

## **Imitational Simulating of the Dynamic Characteristics of a Fuel Tank During Longitudinal Oscillations**

**Denys Zosymovych**

*Shidna Street, 9, 66, Zhytomyr, Ukraine, 10012. Tel: +38-066-056-2660.*

**Abstract:** In the article has been chosen and simulated the dynamic characteristics of a fuel tank during longitudinal oscillations. In this study we shall proposed methods for determining the dynamic properties during longitudinal oscillations of fuel tanks for structurally similar models. The computation results enable to determine load in attachment points of fuel tank, and reduced masses of the liquid in the fuel tank for different natural oscillation frequencies.

The calculations show that in a steady state condition in the absence of longitudinal accelerations, and small amplitudes of liquid oscillations, when we exclude temperature changes (adiabatic thermal insulation), the decrement of free oscillations obtained as a result of computations remains unchanged and is equal to 1.

**Keywords:-** Rocket, space rocket block, fuel, pressure, pulsation, thrust, dynamics, property, module, mathematical simulation, service simulating test, stability, system, feed line, oscillation, frequency, amplitude, oscillator, damping, fuel tank, liquid, launch vehicle (LV), liquid-propellant engine (LPE), spacecraft.

### **I. INTRODUCTION**

In the process of development of sophisticated pieces of technology such as rockets, space rocket blocks and spacecraft, it is important to analyze dynamic processes which occur in them and in their partially fuelled tanks in order to establish dynamic properties of the product in general, as well as of its sections and assembly components.

If a launch vehicle or a spacecraft hasn't been made yet, it is possible to explore its operational reliability in actual use environment only with methods of mathematical simulation, service simulating tests or physical simulation.

Characterization of dynamic properties and provision of perturbed motion stability of an aircraft with due account to its body toughness and fluidity of liquid fuel is a challenging task, even if simplified, when linearization of equations is possible. Numerous solutions of the task in nonlinear formulation, which might be used to check the calculations, are of little use in rocket design. The best way to obtain its dynamic characteristics is simulation.

In the process of design of a launch vehicle system we have to solve a task of an optimal synthesis, i.e. we have to select a structure and parameters of the system, which would provide its optimal performance. This is described in details in [1-6]. Thus, in the process of design a solution of the task of synthesis will be split into several successive steps and present an iterative process with characteristic for each step cycles: theory, calculations, experiment, analysis [4].

Each powered portion of flight of the launch vehicle with a liquid-propellant engine may be characterized with unstable longitudinal oscillations, when dynamic load on the launch vehicle rapidly grows.

### **II. PROBLEM SETTING**

The most unfavorable are low frequency (up to 50 Hz) spring-type pitch motions of the launch vehicle which are accompanied by longitudinal oscillations of liquid fuel in tanks and feed lines. Pressure pulsations in the tanks and feed lines cause changes in consumption of fuel components, pressure ripple inside the combustion chamber and pressure disturbance inside the liquid-propellant engine. This disturbance will be conveyed in its turn to the body of the launch vehicle, and form a closed oscillation system, in which initial slight disturbance may turn into rapidly growing in amplitude oscillations in the system and in all its elements [1]. This is what makes us formulate the task as an engineering one, and try to solve the problem of fuel tank dynamics using structurally similar models studied during longitudinal oscillations.

**Objectives:** to calculate the frequency of natural longitudinal oscillations of the liquid and the reduced mass of the liquid for a spherical aluminum rocket fuel tank mounted in the orbital stage of a launch vehicle.

**Tasks:** 1. To calculate frequency of natural longitudinal oscillations of liquid in a spherical fuel tank.

2. To calculate reduced mass of liquid in a fuel tank for different natural oscillation frequencies.

**Research target:** imitation model of a spherical fuel tank mounted in the orbital stage of a launch vehicle.

**Subject of research:** dynamic properties and continuous flow control units of the guidance system of the spherical fuel tank mounted in the orbital stage of the launch vehicle.

An experimental study of dynamic processes in the process of design and construction of rockets is traditionally performed using models of the aircrafts or their modules and units. The most accurate dynamic properties may be obtained in experiments with application of structurally similar models, i.e. full-scale models which replicate corresponding objects not only in respect of their geometrical parameters, but also with their mass and stiffness characteristics [1]. Structurally similar models allow us to determine dynamic properties of objects fully and accurately, and are used, for example, to study the behavior of the launch vehicle under the influence of loads practically occurring during its flight, to determine characteristics of its stress-strain behavior, and oscillation mode and frequencies of the body generally, or of its components.

Creation of structurally similar models is a labor-intensive process because a model must accurately reproduce the design of a real object. Therefore, in some cases, when the experiments do not require any simulation of deformation and stress in the spacecraft, it is permitted to use geometrically similar models. They provide no similarity either in properties and thickness of materials, or in stiffness of the spacecraft, but only similarity in geometric parameters: external (or internal) contours, shape and arrangement of structural elements and units. Geometrically similar models shall be typically used to study motion parameters of liquid components of fuel in tanks of the launch vehicle [1].

