International Refereed Journal of Engineering and Science (IRJES)
ISSN (Online) 2319-183X, (Print) 2319-1821
Volume 3, Issue 8 (August 2014), PP.06-17

Inventory Model for Variable Deteriorating Items with Two Warehouses under Shortages, Time Varying Holding Cost, Inflation and Permissible Delay In Payments

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Abstract: A deteriorating items inventory model with two warehouses under time varying holding cost and linear demand under inflation and permissible delay in payments is developed. Shortages are allowed and completely backlogged. A rented warehouse (RW) is used to store the excess units over the capacity of the own warehouse. Numerical examples are provided to illustrate the model and sensitivity analysis is also carried out for parameters.

Keywords: Deterioration, Inflation, Inventory, Permissible delay in payment, Shortages, Two-warehouse

I. INTRODUCTION

Inventory models for deteriorating items were widely studied in past. Ghare and Schrader [1] first developed an EOQ model with constant rate of deterioration. Covert and Philip [2] extended this model by considering variable rate of deterioration. Shah [3] further extended the model by considering shortages. The related work are found in (Nahmias [4], Raffat [5], Goyal and Giri [6], Wu et al. [7], Ouyang et al. [8]).

Goyal [9] first considered the economic order quantity model under the condition of permissible delay in payments. Aggarwal and Jaggi [10] extended Goyal's [9] model to consider the deteriorating items. Aggarwal and Jaggi's [10] model was further extended by Jamal et al. [11] to consider shortages. Teng et al. [12] developed an optimal pricing and lot sizing model by considering price sensitive demand under permissible delay in payments. A literature review on inventory model under trade credit is given by Chang et al. [13]. Min et al. [14] developed an inventory model for exponentially deteriorating items under conditions of permissible delay in payments.

The existing literature on classical inventory model generally deal with single storage facility with the assumption that the available warehouse of the organization has unlimited capacity. But in actual practice many times the supplier provide price discounts for bulk purchases and the retailer may purchase more goods than can be stored in single warehouse (own warehouse). Therefore a rented warehouse (RW) is used to store the excess units over the fixed capacity W of the own warehouse. The rented warehouse is charged higher unit holding cost then the own warehouse, but offers a better preserving facility with a lower rate of deterioration.

Hartley [15] first developed a two-warehouse inventory model. An inventory model with infinite rate of replenishment with two-warehouse was considered by Sarma [16]. Pakkala and Achary [17] extended the two-warehouse inventory model for deteriorating items with finite rate of replenishment and shortages. Related work is also find in (Benkherouf [18], Bhunia and Maiti [19], Kar et al. [20], Chung and Huang [21], Rong et al. [22]).

Ghosh and Chakrabarty [23] developed an order level inventory model with two levels of storage for deteriorating items when demand is time dependent and shortages were allowed and completely backlogged. Madhavilata et al. [24] have developed a deterministic inventory model for a single item having two levels of storage. Demand was assumed to be exponentially increasing function of time. Liang and Zhou [25] considered a two warehouse inventory models for deteriorating items under conditionally permissible delay in payments. Tyagi and Singh [26] considered a two warehouse inventory model with time dependent demand, varying rate of deterioration and variable holding cost. Yang [27] considered a two warehouse inventory problem for deteriorating items with constant rate of demand under inflation in two alternatives when shortages are completely backordered. Yadav and Swami [28] studied the effect of permissible delay on two warehouse inventory model for deteriorating items with shortages. Bhunia et al. [29] deals with a deterministic inventory model for linear trend in demand under inflationary conditions with different rates of deterioration in two warehouses.

In this paper we have developed a two-warehouse inventory model under time varying holding cost and linear demand under inflation and permissible delay in payments. Shortages are allowed and completely backlogged. Numerical examples are provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.

II. ASSUMPTIONS AND NOTATIONS

NOTATIONS:

The following notations are used for the development of the model:

$D(t)$: Demand rate is a linear function of time t ($a+bt$, $a>0$, $0<b<1$)

A : Replenishment cost per order for two warehouse system

c : Purchasing cost per unit

p : Selling price per unit

c_2 : Shortage cost per unit

HC(OW): Holding cost per unit time is a linear function of time t (x_1+y_1t, $x_1>0$, $0<y_1<1$) in OW

HC(RW): Holding cost per unit time is a linear function of time t (x_2+y_2t, $x_2>0$, $0<y_2<1$) in RW

I_e : Interest earned per year

I_p : Interest charged per year

M : Permissible period of delay in settling the accounts with the supplier

T : Length of inventory cycle

$I(t)$: Inventory level at any instant of time t, $0 \le t \le T$

W : Capacity of owned warehouse

$I_0(t)$: Inventory level in OW at time t

$I_r(t)$: Inventory level in RW at time t

Q_1 : Inventory level initially

Q_2 : Shortage of inventory

Q : Order quantity

R : Inflation rate

t_r : Time at which the inventory level reaches zero in RW in two warehouse system

$\theta_1 t$: Deterioration rate in OW, $0<\theta_1<1$

$\theta_2 t$: Deterioration rate in RW, $0<\theta_2<1$

TC_i : Total relevant cost per unit time (i=1,2,3)

ASSUMPTIONS:

The following assumptions are considered for the development of two warehouse model.

