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Comparative error of the phenomena model

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Abstract:- The physical-mathematical model of the actual natural or technological phenomena can include different variables, the finite amount of which is defined by a researcher/conscious observer. The *a priori* overall error inherent this model due its finiteness could be compared with the actual experimental measurement error and should be useful in guiding future investigations. In this context, we propose a strategy relying on thermodynamic theory of information processes, to estimate this error that cannot be done an arbitrarily small. For the considered assumptions, the calculated error of the main researched variable, measured in conventional field studies, should not be less than the error caused by the limited number of dimensional variables of the physical-mathematical model. Examples of practical application of the proposed concept for spacecraft heating, climate prediction, thermal energy storage and food freezing are discussed.

Keywords: - A priori error analysis, physical-mathematical model, information theory, theory of similarity, heat- and mass-transfer

I. INTRODUCTION

All models are wrong; some are useful [1]. Any mathematical, computerized model, regardless of physical level of detail, and which can be described symbolically and discussed deductively, by definition, gives only assumption of accuracy. Model is a simplification of the object being studied. It is limited by its own purposes, like experiments that model should generalize. Laws, which are the product of the imagination of scientists, give results that are correct to a certain extent. Even the complicated engineering system, and the more natural process, involves phenomena that are not represented in the models. If the researcher creates detailed models that require a significant amount of input data and the large number of variables, the probability of reaching all the processes occurring in the multi-parametric system is small. In addition, errors and approximations in measured input data, values of parameters, structure and algorithm's solution of a physic-mathematical model (PMM) are all sources of uncertainty.

In the future, PMM is understood as the basis of the ideas and concepts through which the conscious observer explains his observations and experimental data. PMM includes physical model (PM) and a mathematical model (MM). PM interprets MM, including assumptions and limitations. MM is a set of equations using a symbolic representation of quantitative variables in a simplified physical system. MM helps the researcher to understand and quantify PM.

Although there are reasonable ways to quantify and reduce these errors due to which the uncertainty range of different values, changes of the main variable of the studied system narrow, they cannot be completely eliminated. Detailed analysis of various sources of errors is presented in [2] with vivid examples and impressive results. In any case, decisions will continue to be made in terms of risk and theoretically uncertain future. They can be used as new data and knowledge for the modified model.

Whether there is a theoretically substantiated and/or technically expedient limit of increasing complexity of a model and improving accuracy of the used measuring instruments? Fundamental limits on the maximum accuracy with which we can determine the physical variables are defined by the principle of Heisenberg's uncertainty. However, Planck's constant is vanishing small with respect to macro bodies. That is why this uncertainty in the macroscopic measurements cannot be used for practical application. Uncertainties of position and momentum, calculated in accordance with the Heisenberg's principle, do not show themselves in practice and lie far beyond the achievable accuracy of experiments.

According to our data, in the existing literature there is not presented any physical relationship, which would be applicable in the macrocosm and could formulate the interaction between the level of detailed descriptions of the material object (the number of recorded variables) and the lowest achievable total experimental error of the main parameter chosen for the study, and that describes the behavior of the object. The absence of such general physical relations seems unnatural. Human intuition and experience suggest the simple, at first glance, truth. For a small number of variables, the researcher gets a very rough picture of the process being studied. In turn, the huge number of accounted variables allows deeply and thoroughly to understand the structure of the phenomenon. However, with the apparent attractiveness, each variable brings its own error in the integral (theoretical or experimental) error of model/experiment. In addition, the complexity and cost of computer simulations and field tests enormously increase. Therefore, would seem, must be some optimal/rational number of variables specific to each of the studied processes.

On another side, the issue of error existing because of a finite number of considered variables into PMM is generally ignored in the theory of measurements. It covers only aspects of the *measuring* procedure and data analysis for the value of the main variable, which describes the observed phenomenon.

Any MM is a strict structure suitable in any field without any restrictions [3]. When this model is used as a PM, it becomes an object of imposing of two constraints: limits of the applicability of the actual PMM to its area of well-known predictions and the area where the PMM begins to lose strength; the limits of errors at experimental testing with its area of the successful applicability. These constraints force to admit the possibility of the existence of a certain error before starting to apply the PMM for the investigated phenomenon or process. In fact, it is a methodological error caused by the mismatch threshold [4] between the model of the subject and the studied material object itself.

In this paper, we develop a general approach to the calculation of priori estimates of the error arising due to natural imperfections of the abstract multivariable (large dimension) PMM.