If while selecting parameters for models of real objects, designing test benches, and developing experimental techniques and modes we meet specific conditions, the results of experiments with the models may be transferred to full-scale objects with a high degree of reliability [1].

### **Oscillation generators.**

In the present research we will use both a mechanical vibrator which converts rotations into back and forward motions via a crank mechanism, and an electrodynamics' one. The frequency of electric current shall be measured with a sound generator.

### **Data acquisition equipment.**

When performing tests with the models, it is necessary to specify the following characteristics: frequency, amplitude, oscillation mode. As a rule, analyzed oscillation frequencies of a structurally similar model range between 0 and 400 Hz. Oscillation amplitudes are less than 1 mm for components, and 300 mm for liquids [2].

To compensate a temperature error the circuit of the instrumentation bridge shall include besides the main sensor an identical compensating sensor, which shall be attached to a specimen made of the same material as the unit under study. Both will be provided with same temperature conditions. A level sensor is to be included into the bridge circuit by analogy with resistance sensor, and in order to create a balance, the other bridge arm shall include a capacitor.

**Methods for determining the dynamic properties during longitudinal oscillations of fuel tanks for structurally similar models.** The body of the launch vehicle, feed lines, liquid-propellant engines make a closed oscillation circuit, where disturbance of engine thrust may result in oscillations of body, and of fuel in tanks and in lines, etc., and ultimately in oscillatory thrust. Initial thrust oscillations may increase. Elastic shell of the tank filled with liquid will make a separate oscillatory system. Frequencies and modes of longitudinal oscillations are general parameters of rocket body oscillations. It is imperative that such characteristics of longitudinal oscillations as dynamic reactions in attachment points of fuel tanks and pressure pulsations on entering service line be calculated [1].

Pressure pulsations, when increasing in some cases will be passed to the engine and cause thrust oscillations.

### **Reduced mass calculation.**

The liquid, which oscillates in the liquid tank, may be presented as an oscillator having a mass  $m$ , suspended to a support on a spring with certain stiffness  $k$ .

Oscillator equation shall be presented as follows [1]:

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = P, \quad (1)$$

where  $x$  is motion of mass  $m$  of the oscillator;  $b$  is damping ratio of oscillations;  $P$  is a force imposed on the oscillating support.

$u$  is movement of tank attachment points, thus [1]:

If

$$P = m \frac{d^2 u}{dt^2}. \quad (2)$$

Let's assume that oscillator attachment point harmonically oscillates [1]:

$$u = U e^{i\omega t}. \quad (3)$$

Then natural oscillations of the oscillator will be harmonic as well:

$$u = X e^{i\omega t}, \quad (4)$$

where  $\omega$  is a frequency of support oscillations.

If we substitute (1) with (2) – (4), we will receive:

$$mX(-\omega^2 + \frac{b}{m}\omega i + \omega_0^2) = mU\omega^2. \quad (5)$$

In this equation  $\omega_0$  - is frequency of oscillator's natural oscillations, which shall be determined according to the formula  $\omega_0^2 = k/m$ .

If we assume that the frequency of forced oscillations  $\omega$  is equal to the frequency of natural oscillations  $\omega_0$ , using (5) we will receive

$$iX \frac{b}{m} \omega_0 = U \omega_0^2. \quad (6)$$

$$\text{Where } X = i \frac{U \omega_0 m}{b}. \quad (7)$$

On the other hand, supporting force  $Q^*$  will be determined using [1]:

$$Q^* = m \frac{d^2 x}{dt^2}. \quad (8)$$

Let's write down the formula (8) using amplitude values, then

$$Q = -m\omega_0^2 X, \quad (9)$$

where  $Q$

is support response amplitude.

From (9) we will receive

$$X = -\frac{Q}{m\omega_0^2}. \quad (10)$$

If we compare the equation (7) and (10) we will receive next expression:

$$m^2 = \frac{Qb}{\omega_0^3 U}. \quad (11)$$

The damping ratio  $b/m$  maintains a reference to the logarithmic decrement of oscillations  $d$  through the equation [3]:

$$b = m \frac{d}{\pi} \omega_0. \quad (12)$$

From equations (11) and (12) we will receive a formula for calculation of the mass

$$m = \frac{1}{\pi} \frac{Qd}{\omega_0^2 U}. \quad (13)$$

Mass  $m$

is a part of mass of the liquid inside the tank, which oscillates as well with natural frequency  $\omega_0$ . This mass is called reduced.