- The demand of the product is declining as a linear function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and completely backlogged.
- OW has a fixed capacity W units and the RW has unlimited capacity.
- The goods of OW are consumed only after consuming the goods kept in RW.
- The unit inventory costs per unit in the RW are higher than those in the OW.
- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

III. THE MATHEMATICAL MODEL AND ANALYSIS

At time t=0, a lot size of certain units enter the system. W units are kept in OW and the rest is stored in RW. The items of OW are consumed only after consuming the goods kept in RW. In the interval $[0,t_r]$, the inventory in RW gradually decreases due to demand and deterioration and it reaches to zero at $t=t_r$. In OW, however, the inventory W decreases during the interval $[0,t_r]$ due to deterioration only, but during $[t_r, t_0]$, the inventory is depleted due to both demand and deterioration. By the time to t_0, both warehouses are empty. Shortages occur during (t_0,T) of size Q_2 units. The figure describes the behaviour of inventory system.

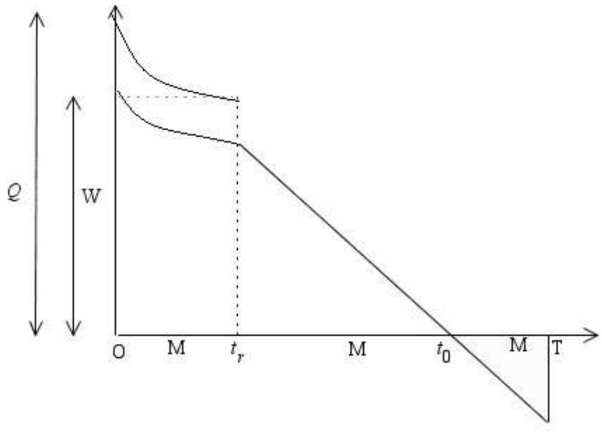


Figure 1

Hence, the inventory level at time t at RW and OW are governed by the following differential equations:

$$\frac{dI_r(t)}{dt} + \theta_2 t I_r(t) = -(a+bt), \qquad 0 \le t \le t_r \qquad (1)$$

with boundary conditions $I_r(t_r) = 0$ and

$$\frac{dI_0(t)}{dt} + \theta_1 t I_0(t) = 0, \qquad 0 \le t \le t_r \qquad (2)$$

with initial condition $I_0(0) = W$, respectively.

While during the interval (t_r, t_0), the inventory in OW reduces to zero due to the combined effect of demand and deterioration both. So the inventory level at time t at OW, $I_0(t)$, is governed by the following differential equation:

$$\frac{dI_0(t)}{dt} + \theta_1 t I_0(t) = -(a+bt), \qquad t_r \le t \le t_0 \qquad (3)$$

with the boundary condition $I_0(t_0)=0$.

Similarly during (t_0, T) the shortage level at time t, $I_s(t)$ is governed by the following differential equation:

$$\frac{dI_s(t)}{dt} = -(a+bt), \qquad t_0 \le t \le T, \qquad (4)$$

with the boundary condition $I_s(t_0)=0$.

The solutions to equations (1) to (4) are given by:

$$I_r(t) = \left[\begin{array}{l} a(t_r - t) + \frac{1}{2}b(t_r^2 - t^2) + \frac{1}{6}a\theta_2(t_r^3 - t^3) \\ + \frac{1}{8}b\theta_2(t_r^4 - t^4) - \frac{1}{2}a\theta_2 t^2(t_r - t) - \frac{1}{4}b\theta_2 t^2(t_r^2 - t^2) \end{array}\right] \qquad 0 \le t \le t_r \qquad (5)$$

$$I_o(t) = W(1 - \theta_1 t^2), \qquad 0 \le t \le t_r \qquad (6)$$

$$I_o(t) = \left[\begin{array}{l} a(t_0 - t) + \frac{1}{2}b(t_0^2 - t^2) + \frac{1}{6}a\theta_1(t_0^3 - t^3) \\ + \frac{1}{8}b\theta_1(t_0^4 - t^4) - \frac{1}{2}a\theta_1 t^2(t_0 - t) - \frac{1}{4}b\theta_1 t^2(t_0^2 - t^2) \end{array}\right] \qquad t_r \le t \le t_0 \qquad (7)$$

$$I_s(t) = \left[a(t_0 - t) + \frac{1}{2}b(t_0^2 - t^2)\right] \qquad t_0 \le t \le T \qquad (8)$$

(by neglecting higher powers of θ_1, θ_2)

Using the condition $I_r(t) = Q_1 - W$ at t=0 in equation (5), we have

$$Q_1 - W = \left[at_r + \frac{1}{2}bt_r^2 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{8}b\theta_2 t_r^4\right],$$

$$\therefore\ Q_1 = W + \left[at_r + \frac{1}{2}bt_r^2 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{8}b\theta_2 t_r^4\right]. \qquad (9)$$

Using the condition $I_s(t) = Q - Q_1$ at t=T in equation (8), we have

$$Q - Q_1 = -\left[a(T - t_0) + \frac{1}{2}b(T^2 - t_0^2)\right]$$

$$\therefore\ Q = Q_1 - \left[a(T - t_0) + \frac{1}{2}b(T^2 - t_0^2)\right]. \qquad (10)$$