The purpose of this paper is to present the objective existence of *a priori* fundamental error due to the limited number of variables in PMM and justification to the use of the proposed concept to improve the efficiency of research and development works.

In the frame of this approach, "error" is the actual difference between the formulated model and the real object that is intended to identify. This error is generally *known* at the time of computer simulation and experiment activities by one condition that amount of recorded variables is already calculated.

It should be noted that increasing the complexity of the model in order to describe more precisely the studied real system can increase not only the cost of data collection. This will expand the list of potential sources of errors in the output pattern. We are going to propose a suitable level of complexity of the model and to assess the magnitude of uncertainty associated with different assumptions about the model structure and the number of variables under consideration.

II. BACKGROUND

Human desire to learn about the macrocosm, to understand the laws of the invisible microcosm, enhance the quality of everyday life, to protect against natural disasters and prevent an ecological catastrophe on our planet stimulate researchers and designers in a bold and ambitious search, generate desire of scientists and engineers to create energy-efficient appliances and equipment. This equipment is compact, characterized by high computerization and robotics, and can implement complex algorithms.

All the above causes systematic research of processes and phenomena (the material objects - MO) by methods of physical and mathematical modeling. In addition, demands increase for a clear understanding of the results obtained using these methods.

Common in the scientific community the point of view assumes that the observer creates a model of MO, armed with the known laws of using available information, based on her/his experience and intuition. Thus, from the viewpoint of developers, if the results of theoretical calculations differ from the data obtained during the experiments on the value which is less than the reached certain measurement error, the selected PMM is considered as acceptable.

In recent years, new tools and methods are being developed to detect the proximity between the researched MO and the designed PMM, to evaluate modeling errors, as well as to quantify the uncertainties inherent in the numerical calculations, and for choosing the appropriate and adequate PMM [5-7].

The commonly used procedure for the model building is to develop the conceptual ideas and interactions, and then perform the functionalization of variables of the model with suitable data.

However, comprehensive testing is impossible [8]. Exhaustive checking is realized only upon receipt of all results from a model sweep for all possible variants of the input data. In practice, model validation aims to increase confidence in the accuracy of the model. Estimation of the model can be made with different levels of detail, but there is no generally accepted/standard procedure, which would establish the minimum quantitative requirements for the making of a model testing [9].

Over the past ten years, many studies have been conducted to identify what method will demonstrate the most accurate agreement between observation and prediction. Unfortunately, the confirmation is only inherently partial. Complete confirmation is logically precluded by the incomplete access to MO. At the same time, the general strategies of matching models and MO that have been particularly popular from both a theoretical and applied perspective are *verification* and *validation* (V&V) techniques [10].

In [11] the following definition is proposed: *verification* is the process of determining that a computational model accurately represents the basic mathematical model and its solution; *validation* is the process of determining to what degree a model is an accurate representation of the real world from the perspective of the intended use of PMM.

Given the above definition, we can say that the quality validation may be useful in certain scenarios, especially when identifying possible causes of errors in the model. However, at the moment, the validation is not able to provide a quantitative measure of the consent of the agreement/difference between the experimental and computer data. This makes it difficult to use in determining at what point the accuracy requirements are made [10]. We refer the reader to [12, 13] for a more detailed discussion of the existing developments in V&V.

However, some scholars suggest that the V&V of numerical models of natural systems is impossible [14, 15]. The authors argue that the model can never fully simulate reality in all conditions and, therefore, cannot be confirmed. Specific examples of selection of the expedient PMM to describe the studied MO are presented in [16-19]. In addition, it should be noted [20], in which a study of quantum gates is developed. The authors consider these gates as physical devices which are characterized by the existence of random error. Reliability of quantum gates is looked from the perspective of information complexity. In turn, the complexity of the gate's operation is determined by the difference between the entropies of the variables characterizing the initial and final states. In this paper, it is explained that the gate operation may be associated with unlimited entropy, implying the impossibility of realization of the quantum gates function under certain conditions.

Thus, there is no shortage of methods and techniques to identify matching of PMM and studied natural phenomena or processes. At the same time, nobody tried to solve the unusual task of quantifying the conceptual error of the limited dimension PMM caused by the choice of System of Primary Variables (SPV). This is a unique channel (generalizing carrier of information) through which information is transmitted to the observer or the observer extracts information about MO from SPV.