Thus, a reduced mass  $m$  may be determined, if we know values of the logarithmic decrement of oscillations  $d$ ,

natural frequency  $\omega_0$ , an amplitude in the agitation point  $U$  and dynamic reaction in the tank supports  $Q$ .

### **Calculating the frequencies of natural oscillations.**

Free oscillations of the tank with liquid in the absence of dissipative forces are called natural oscillations. These oscillations have an infinite spectrum of frequencies, which are called frequencies of natural oscillations. The most interesting frequencies within the range are first few ones (mostly 1st, 2nd and 3<sup>rd</sup> natural frequencies).

These frequencies can be both calculated and defined experimentally [2].

Natural frequencies of longitudinal vibrations of a full spherical tank can be calculated using the following empirical formula [1]:

$$\omega_{0j} = \frac{1}{2\pi} \sqrt{\frac{\lambda_j E \delta}{\rho R^3}}, \quad (14)$$

where  $\lambda_j$  – are characteristic values: for first natural frequency  $\lambda_1 = 1,35$ ; for second natural frequency  $\lambda_2 = 3,33$ ; for third natural frequency  $\lambda_3 = 5,04$ ;

$E$  – is elasticity module of the tank material;  $\delta$  – is thickness of the tank walls;  $\rho$  – is liquid density;  $R$  – is tank radius.

According to calculations and experimental data [6], the natural longitudinal oscillations of the liquid in the tank, filled to 60...100 per cent of its volume, is very little dependent from the level of the liquid in the tank.

Therefore, we may use the formula (14) for arbitrary tank charge levels in order to determine initial (designed) natural frequencies.

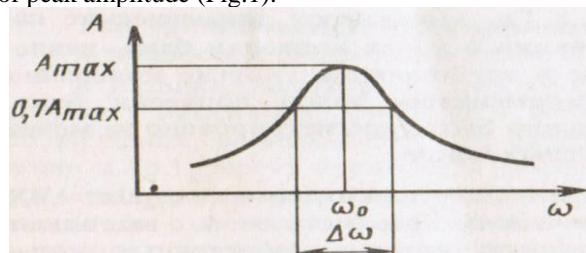
The method of experimental determination of natural frequencies is described below.

#### **Determination of decrement of natural oscillations of liquid in a tank.**

The decrement of free oscillations is determined by experimental calculations [1]. For this purpose, an amplitude-frequency characteristic (AFC) of elastic vibrations of the shell is used. It is to be determined experimentally. The decrement of free oscillations is to be calculated using the formula [2]:

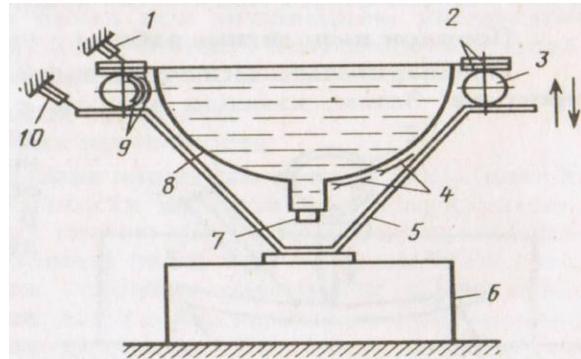
$$d = \frac{\Delta\omega\pi}{\omega_0}, \quad (15)$$

where  $\omega_0$  – is natural frequency at which the frequency decrement is determined;  $\Delta\omega$  – is width of resonance peak at its height, which makes 0,7 of peak amplitude (Fig.1).



**Fig. 1. Determination of the width of resonance peak [1]**

**Unit description.** In the present study, experiments are conducted with a hemispherical shell 8 (Fig. 2) made of aluminum-based material AM-16B (a model version). A section of pipe duct is affixed to the pole of the shell, and is provided with a pressure pulsation sensor 7 performing the function of a cap. In order to measure elastic vibrations of the walls, several tension sensors 4 are attached to the shell along the meridian.



**Fig. 2. Experimental hemispherical tank**

The hemispherical shell is provided with a bead along its equator, which is clamped with two hard steel rings 2 with shell components attached. Generally, there are 6 attachment points.

Axisymmetric shell vibrations shall be excited by a vibro-bench through frame 5 and attachment points of the shell, which have a certain rigidity and enable to measure the force conveyed by the vibro-bench to the shell. They are made in the form of rings 3 with tension sensors 9 attached on both sides (external and internal).

To measure displacement of the base of the vibro-bench to which the tested shell is attached, we will use probes 1, 10 made of elastic plates with tension sensors attached on their both sides. One end of the plate shall be firmly fixed to the stationary base and the other one shall rest upon the tested model, or upon the table of the vibro-bench, if displacement is to be measured.

### III. RESULTS AND DISCUSSION

Computation of reduced masses, natural oscillations and the decrement of free oscillations of the liquid were done in MS Excel. Service simulating test was performed in MATLAB/ Simulink.