Using the continuity of $I_0(t)$ at t=tr in equations (6) and (7), we have

$$I_o(t_r) = W\left(1 - \theta_1 t^2\right) = \left[\begin{matrix} a\left(t_0 - t_r\right) + \frac{1}{2}b\left(t_0^2 - t_r^2\right) + \frac{1}{6}a\theta_1\left(t_0^3 - t_r^3\right) \\ + \frac{1}{8}b\theta_1\left(t_0^4 - t_r^4\right) - \frac{1}{2}a\theta_1 t_r^2\left(t_0 - t\right) - \frac{1}{4}b\theta_1 t_r^2\left(t_0^2 - t_r^2\right) \end{matrix}\right] \qquad (11)$$

which implies that

$$t_0 = \frac{-a + \sqrt{a^2 + 2bW - bW\theta_1 t_r^2 + b^2 t_r^2 + 2abt_r}}{b} \qquad (12)$$

(by neglecting higher powers of tr and t_0)

From equation (12), we note that t_0 is a function of t_r, therefore t_0 is not a decision variable.

Based on the assumptions and descriptions of the model, the total annual relevant costs TC_i, include the following elements:

(i) Ordering cost (OC) = A (13)

$$\text{(ii)}\ \ HC(RW) = \int_0^{t_r}(x_2+y_2t)I_r(t)e^{-Rt}dt = \int_0^{t_r}(x_2+y_2t)\left[\begin{matrix} a\left(t_r - t\right) + \frac{1}{2}b\left(t_r^2 - t^2\right) + \frac{1}{6}a\theta_2\left(t_r^3 - t^3\right) \\ + \frac{1}{8}b\theta_2\left(t_r^4 - t^4\right) - \frac{1}{2}a\theta_2 t^2\left(t_r - t\right) - \frac{1}{4}b\theta_2 t^2\left(t_r^2 - t^2\right) \end{matrix}\right]e^{-Rt}dt$$

$$= -\frac{1}{56}y_2R\theta_2bt_r^7 + \frac{1}{6}\left(\frac{1}{8}\left(y_2 - x_2R\right)\theta_2b - \frac{1}{3}y_2R\theta_2a\right)t_r^6 + \frac{1}{5}\left(\frac{1}{8}x_2\theta_2b + \frac{1}{3}\left(y_2 - x_2R\right)\theta_2a - y_2R\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right)\right)t_r^5$$
$$+ \frac{1}{4}\left(\frac{1}{3}x_2\theta_2a + \left(y_2 - x_2R\right)\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right) + y_2Ra\right)t_r^4$$
$$+ \frac{1}{3}\left(x_2\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right) - \left(y_2 - x_2R\right)a - y_2R\left(\frac{1}{8}b\theta_2t_r^4 + \frac{1}{6}a\theta_2t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)\right)t_r^3$$
$$+ \frac{1}{2}\left(-x_2a + \left(y_2 - x_2R\right)\left(\frac{1}{8}b\theta_2t_r^4 + \frac{1}{6}a\theta_2t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)\right)t_r^2 + x_2\left(\frac{1}{8}b\theta_2t_r^4 + \frac{1}{6}a\theta_2t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)t_r \qquad (14)$$

(by neglecting higher powers of R)

$$\text{(iii)}\ \ HC(OW) = \int_0^{t_0}(x_1+y_1t)I_0(t)e^{-Rt}dt = \int_0^{t_r}(x_1+y_1t)I_0(t)e^{-Rt}dt + \int_{t_r}^{t_0}(x_1+y_1t)I_0(t)e^{-Rt}dt$$

$$= \int_0^{t_r} (x_1+y_1t)W\left(1-\theta_1 t^2\right)e^{-Rt}dt + \int_{t_r}^{t_0}(x_1+y_1t)\begin{bmatrix} a\left(t_0-t\right) + \frac{1}{2}b\left(t_0^2-t^2\right) + \frac{1}{6}a\theta_1\left(t_0^3-t^3\right) \\ + \frac{1}{8}b\theta_1\left(t_0^4-t^4\right) - \frac{1}{2}a\theta_1 t^2\left(t_0-t\right) - \frac{1}{4}b\theta_1 t^2\left(t_0^2-t^2\right)\end{bmatrix}e^{-Rt}dt$$

$$= W\left(\frac{1}{10}y_1Rb\theta_1 t_r^5 - \frac{1}{8}\left(y_1-x_1R\right)\theta_1 t_r^4 + \frac{1}{3}\left(-\frac{1}{2}x_1\theta_1 - y_1R\right)t_0^3 + \frac{1}{2}\left(y_1-x_1R\right)t_0^2\ x_1t_r\right)$$

$$+\begin{pmatrix} -\frac{1}{56}y_1Rb\theta_1 t_0^7 + \frac{1}{6}\left(\frac{1}{8}\left(y_1-x_1R\right)b\theta_1 - \frac{1}{3}y_1Ra\theta_1\right)t_0^6 \\ + \frac{1}{5}\left(\frac{1}{8}x_1b\theta_1 + \frac{1}{3}\left(y_1-x_1R\right)a\theta_1 - y_1R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2+at_0\right)-\frac{1}{2}b\right)\right)t_0^5 \\ + \frac{1}{4}\left(\frac{1}{3}x_1a\theta_1 + \left(y_1-x_1R\right)\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2+at_0\right)-\frac{1}{2}b\right)+y_1Ra\right)t_0^4 \\ + \frac{1}{3}\left(x_1\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2+at_0\right)-\frac{1}{2}b\right) - \left(y_1-x_1R\right)a + y_1R\left(\frac{1}{8}b\theta_1 t_0^4+\frac{1}{6}a\theta_1 t_0^3+\frac{1}{2}bt_0^2+at_0\right)\right)t_0^3 \\ + \frac{1}{2}\left(-x_1a - \left(y_1-x_1R\right)\left(\frac{1}{8}b\theta_1 t_0^4+\frac{1}{6}a\theta_1 t_0^3+\frac{1}{2}bt_0^2+at_0\right)\right)t_0^2 + x_1\left(\frac{1}{8}b\theta_1 t_0^4+\frac{1}{6}a\theta_1 t_0^3+\frac{1}{2}bt_0^2+at_0\right)t_0 \end{pmatrix}$$