III. PROBLEM FORMULATION

During the process of formulating and constructing of PMM, the whole groups of developers can be engaged. They are with different skill levels and specialize in a narrow field of science and technology: physics, mathematics, chemistry, computer programmers, etc. Abstract mathematical characteristics of the studied MO used in the simulation and recording the rigorous relations between these characteristics, express the modeler's "intuitive," inaccurate, vague ideas about the subject of study, taken from experience, observation, common sense and acquired knowledge of researchers.

The only category that combines all stages and elements of the model construction, and which is like a magician's sword with the strung playing cards, is the system of primary variables (SPV). Any scientific knowledge and all, without exception, formulated physical laws are discovered due to information contained in SPV. We define SPV as "a limited set of circumstances in which the world around us and, of course, any real natural system or process could be observed, tested or subjected to experimental verification." SPV is a set of dimensional variables (DL), primary and calculated on their basis secondary, which are necessary and sufficient to describe the known laws of nature, as in the physical content and quantitatively [21]. As an example of SPV it may be served SGS or SI (International System of Units).

This article aims to look into the feasibility of the idea that the observed/recorded variables are random in terms of information complexity in SPV. From the standpoint of the thermodynamic theory of information processes [22], it is proposed to consider each recorded variable as a kind of readout [23]. This readout allows the researcher to get a certain amount of information about the studied MO. Total number of variables/readouts can be calculated (see below), and it corresponds to the maximum amount of information contained in the SPV. It should be taken into account that the appearance (registration) of readouts is equiprobably.

This time there is completely ignored the human evaluation of information. In accordance with the proposed approach, the set of 100 notes played chimpanzees, and the melody of Mozart's 100 notes in the Piano Concerto No. 21 - Andante, have exactly the same amount of information. However, the basic definitions and estimates of the amount of information during the experiment were clearly formulated by L. Brillouin [24].

The idea may be challenged in terms of philosophy or theory of measurements. At the same time, as we shall see, under the proposed approach, it is possible prior to the start of field studies of mechanical, heat- and mass-transfer processes, find the minimum value of the estimated experimental error in order to confirm the acceptability of the selected model or revise it before the experiment. This error will correspond to the error inherent in the model and caused only because its extremity.

SPV such as SI, includes the following seven ($\xi = 7$) basic primary variables: $\boldsymbol{L}$–length, $\boldsymbol{M}$–weight, $\boldsymbol{T}$–time, $\boldsymbol{I}$–powered by electric current, $\boldsymbol{\Theta}$–thermodynamic temperature, $\boldsymbol{J}$–force of light, $\boldsymbol{F}$–number of substances. The unique combination of dimensions of the basic primary variables in different degrees allows us to represent the dimension of each secondary variable $\mathbf{q}$ [21]:

$$\mathbf{q} \supset \boldsymbol{L}^{l} \cdot \boldsymbol{M}^{m} \cdot \boldsymbol{T}^{t} \cdot \boldsymbol{I}^{i} \cdot \boldsymbol{\Theta}^{\Theta} \cdot \boldsymbol{J}^{j} \cdot \boldsymbol{F}^{f} \qquad (1)$$

where sign "$\supset$" - means "corresponds to a dimension"; l, m ... f - are integers, and in accordance with [25]:

$$-3 \le l \le +3, \quad -1 \le m \le +1, \quad -4 \le t \le +4, \quad -2 \le i \le +2$$
$$-4 \le \Theta \le +4, \quad -1 \le j \le +1, \quad -1 \le f \le +1 \qquad (2)$$

In SPV frames, every analyst/engineer/researcher selects a particular class of phenomena (COP) to study MO. COP is a set of physical phenomena and processes described by a finite number of primary and secondary variables that characterize certain features of MO from the position with qualitative and quantitative aspects [26]. In studying mechanics, for example, the base units of SI are typically used: **L**, ***M***, ***T*** (***LMT***). In studying the phenomena of electromagnetism, the basic set often includes **L**, **M**, **T** and ***I*** (***LMTI***).Maximum number of choices of dimensions of physical variables characterizing the interaction of MO with the environment reaches $\breve{\mathbf{G}}=\mathbf{\Pi}\boldsymbol{e}_n-1$, where "-1" corresponds to the case when all indicators of primary variables equal zero. $\mathbf{\Pi}$ denotes the multiplication of elements $\boldsymbol{e}_n$.