Let's assume as known the following design characteristics accepted in accordance with findings [4-5]:

$$r = 0,86 \text{ m}; \quad p = 1,25 \cdot 10^5 \text{ Pa}; \quad m = 42 \text{ kg}; \quad k = 0,068 \text{ GPa} \text{ (for Al)}; \quad E = 70 \text{ GPa} \text{ (for Al)}; \quad \delta = 2 \cdot 10^{-3} \text{ m}; \quad \rho_p = 1,429 \frac{\text{kg}}{\text{m}^3}$$

(for  $O_{2_p}$ ).

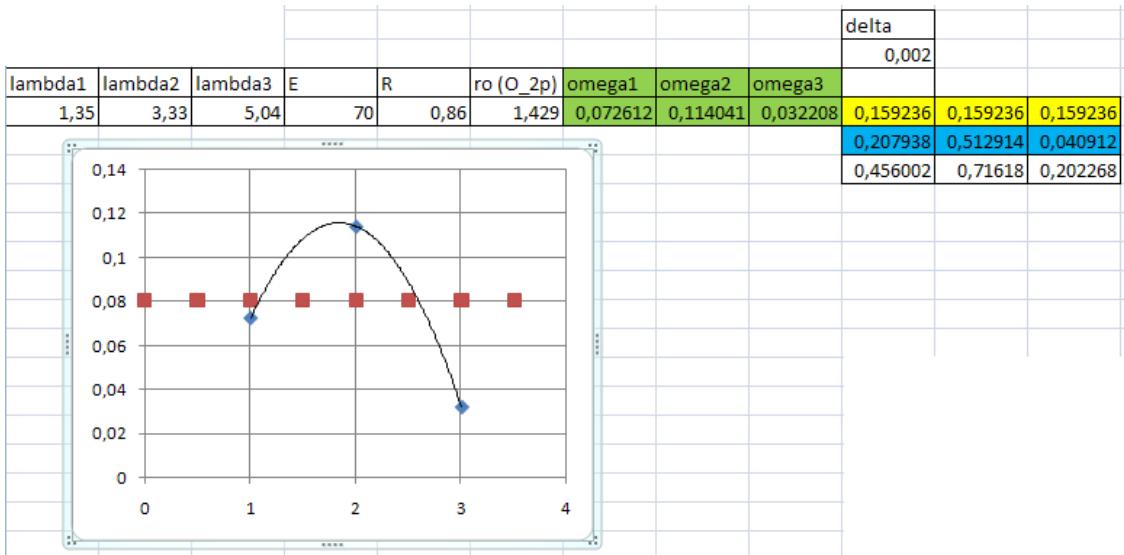
In the case with damped free oscillations for different values  $A_{\tan k}$  we shall obtain logarithmic decrements:

$$A_{\tan k} = 1,2 \cdot 10^{-2}; 1,4 \cdot 10^{-2} \dots 10^{-1}; 1,2 \cdot 10^{-1} \dots 1,8 \cdot 10^{-1}.$$

$$d = \ln \frac{x_{1_{\max}}}{x_{2_{\max}}} = \frac{\pi r A_0 p^2}{g a_0}; \quad g a_0 = 9,81 \frac{\text{M}}{\text{c}^2};$$

0,068	42	omega_0^2
		0,00161905
		omega_0
		0,04023739

For proper values of frequencies [6]:  $\lambda_1 = 1,35; \lambda_2 = 3,33; \lambda_3 = 5,04$  we will calculate natural frequencies of longitudinal oscillations of the spherical tank according to the formula (14).



The frequencies of natural oscillations of the tank with liquid ( $H_2O_{2\text{liquid}}$ ) will be as follows:

1,1	2,6	1,5
0,228001	0,35809	0,101134

The decrements of free oscillations for different natural frequencies according to formula (15) will be as follows:

delta omega 1	delta omega 2	delta omega 3
0,072611844	0,114041442	0,032208298
omega_01	omega_02	omega_03
0,228001191	0,358090127	0,101134055
d1	d2	d3
1	1	1

Thus, the calculations show that in a steady state condition in the absence of longitudinal accelerations, and small amplitudes of liquid oscillations, when we exclude temperature changes (adiabatic thermal insulation), the decrement of free oscillations obtained as a result of computations remains unchanged and is equal to 1.

#### IV. CONCLUSIONS

- Was proposed methods for determining the dynamic properties during longitudinal oscillations of fuel tanks for structurally similar models.
- The computation results enable to determine load in attachment points of fuel tank, and reduced masses of the liquid in the fuel tank for different natural oscillation frequencies.
- Were calculated frequencies of natural longitudinal oscillations, and the reduced mass of  $(H_2O_2)_{\text{liquid}}$  - based liquid for a spherical rocket fuel tank to be installed on the orbital stage of a launch vehicle.

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