$$+\begin{pmatrix} \frac{1}{56}y_1Rb\theta_1 t_r^7 - \frac{1}{6}\left(\frac{1}{8}\left(y_1-x_1R\right)b\theta_1 - \frac{1}{3}y_1Ra\theta_1\right)t_r^6 \\ - \frac{1}{5}\left(\frac{1}{8}x_1b\theta_1 + \frac{1}{3}\left(y_1-x_1R\right)a\theta_1 - y_1R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2+at_0\right)-\frac{1}{2}b\right)\right)t_r^5 \\ - \frac{1}{4}\left(\frac{1}{3}x_1a\theta_1 + \left(y_1-x_1R\right)\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2+at_0\right)-\frac{1}{2}b\right)+y_1Ra\right)t_r^4 \end{pmatrix}$$

$$+\begin{pmatrix} -\frac{1}{3}\left(x_1\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2+at_0\right)-\frac{1}{2}b\right) - \left(y_1-x_1R\right)a - y_1R\left(\frac{1}{8}b\theta_1 t_0^4+\frac{1}{6}a\theta_1 t_0^3+\frac{1}{2}bt_0^2+at_0\right)\right)t_r^3 \\ -\frac{1}{2}\left(-x_1a + \left(y_1-x_1R\right)\left(\frac{1}{8}b\theta_1 t_0^4+\frac{1}{6}a\theta_1 t_0^3+\frac{1}{2}bt_0^2+at_0\right)\right)t_r^2 - x_1\left(\frac{1}{8}b\theta_1 t_0^4+\frac{1}{6}a\theta_1 t_0^3+\frac{1}{2}bt_0^2+at_0\right)t_r \end{pmatrix} \quad (15)$$

(iv) Shortage cost:

$$SC = -c_2\int_{t_0}^{T} I(t)e^{-Rt}dt = -c_2\int_{t_0}^{T}\left[a(t_0-t)+\frac{1}{2}b(t_0^2-t^2)\right]e^{-Rt}dt$$

$$= -c_2\left[\frac{1}{8}bRT^4 + \frac{1}{3}(aR-\frac{1}{2}b)T^3 + \frac{1}{2}(-(at_0+\frac{1}{2}bt_0^2)R-a)T^2 + at_0T + \frac{1}{2}bt_0^2T\right]$$
$$+ c_2\left[\frac{1}{8}bRt_0^4 + \frac{1}{3}(aR-\frac{1}{2}b)t_0^3 + \frac{1}{2}(-(at_0+\frac{1}{2}bt_0^2)R-a)t_0^2 + at_0^2 + \frac{1}{2}bt_0^3\right] \quad (16)$$

(v) Deterioration cost:

The amount of deterioration in both RW and OW during $[0,t_0]$ are:

$$\int_0^{t_r}\theta_2 tI_r(t)dt \text{ and } \int_0^{t_0}\theta_1 tI_0(t)dt$$

So deterioration cost

$$DC = c\left[\int_0^{t_r}\theta_2 tI_r(t)e^{-Rt}dt + \int_0^{t_0}\theta_1 tI_0(t)e^{-Rt}dt\right]$$

$$= c\left[\int_0^{t_r}\theta_2 tI_r(t)e^{-Rt}dt + \int_0^{t_r}\theta_1 tI_0(t)e^{-Rt}dt + \int_{t_r}^{t_0}\theta_1 tI_0(t)e^{-Rt}dt\right]$$

$$= c\theta_2\left[\begin{array}{l} -\frac{1}{56}R\theta_2 bt_r^7 + \frac{1}{6}\left(\frac{1}{8}b\theta_2 - \frac{1}{3}Ra\theta_2\right)t_r^6 + \frac{1}{5}\left(\frac{1}{3}a\theta_2 - R\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right)\right)t_r^5 \\ + \frac{1}{4}\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b + Ra\right)t_r^4 + \frac{1}{3}\left(-a - R\left(\frac{1}{8}b\theta_2 t_r^4 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)\right)t_r^3 \\ + \frac{1}{2}\left(\frac{1}{8}b\theta_2 t_r^4 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)t_r^2 + c\theta_1 W\left[\frac{1}{10}R\theta_1 t_r^5 - \frac{1}{8}\theta_1 t_r^4 - \frac{1}{3}Rt_r^3 + \frac{1}{2}t_r^2\right] \end{array}\right]$$