Taking into account (2), we get

$$\breve{\mathbf{G}} = 7 \cdot 3 \cdot 9 \cdot 5 \cdot 9 \cdot 3 \cdot 3 - 1 = 76{,}544 \qquad (3)$$

The information quantity about MO depends on its symmetry [27]. Obviously, equivalent parts of a symmetrical object have an identical structure. Consequently, the characteristics of MO can be judged, having been informed of only one part of it. At the same time, other parts that structurally duplicate it, may be considered as information empty. So you can reduce the number $\breve{\mathbf{G}}$ in $\boldsymbol{\omega}$ times (the number of equivalent parts of researched MO): $\mathbf{G} = \breve{\mathbf{G}}/\boldsymbol{\omega}$.

For the case of SI use in the modeling process, the value $\breve{G}$ includes both direct and inverse dimensional (DL) variables (for example, L^1 - length, L^{-1} - running length). Therefore, the number of variants of dimensions can be reduced by $\omega = 2$ times. This means $\mathbf{G} = \breve{\mathbf{G}}/2 = 38{,}272$. According to $\boldsymbol{\pi}$-theorem [21], the number $\aleph_{SI}$ of possible dimensionless (DS) complexes (criteria) with $\boldsymbol{\xi}=7$ basic DL variables equals

$$\aleph_{SI} = \mathbf{G} - \boldsymbol{\xi} = 38{,}265. \qquad (4)$$

Consideration of the *dimensionless* (DS) field for the researched object is permissible in view of the similarity of the arguments for each dimensional field. In addition, it is caused by the desire to generalize obtained results in the future for the different areas of physical applications. The value of $\aleph_{SI}$ can only increase with the deepening of knowledge about the physical world.

In turn, generally, the research team/observer in the study of MO selects a small number of variables taken into account, sometimes dozens, and in rare cases – hundreds. Thus, in the case of study of the Martian atmosphere, 130 input variables have been used [28]. 10 or 20 or 130 variables - is "a lot" or "a little" for the study of a particular process or a natural phenomenon? As they say, everything is relative: two hairs in a cup of coffee - a lot, and two hairs on his bald head - it's not enough. In other words, what should be the accuracy of the measuring equipment in order to assert the correctness of the proposed PMM and its "closeness" to MO at the chosen criteria of verisimilitude and a given number of selected physical dimensional variables? The clear trend, observed in most areas of science and technology, is the use of very complex models and conducting numerous computer simulations. As a rule, at the end of the article each author claims a "good agreement" or "sufficient accuracy" between the numerical predictions (NP) of behavior of the investigated MO and the experimental results (ER). *In fact, none of the authors counts the total absolute error (AE) of the main indicator of system behavior, and does not compare the difference NP - ER with AE.*

Thus, it becomes necessary to select the appropriate/acceptable level of detail of MO and to formulate the requirements for the accuracy of the input data and the comparative error [23] of the specific target function (similarity criterion), which describes the "livelihoods" and characterizes the behavior of the observed MO. This function consists of physical DL variables, which are measured with some accuracy.

IV. CALCULATION OF COMPARATIVE ERROR OF MODEL WITH FINITE NUMBER OF VARIABLES

For the next reasoning we will specify the concept of "comparative error."

Taking into account the fact that the debates and discussions on the replacement of term "error" with the term "uncertainty" are continuing in nature and not mandatory [29, 30], we used both terms in the work. We understand the "error" as a minimum discrepancy between the tested MO and PMM, which caused only because of a finite number of variables taken into account.

We shall henceforth use the term "comparative error" - the ratio $\boldsymbol{\tau}$ between the alleged error $\boldsymbol{\Delta}\mathbf{U}$ and the considered range of changes $\mathbf{S^*}$ of the measured DL variable $\mathbf{U}$, proposed by Brillouin [23]:

$$\boldsymbol{\tau} = \boldsymbol{\Delta}\mathbf{U}/\mathbf{S^*} \quad (5)$$

where $\boldsymbol{\Delta}\mathbf{U}$ – error in determining the DL variable **U**;

S*- DL range of values in which the DL variable **U** is measured/changed.

Note that, if **S*** is not declared, the information obtained in the measurement, is impossible to define. This full priori range of changes depends on the previous knowledge which the developer had before research. If nothing is known about the studied system, then S* is defined by the limits of the used measuring devices. For this reason, it would be useful to express the closeness of PMM to the studied MO by the comparative error.

Note also, that comparative errors of the DL variable **U** and the DS variable ***u*** are equaled

$$(\boldsymbol{\Delta u}/\boldsymbol{S}) = (\boldsymbol{\Delta}\mathbf{U}/r^*)/(\boldsymbol{S^*}/r^*) = (\boldsymbol{\Delta}\mathbf{U}/\boldsymbol{S^*}) \quad (6)$$

where ***S***, $\boldsymbol{\Delta u}$ - DS variables, respectively, range of variations and total error in determining the DS variable ***u***; r* - DL scale parameter with the same dimension that **U** and ***S**** have.

