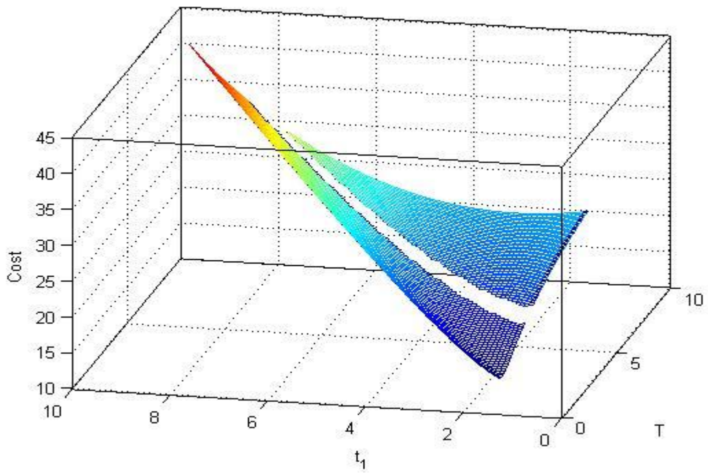

Figure 8.2: surface plot for the order segment dependent inventory cost type model. With respect to t_1 (the time at which shortage starts) and T(ordering cycle time) the figure shows much abrupt variations and many discontinuities in the curve and hence can be very difficult to solve by analytical techniques.

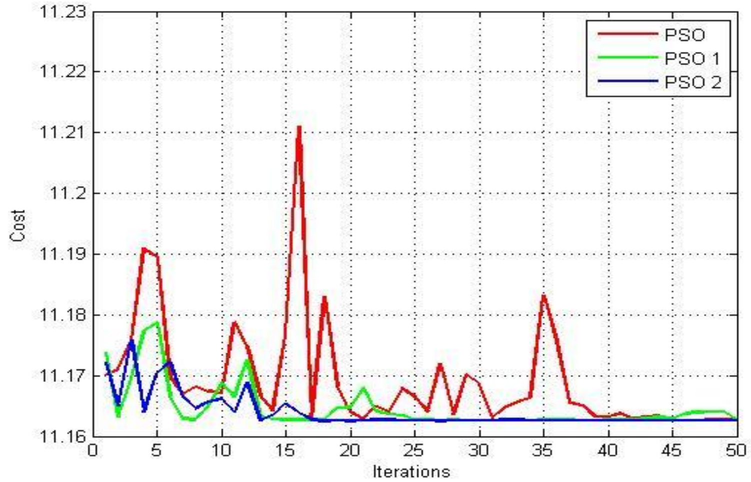

Figure 8.3: the value of objective function (fitness value or TVC) at every iteration of PSO for model 1.

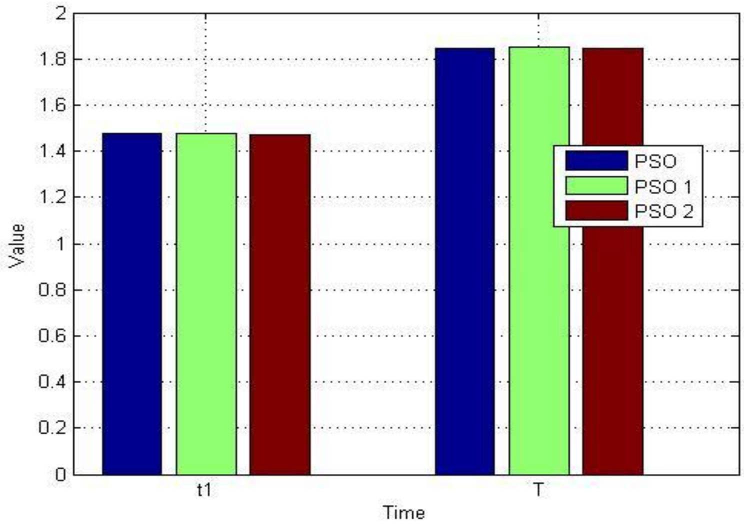

Figure 8.4: the best values of variables t_1 and T for all three PSO for model 1.

Table 3: Best Fitness Values by all three PSO for model 1.

Type of PSO	Best Fitness (TVC)
PSO	11.6625
PSO1	11.4125
PSO2	11.2736

IX. CONCLUSION AND FUTURE SCOPE

In this paper presents the mathematical model for inventories systems Considering Order Size Limits, Stock Limits and Prohibited Ordering Segments the paper also presents the derivations for evaluation of the function parameters for practical applications and finally it proposes an efficient approach for solving EOQ problem under the mentioned constrains applied simultaneously. Which may not be solved by analytical approach hence the meta-heuristic approach has been accepted in the form of standard PSO furthermore the performance of standard PSO is also enhanced by alternative use of two different chaotic maps for velocity updating finally it is applied to the EOQ problem for the inventory models discussed above and tested for different systems and objectives. The simulation result shows the proposed approach finds the solution very quickly with much lesser mathematical complexity. The simulation also verifies the superiority of proposed PSO over the standard PSO algorithm and supports the idea that switching between different chaotic pseudorandom number generators for updating the velocity of particles in the PSO algorithm improves its performance and the optimization process. The results for different experiments are collected with different settings and results compared with other methods which shows that the proposed algorithm improves the results by considerable margin.

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