$$+ c\theta_1\left[\begin{array}{l} -\frac{1}{56}R\theta_1 bt_0^7 + \frac{1}{6}\left(\frac{1}{8}b\theta_1 - \frac{1}{3}Ra\theta_1\right)t_0^6 + \frac{1}{5}\left(\frac{1}{3}a\theta_1 - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b\right)\right)t_0^5 \\ + \frac{1}{4}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b + Ra\right)t_0^4 + \frac{1}{3}\left(-a - R\left(\frac{1}{8}b\theta_1 t_0^4 + \frac{1}{6}a\theta_1 t_0^3 + \frac{1}{2}bt_0^2 + at_0\right)\right)t_0^3 \\ + \frac{1}{2}\left(\frac{1}{8}b\theta_1 t_0^4 + \frac{1}{6}a\theta_1 t_0^3 + \frac{1}{2}bt_0^2 + at_0\right)t_0^2 \end{array}\right]$$

$$- c\theta_1\left[\begin{array}{l} -\frac{1}{56}R\theta_1 bt_r^7 + \frac{1}{6}\left(\frac{1}{8}b\theta_1 - \frac{1}{3}Ra\theta_1\right)t_r^6 + \frac{1}{5}\left(\frac{1}{3}a\theta_1 - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b\right)\right)t_r^5 \\ + \frac{1}{4}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b + Ra\right)t_r^4 + \frac{1}{3}\left(-a - R\left(\frac{1}{8}b\theta_1 t_0^4 + \frac{1}{6}a\theta_1 t_0^3 + \frac{1}{2}bt_0^2 + at_0\right)\right)t_r^3 \\ + \frac{1}{2}\left(\frac{1}{8}b\theta_1 t_0^4 + \frac{1}{6}a\theta_1 t_0^3 + \frac{1}{2}bt_0^2 + at_0\right)t_r^2 \end{array}\right] \quad (17)$$

(vi) Interest Earned: There are two cases:

Case I : ($M \le t_r \le T$):

In this case interest earned is:

$$IE_1 = pI_e\int_0^M (a + bt)te^{-Rt}dt = pI_e\left[-\frac{1}{4}bRM^4 + \frac{1}{3}(-Ra + b)M^3 + \frac{1}{2}aM^2\right] \quad (18)$$

Case II : ($t_r \le M \le T$):

In this case interest earned is:

$$IE_2 = pI_e\left(\int_0^{t_0}(a+bt)te^{-Rt}dt + (a + bt_0)t_0(M - t_0)\right)$$

$$= pI_e\left[-\frac{1}{4}bRt_0^2 + \frac{1}{3}(-Ra + b)t_0^3 + \frac{1}{2}at_0^2 + (a+bt_0)t_0(M-t_0)\right] \quad (19)$$

(vii) Interest Payable: There are three cases described as in figure:

Case I : ($M \le t_r \le T$):

In this case, annual interest payable is:

$$IP_1 = cI_p\left[\int_M^{t_r} I_r(t)e^{-Rt}dt + \int_M^{t_r} I_0(t)e^{-Rt}dt + \int_{t_r}^{t_0} I_0(t)e^{-Rt}dt\right]$$

$$= cI_p \left[\begin{array}{l} - \frac{1}{48} R\theta_2 bt_r^6 + \frac{1}{5}\left(\frac{1}{8}\theta_2 b - \frac{1}{3}R\theta_2 a\right)t_r^5 + \frac{1}{4}\left(\frac{1}{3}\theta_2 a - R\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right)\right)t_r^4 \\ + \frac{1}{3}\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b + Ra\right)t_r^3 + \frac{1}{2}\left(-a - R\left(\frac{1}{8}b\theta_2 t_r^4 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)\right)t_r^2 \\ + \frac{1}{8}\theta_2 bt_r^5 + \frac{1}{6}a\theta_2 t_r^4 + \frac{1}{2}bt_r^3 + at_r^2 \end{array} \right]$$

$$- cI_p \left[\begin{array}{l} - \frac{1}{48} R\theta_2 bM^6 + \frac{1}{5}\left(\frac{1}{8}\theta_2 b - \frac{1}{3}R\theta_2 a\right)M^5 + \frac{1}{4}\left(\frac{1}{3}\theta_2 a - R\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right)\right)M^4 \\ + \frac{1}{3}\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b + Ra\right)M^3 + \frac{1}{2}\left(-a - R\left(\frac{1}{8}b\theta_2 t_r^4 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)\right)M^2 \\ + \frac{1}{8}\theta_2 bt_r^4 M + \frac{1}{6}a\theta_2 t_r^3 M + \frac{1}{2}bt_r^2 M + at_r M \end{array} \right]$$

$$+ cI_p W\left[t_r + \frac{1}{8}R\theta_1 t_r^4 - \frac{1}{6}\theta_1 t_r^3 - \frac{1}{2}Rt_r^2\right] - cI_p W\left[M + \frac{1}{8}R\theta_1 M^4 - \frac{1}{6}\theta_1 M^3 - \frac{1}{2}RM^2\right]$$

$$+ cI_p \left[\begin{array}{l} - \frac{1}{48} R\theta_1 bt_0^6 + \frac{1}{5}\left(\frac{1}{8}\theta_1 b - \frac{1}{3}R\theta_1 a\right)t_0^5 + \frac{1}{4}\left(\frac{1}{3}\theta_1 a - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b\right)\right)t_0^4 \\ + \frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b + Ra\right)t_0^3 + \frac{1}{2}\left(-a - R\left(\frac{1}{8}b\theta_1 t_0^4 + \frac{1}{6}a\theta_1 t_0^3 + \frac{1}{2}bt_0^2 + at_0\right)\right)t_0^2 \\ + \frac{1}{8}\theta_1 bt_0^5 + \frac{1}{6}a\theta_1 t_0^4 + \frac{1}{2}bt_0^3 + at_0^2 \end{array} \right]$$