For further discussion we use the results obtained in [31]: DS comparative error $\boldsymbol{\Delta u}_{pmm}/\boldsymbol{S}$ of the DS variable ***u***, which varies in a predetermined DS range of values ***S***, for a given number of selected physical DL variables ***z***", and ***β***" (the number of the recorded primary physical variables) can be determined from the relation:

$$\boldsymbol{\Delta u}_{pmm}/\boldsymbol{S} \le [(\mathbf{z}' - \boldsymbol{\beta}')/(\boldsymbol{G} - \xi) + (\mathbf{z}'' - \boldsymbol{\beta}'')/(\mathbf{z}' - \boldsymbol{\beta}')] \quad (7)$$

where $\boldsymbol{\Delta u}_{pmm}$ – DS error of PMM at the determining of ***u***; ***G*** – total number of DL physical variables; ***ξ*** - the number of primary physical variables with independent dimension; ***z***'- total number of DL physical variables in the chosen COP; ***β***'- the number of primary physical variables in the chosen COP.

Equation (7) quantifies the comparative error of PMM caused by the limited number of variables in the theoretical analysis of physical phenomena. On the other hand, it also sets a limit on the expedient increasing of the measurement accuracy in conducting experimental studies.

For the specific COP (***z***", ***β***"), conditions to achieve the minimum comparative error $\boldsymbol{\Delta u}_{pmm}/\boldsymbol{S}$ are calculated as follows:

$$\begin{aligned}(\boldsymbol{\Delta u}_{pmm}/\boldsymbol{S})'_{z'-\beta'} &= [(\mathbf{z}'-\boldsymbol{\beta}')/ (\boldsymbol{G}-\xi) + (\mathbf{z}''-\boldsymbol{\beta}'')/(\mathbf{z}'-\boldsymbol{\beta}')]' = \\ &= [1/(\boldsymbol{G}-\xi) - (\mathbf{z}''-\boldsymbol{\beta}'')/ (\mathbf{z}'-\boldsymbol{\beta}')^2]\end{aligned} \quad (8)$$

$$[1/(\boldsymbol{G}-\xi) - (\mathbf{z}''-\boldsymbol{\beta}'')/ (\mathbf{z}'-\boldsymbol{\beta}')^2] = 0 \quad (9)$$

$$(\mathbf{z}'-\boldsymbol{\beta}')^2/(\boldsymbol{G}-\xi) = (\mathbf{z}''-\boldsymbol{\beta}'') \quad (10)$$

where sign " ′ " denotes the derivative.

According to (10), and taken into account (2), for SI and the chosen COP, for example, ***LMTI***, a lowest comparative error can be reached at (***z***"- ***β***") ≈ 6; for ***LMTΘ***, where ***Θ***-thermodynamic temperature, the number of DS parameters causing a minimum value of $\boldsymbol{\Delta}_{pmm}/\boldsymbol{S}$ is about 19 (specific explanation of calculations - see below).

Based on (7), the situation described above, can be regarded as an uncertainty principle for the process of the PMM formulation. Namely, any change in the level of detailed description of MO (***z***"-***β***"; ***z***'-***β***') causes a change in the comparative error of PMM ($\boldsymbol{\Delta u}_{pmm}/\boldsymbol{S}$), and in the accuracy calculation of each main variable characterizing the features of the internal structure of MO or the interaction of MO with the environment.

Within the above-mentioned approach and for a given COP, one could define the actual value of the minimum comparative error inherent PMM having a chosen finite number of variables. Let’s calculate the achievable minimum comparative error for several examples.

Electrodynamics, COP – ***LMTI***, taking into account (2)

$$(z'-\beta') = (7\cdot3\cdot9\cdot5-1)/2 = 472 \quad (11)$$

Then, using (4) и (10), we get

$$(z''-\beta'') = (z'-\beta')^2 / \aleph_{SI} = 472^2 /38,265 = 6 \quad (12)$$

Substituting (4), (11) and (12) into (7), we find

$$(\boldsymbol{\Delta u}_{\mathrm{pmm}}/\boldsymbol{S}) = 0.0247 \quad (13)$$

Heat-transfer, COP - ***LMTΘ***, taking into account (2)

$$(z'-\beta') = (7 \cdot 3 \cdot 9 \cdot 9 - 1)/2 = 850 \quad (14)$$

Then, using (4) и (10), we get

$$(z''-\beta'') = (z'-\beta')^2 / \aleph_{SI} = 472^2 / 38{,}265 = 19 \quad (15)$$

Substituting (4), (14) and (15) into (7), we find

$$(\boldsymbol{\Delta u}_{\mathrm{pmm}}/\boldsymbol{S}) = 0.0444 \quad (16)$$

V. EXAMPLES OF APPLICATION

Unfortunately, in many publications the sufficient basic data are not provided. These data are needed for the calculation and verification of the results obtained by the equation (7).