$$- cI_p \left[\begin{array}{l} - \frac{1}{48} R\theta_1 bt_r^6 + \frac{1}{5}\left(\frac{1}{8}\theta_1 b - \frac{1}{3}R\theta_1 a\right)t_r^5 + \frac{1}{4}\left(\frac{1}{3}\theta_1 a - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b\right)\right)t_r^4 \\ + \frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b + Ra\right)t_r^3 + \frac{1}{2}\left(-a - R\left(\frac{1}{8}b\theta_1 t_0^4 + \frac{1}{6}a\theta_1 t_0^3 + \frac{1}{2}bt_0^2 + at_0\right)\right)t_r^2 \\ + \frac{1}{8}\theta_1 bt_0^4 t_r + \frac{1}{6}a\theta_1 t_0^3 t_r + \frac{1}{2}bt_0^2 t_r + at_0 t_r \end{array} \right] \quad (20)$$

Case II : ($t_r \le M \le T$):

In this case interest payable is:

$$IP_2 = cI_p \int_M^{t_0} I_0(t) e^{-Rt} dt$$

$$= cI_p \left[\begin{array}{l} - \frac{1}{48} R\theta_1 bt_0^6 + \frac{1}{5}\left(\frac{1}{8}\theta_1 b - \frac{1}{3}R\theta_1 a\right)t_0^5 + \frac{1}{4}\left(\frac{1}{3}\theta_1 a - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b\right)\right)t_0^4 \\ + \frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b + Ra\right)t_0^3 + \frac{1}{2}\left(-a - R\left(\frac{1}{8}b\theta_1 t_0^4 + \frac{1}{6}a\theta_1 t_0^3 + \frac{1}{2}bt_0^2 + aT\right)\right)t_0^2 \\ + \frac{1}{8}\theta_1 bt_0^5 + \frac{1}{6}a\theta_1 t_0^4 + \frac{1}{2}bt_0^3 + at_0^2 \end{array} \right]$$

$$- cI_p \left[\begin{array}{l} - \frac{1}{48}R\theta_1 bM^6 + \frac{1}{5}\left(\frac{1}{8}\theta_1 b - \frac{1}{3}R\theta_1 a\right)M^5 + \frac{1}{4}\left(\frac{1}{3}\theta_1 a - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b\right)\right)M^4 \\ + \frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b + Ra\right)M^3 + \frac{1}{2}\left(-a - R\left(\frac{1}{8}b\theta_1 t_0^4 + \frac{1}{6}a\theta_1 t_0^3 + \frac{1}{2}bt_0^2 + at_0\right)\right)M^2 \\ + \frac{1}{8}\theta_1 bt_0^4 M + \frac{1}{6}a\theta_1 t_0^3 M + \frac{1}{2}bt_0^2 M + at_0 M \end{array} \right] \quad (21)$$

Case III : ($t_0 \le M \le T$):

In this case, no interest charges are paid for the item. So,

$IP_3 = 0.$ (22)

The retailer's total cost during a cycle, $TC_i(t_r,T)$, i=1,2,3 consisted of the following:

$$TC_i = \frac{1}{T}\left[A + HC(OW) + HC(RW) + SC + DC + IP_i - IE_i\right] \quad (23)$$

and t_0 is approximately related to t_r through equation (12).

Substituting values from equations (13) to (17) and equations (18) to (22) in equation (23), total costs for the three cases will be as under:

$$TC_1 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + SC + DC + IP_1 - IE_1\right] \quad (24)$$

$$TC_2 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + SC + DC + IP_2 - IE_2\right] \quad (25)$$

$$TC_3 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + SC + DC + IP_3 - IE_2\right] \quad (26)$$

The optimal value of tr = tr*, T=T* (say), which minimizes TCi can be obtained by solving equation (24), (25) and (26) by differentiating it with respect to t_r and T and equate it to zero i.e.

$$\text{i.e. } \frac{\partial TC_i(t_r,T)}{\partial t_r} = 0,\ \frac{\partial TC_i(t_r,T)}{\partial T} = 0,\ i=1,2,3, \quad (27)$$

provided it satisfies the condition

$$\frac{\partial^2 C_i(t_r,T)}{\partial^2 t_r} > 0,\ \frac{\partial^2 C_i(t_r,T)}{\partial^2 T} > 0 \text{ and } \left[\frac{\partial^2 C_i(t_r,T)}{\partial^2 t_r}\right]\left[\frac{\partial^2 C_i(t_r,T)}{\partial^2 T}\right] - \left[\frac{\partial^2 C_i(t_r,T)}{\partial t_r \partial T}\right]^2 > 0,\ i=1,2,3. \quad (28)$$