In different scientific and technical articles known to the author, there was not actually simultaneously provided the information about the value of the resulting total uncertainty and about the changes range of the main variable characterizing the studied MO. At this moment, the author attempts to analyze the published results and compare them with data obtained according to the introduced approach taken by the equation (7).

5.1 Convective heating the outer surface of the spacecraft

The first example, we are going to use, is the forecast of laminar convective heating the outer surface of the spacecraft during its entry into the Martian atmosphere. As the basis of calculations there was used the Navier-Stokes code with non equilibrium interaction for computational fluid dynamics [28]. The main objectives are to identify mechanisms speed limits and definition of the main sources of uncertainty in aerodynamic heating. A total of 130 input DL parameters is recorded in this study for the efficient prediction of heat flux to the surface of the device.

Limits of changes for these variables are selected to represent roughly their typical uncertainties. Uncertainties of these key input parameters are estimated, and the full analysis of uncertainties is provided by Monte Carlo method. The range of variation of the heat flux is in (40-115) W/cm². The results show the quantitative contribution of uncertainties of the modeling key parameters on the uncertainty of the final heat flow.

It should be noted the number of the taken into account variables is $z^*=130$. If, for example, the number of the recorded primary physical variables is $\beta^*=5$, then $z^*- \beta^*=125$. Therefore this value is not closed to the quantity of variables $z''-\beta''$ calculated by the equation (10): COP – ***LMTΘF,***

$$(z'-\beta') = (7 \cdot 3 \cdot 9 \cdot 3 \cdot 9 - 1)/2 = 2{,}551 \quad (17)$$

$$(z''-\beta'') = (z'-\beta')^2/\aleph_{SI} = 2{,}551^2/38{,}265 = 170 \quad (18)$$

$$(\boldsymbol{\Delta u}_{\mathrm{pmm}}/\boldsymbol{S}) = 0.13 \quad (19)$$

The authors did not provide data on the limits of observation (size of change range) and the alleged error of measurement for each parameter, and do not calculate the total relative or comparative error of key variables of the investigated process.

Taking into account the above data, we can make the following assumption. In a frame of the proposed approach, the potential prediction of heat flux to the outer surface of the spacecraft, at its entry into the atmosphere of Mars, can be improved with the proviso that the number of the taken into account variables will be increased on

$$((170 - 125)/125) \cdot 100\% = 36\% \quad (20)$$

5.2 Thermal energy storage system

The similarity theory was used the first time to build a PMM for an ideal thermal energy storage system (TESS) [32]. The three types of TESS were discussed, and generalized energy storage governing equations were introduced in both dimensional and dimensionless forms. Authors studied the temperatures of the heat transfer fluid during the energy charge and discharge processes and the overall energy storage efficiencies through solution of the energy storage governing equations.

Finally, provided in the paper are a series of generalized charts bearing curves for energy storage effectiveness against four dimensionless parameters grouped up from many of the thermal storage system properties including dimensions, fluid and thermal storage material properties, as well as the operational conditions including mass flow rate of the fluid, and the ratio of the energy charge and discharge time periods.

The number of the taken into account variables is $z^*=40$. There were used $\lambda=20$ dimensionless criteria and numbers. Unfortunately, there is not provided a comparison between the theoretical model calculations and experiments results.

The COP of researched phenomena is ***LMTΘ***. In this case, a suitable number of the recorded variables $z''-\beta''$ according to equation (10):

$$(z'-\beta') = (7 \cdot 3 \cdot 9 \cdot 9-1)/2 = 850 \quad (21)$$

$$(z''-\beta'') = (z'-\beta')^2/\aleph_{SI} = 850^2/38{,}265 = 19 \quad (22)$$

Thus, the recommended number $z''-\beta'' = 19$ is closed to $\lambda=20$ of dimensionless criteria and numbers taken into account by authors. It means the minimum achievable comparative error had almost reached. Therefore, engineers can, with high plausibility, conveniently look up the charts to design and calibrate the size of thermal storage tanks and operational conditions without doing complicated individual modeling and computations.