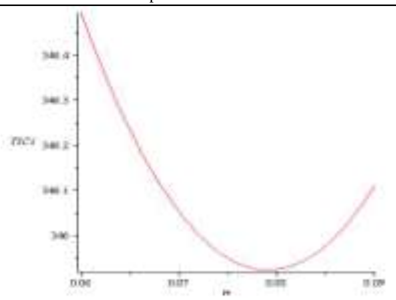
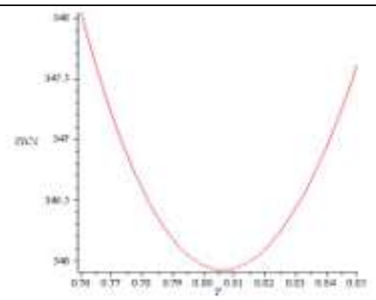
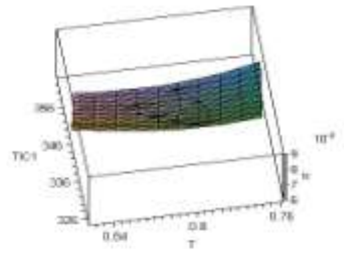
IV. NUMERICAL EXAMPLES

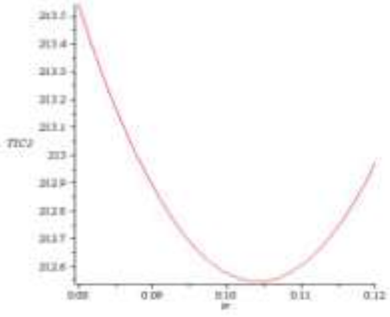
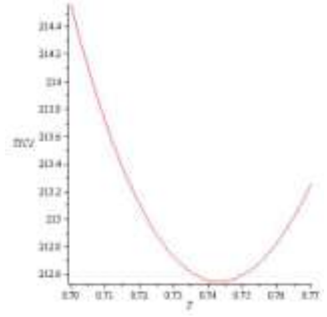
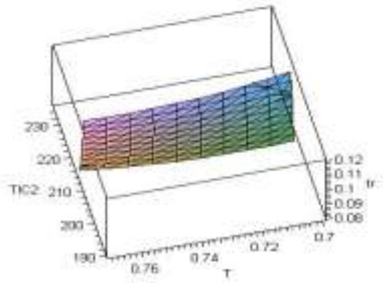
Case I: Considering A= Rs.150, W = 100, a = 200, b=0.05, c=Rs. 10, p= Rs. 15, θ_1=0.1, θ_2 =0.06, x_1 = Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, Ip= Rs. 0.15, Ie= Rs. 0.12, R = 0.06, c_2 = Rs. 8, M=0.01 year, in appropriate units. The optimal value of t_r^*=0.0791, T*=0.8062 and TC_1^*= Rs. 345.9229.

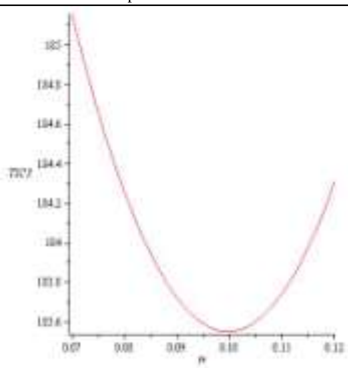
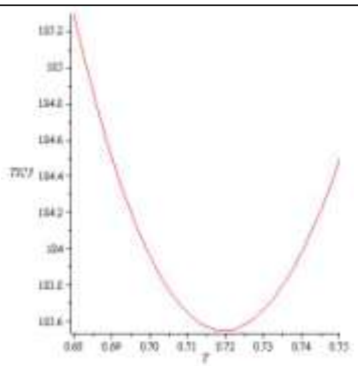
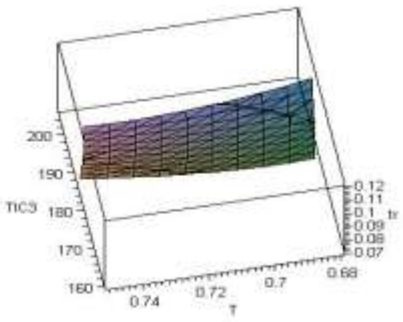
Case II: Considering A= Rs.150, W = 100, a = 200, b=0.05, c = Rs. 10, p= Rs. 15, θ_1=0.1, θ_2 =0.06, x_1= Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, Ip= Rs. 0.15, Ie = Rs. 0.12, R= 0.06, c_2 = Rs. 8, M=0.55 year, in appropriate units. The optimal value of t_r^*=0.1041, T*=0.7431 and TC_2^* = Rs. 212.5456.

Case III: Considering A= Rs.150, W = 100, a = 200, b=0.05, c = Rs. 10, p= Rs. 15, θ_1=0.1, θ_2 =0.06, x_1= Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, Ip= Rs. 0.15, Ie= Rs. 0.12, R = 0.06, c_2 = Rs. 8, M = 0.65 year, in appropriate units. The optimal value of t_r^*=0.0996, T*=0.7195 and TC_1^* = Rs. 183.5503.

The second order conditions given in equation (28) are also satisfied. The graphical representation of the convexity of the cost functions for the three cases are also given.

Case I		
t_r and cost	T and cost	t_r, T and cost
 Graph 1	 Graph 2	 Graph 3

Case II		
t_r and cost	T and cost	t_r, T and cost
 Graph 4	 Graph 5	 Graph 6

Case III		
t_r and cost	T and cost	t_r, T and cost
 Graph 7	 Graph 8	 Graph 9

V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

		Case I			Case II			Case III		
		$(M \le t_r \le T)$			$(t_r \le M \le T)$			$(t_0 \le M \le T)$		
Para-meter	%	t_r	T	Cost	t_r	T	Cost	t_r	T	Cost
a	+10%	0.0909	0.7618	363.4295	0.1154	0.6979	215.9379	0.1133	0.6765	183.5556
	+5%	0.0854	0.7831	354.7541	0.1101	0.7197	214.3379	0.1069	0.6971	183.5526
	-5%	0.0718	0.8314	336.9279	0.0972	0.7687	210.5526	0.0914	0.7439	183.3606
	-10%	0.0635	0.8590	327.7607	0.0892	0.7967	208.3502	0.0819	0.7706	182.9752

x_1	+10%	0.0732	0.8029	349.9459	0.0990	0.7411	217.2443	0.0951	0.7181	188.3437
	+5%	0.0762	0.8046	347.9413	0.1016	0.7421	214.9018	0.0974	0.7188	185.9536
	-5%	0.0819	0.8078	343.8908	0.1068	0.7442	210.1758	0.1019	0.7202	181.1337
	-10%	0.0848	0.8094	341.8451	0.1092	0.7451	207.7925	0.1042	0.7209	178.7040
x_2	+10%	0.0747	0.8020	346.1431	0.0986	0.7378	212.9609	0.0948	0.7149	183.9448
	+5%	0.0768	0.8041	346.0359	0.1013	0.7404	212.7585	0.0972	0.7172	183.7521
	-5%	0.0814	0.8085	345.8034	0.1071	0.7460	212.3212	0.1023	0.7220	183.5556
	-10%	0.0838	0.8109	345.6772	0.1103	0.7491	212.0846	0.1050	0.7246	183.1163
θ_1	+10%	0.0766	0.8042	346.7553	0.1016	0.7413	213.5367	0.0974	0.7179	184.5525
	+5%	0.0778	0.8052	346.3401	0.1029	0.7422	213.0422	0.0985	0.7187	184.0523
	-5%	0.0803	0.8072	345.5038	0.1054	0.7441	212.0468	0.1008	0.7203	183.0463
	-10%	0.0816	0.8082	345.0827	0.1067	0.7451	211.5459	0.1019	0.7212	182.5403
θ_2	+10%	0.0790	0.8062	345.9241	0.1041	0.7431	212.5487	0.0996	0.7195	183.5531
	+5%	0.0790	0.8062	345.9235	0.1041	0.7431	212.5471	0.0996	0.7195	183.5517
	-5%	0.0791	0.8062	345.9223	0.1042	0.7432	212.5441	0.0997	0.7196	183.5489
	-10%	0.0791	0.8063	345.9210	0.1042	0.7432	212.5425	0.0997	0.7196	183.5475
R	+10%	0.0805	0.8086	345.5622	0.1046	0.7443	212.5380	0.0994	0.7199	183.6207
	+5%	0.0793	0.8069	345.7424	0.1045	0.7437	212.5419	0.0995	0.7197	183.5856
	-5%	0.0788	0.8055	346.1028	0.1039	0.7426	212.5489	0.0998	0.7194	183.5142
	-10%	0.0785	0.8048	346.2822	0.1037	0.7420	212.5520	0.0999	0.7192	183.4789
A	+10%	0.0978	0.8374	364.1758	0.1242	0.7764	232.2881	0.1185	0.7519	203.9379
	+5%	0.0885	0.8219	355.1358	0.1143	0.7599	222.5248	0.1092	0.7359	193.8564
	-5%	0.0694	0.7902	336.5269	0.0938	0.7260	202.3356	0.0899	0.7028	173.0040
	-10%	0.0596	0.7739	326.9365	0.0832	0.7084	191.8781	0.0799	0.6856	162.2001
M	+10%	0.0791	0.8062	345.7067	0.1044	0.7332	197.0115	0.1029	0.7098	163.8605
	+5%	0.0791	0.8062	345.8148	0.1044	0.7383	204.8283	0.1013	0.7148	173.7520
	-5%	0.0790	0.8062	346.0309	0.1038	0.7479	220.1654	0.0978	0.7241	193.3603
	-10%	0.0789	0.8062	346.1389	0.1034	0.7524	227.6897	0.0960	0.7286	202.8740

From the table we observe that as parameter a increases/ decreases average total cost increases/ decreases in case I and case II, whereas there very slight increase/ decrease in average total cost due to increase/ decrease in parameter a in case III.

From the table we observe that with increase/ decrease in parameters A, x_1 and θ_1, there is corresponding increase/ decrease in total cost for case I, case II and case III respectively.

From the table we observe that with increase/ decrease in parameter x_2, there is corresponding increase/ decrease in total cost for case I and there is very slight increase/ decrease in total cost for case II and case III respectively.

Also, we observe that with increase and decrease in the value of θ_2, there is corresponding very slight increase/ decrease in total cost for case I, case II and case III.

Also, we observe that with increase and decrease in the value of R, there is corresponding very slight decrease/ increase in total cost for case I and case II, and there is very slight increase/ decrease in total cost for case III.

Also, we observe that with increase and decrease in the value of M, there is corresponding very slight decrease/ increase in total cost for case I, and there is decrease/ increase in total cost for case II and case III respectively.

VI. CONCLUSION

In this model, we have developed a two warehouse inventory model for deteriorating items having linear demand with inflation and permissible delay in payments.

It is assumed that rented warehouse holding cost is greater than own warehouse holding cost but provides a better storage facility and there by deterioration rate is low in rented warehouse.

Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of cost.

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