5.3 Global Climate Models

As a third example consider the results [33]. Based on common indicators identified through several methods of multivariate analysis, the presented study compares the global climate models in terms of their ability to reproduce the climatologically-area average by several variables. The authors attempted to identify a single overall indicator through which the generalized characteristic (metric) of a global climate model could be evaluated. It is proved that the redundancy does not significantly affect the quality of generic metric as this metric is based on a sufficient number of variables. Furthermore, it is argued that the addition of a new variable to the total general metric does not necessarily lead to an effective increase of information provided by the basic principal characteristic, if a new variable is closely connected with any of the variables that have been included in the metric. In this case, the addition of a variable causes receiving of redundant information.

In addition, several methods are proposed to reduce redundancy in varying variables before defining the main metric, which measures the overall response of climate models.

Authors argue that an accurate assessment of the effective number of models and variables may be insignificant. In an attempt to assess the climate models by introducing a common characteristic metric with a reduced redundancy of variables, the authors substantiate the claim that the general ranking model is quite insensitive to the particular definition of the metric. The twenty two (22) variables are used for analysis. For each variable, the comparison calculation of data obtained from different climatic models is organized. In addition, model output data from 24 climate models are compared with observational data. Several methods are also proposed to reduce redundancy in variable metrics before defining a general metric that scores the general performance of climate models.

In this paper also the specific data of the dimensional range of variable changes (for example, "temperature varies from 10ºC to 40ºC," etc.) and an error of measurement for each variable are absent. However, the class of phenomena (COP) can be determined - (***LMTΘ***). A suitable number of recorded variables $z''-\beta''$ equals 19 (look (22)).

If, for example, the number of the recorded primary physical variables is $\beta^*=3$, and the number of considered variables is $z^*=22$, then $z^*- \beta^*=19$ which equals to the number of variables recommended by the formula (22) – 19. Thus, it can be argued that the authors after the results of months of research reached the minimum comparative error.

Following the suggested approach, the required number of recorded variables can be calculated during several minutes.

5.4 Food Freezing

We consider the engineering task [34] of the heat transfer to a thin layer of material frozen on a moving cooled cylindrical wall. In the specified work, the COP is ***LMTΘ.*** Based on (3.2), we find z'-β' = 850. While examining this process, theoretical calculations and experimental data were introduced in the DS form. The final DS temperature of the outer surface of the material $\Theta_s°=(\Theta_s-\Theta_e)/\Theta_{cr}-\Theta_e)$ is presented in the form of a correlation function of multiplication of six (z''-β''=6) independent one-parameter DS complexes where Θ_{cr}, Θ_s, Θ_e are the absolute temperatures respectively of freezing of a material, outer surface of a material layer and evaporating of the refrigerant; $\Delta\Theta_{cr}$, $\Delta\Theta_s$, $\Delta\Theta_e$ are the absolute errors of measurement of these temperatures. Then, considering Θ_{cr}=272ºК, Θ_s=259ºК, Θ_e= 243ºК, $\Delta\Theta_{cr}$=0.1ºК, $\Delta\Theta_s$=$\Delta\Theta_e$=0.5ºК, you can find an absolute DS error of the indirect measurement $(\Delta\Theta_s°)_{exp}$, reached in the experiment according to formulas introduced in [35]:

$$(\Delta\Theta_s°)_{exp} = (\Delta\Theta_s+\Delta\Theta_e) / (|\Theta_{cr}-\Theta_e|) + \\ + |\Theta_s - \Theta_e| / ((\Delta\Theta_{cr}+\Delta\Theta_e)\cdot|\Theta_{cr} - \Theta_e|^2) \approx 0.066 \quad (23)$$

From (7), using calculated values $\aleph_{SI}$ и z'-β', you get a DS error value of $(\Delta\Theta_s°)_{pmm}$ for the chosen PMM:

$$(\Delta\Theta_s°)_{pmm} \leq \Theta_{smax}° \cdot ((z'-\beta')/\aleph_{SI} + (z''-\beta'')/(z'-\beta')) = \\ = 0.93\cdot [850/38,265+6/850] = 0.027 \quad (24)$$

where $\Theta_{smax}°$ is a given range of changes of the DS final temperature [34], allowed by the chosen mathematical model.

From (23) and (24), we get $(\Delta\Theta_s°)_{exp}$>$(\Delta\Theta_s°)_{pmm}$, i.e., an actual error in the experiment is 2.4 times (0.066/0.027) more than the minimum. It means, at the chosen number of DS criteria the existing accuracy of DL variable's measurement is not enough. The further experimental work is required to change devices to a higher grade of accuracy satisfactorily in order to confirm/refine the elaborated PMM.

Hence, the use of the suggested approach helps a researcher to find the minimum value of the *required* experimental error for the confirmation of the eligibility of the chosen PMM. This error will correspond to the error inherent in the model and caused only by its finiteness.

VI. CONCLUSION

Identification and recognition of the legitimacy of a particular physical-mathematical model is an important step in the study and cognition of physical phenomena, as well as the optimization of a technological process. Assessment of the adequacy of mathematical models is just possible via the comparison between an error Δu_{pmm} /S caused by a model's finite amount of variables and the total integrated and calculated (by computer simulation or field tests) experimental error Δu_{exp}/S of the main variable characterizing the process under study. Only after said comparison should use known techniques and principles to demonstrate the correctness of the proposed model.

Within the proposed approach, the error Δu_{pmm} caused by the finite amount of considered variables is a peculiar "firstborn" and least error inherent any actual physical-mathematical model.

The achievable in the field experiments, measured comparative error $\Delta u_{exp}/S$ of the main variable/indicator/complex, describing the livelihoods of the phenomenon under study, should be not less than $\Delta u_{pmm}/S$. Otherwise, it is necessary to revise the model before conducting of experiments. In fact, Δu_{pmm} represents a kind of "model noise" (similar to "thermal noise" also called Schottky noise [7]).

When comparing different physical-mathematical models (according to a value of $\Delta u_{pmm}/S$) describing the same MO, preference should be given to the PMM for which $\Delta u_{pmm}/\Delta u_{exp}$ is closer to 1.

Author hoped that the use of the proposed approach in practice will help scientists and engineers to more accurately analyze the results and reduce the volume and cost of research and engineering projects.

REFERENCES

[1]. G. E. P. Box, Robustness in the Strategy of Scientific Model Building, Robustness in statistics (New York: Academic Press, **1979**) **201-236**.

[2]. M.A. Christie, J. Glimm, J.W. Grove, D.M. Higdon, D.H. Sharp, and M.M. Wood-Schultz, Error analysis and simulations of complex phenomena, Los Alamos Science, 29, **2005**, **6-25**. Available: http://goo.gl/GLD6q4.

[3]. M. Burgin, Theory of Information - Fundamentality, Diversity and Unification (World Scientific Publishing Co., **2009**).

[4]. S.G. Rabinovich, Evaluating Measurement Accuracy (New York: Springer Science+Business Media, **2013**). Available: http://goo.gl/20PYxD.

[5]. J.T. Oden, and S. Prudhomme, Estimation of modeling error in computational mechanics, Journal of Computational Physics, 182, **2002**, **496–515**. Available: http://goo.gl/0OUEt0.

[6]. W.L. Oberkampf, and M.F. Barone, Measures of agreement between computation and experiment: validation metrics, Journal of Computational Physics, 217, **2006**, **5-36**.

[7]. D.G. Cacuci, Sensitivity and Uncertainty Analysis (CHAPMAN & HALL/CRC, **2003**). O. Balci, Principles of simulation model validation, verification, and testing, Transactions of the Society for Computer Simulation International, 14, **1997**, **3-12**.

[8]. Donatelli, Crop modelling and validation: integration of Irene DLL in the warm environment, Italian Journal of Agrometeorology, 3, **2006**, **35-39**. Available: http://goo.gl/9MO8Ie. C-J. Kat, and P. S. Els, Validation metric based on relative error, Mathematical and Computer Modelling of Dynamical Systems: Methods, Tools and Applications in Engineering and Related Sciences, 18(5), **2012**, **487-520**. Available: http://goo.gl/sFk9UR.

[9]. ASME V&V 10-2006 Guide for Verification and Validation in Computational Solid Mechanics, (The American Society of Mechanical Engineers standards, **2006**).

[10]. Maropoulos, and D. Ceglarek, Design verification and validation in product lifecycle, CIRP Annals - Manufacturing Technology, doi:10.1016/j.cirp.2010.05.005, **2010**, **1-20**. Available: http://goo.gl/sJc02G. A.T. Bahill, and S.J. Henderson, Requirements development, verification, and validation exhibited in famous failures systems engineering, Systems Engineering, 8(1), **2005**, **1-14**. Available: http://goo.gl/Oj8zDC.

[11]. N. Oreskes, K. Shrader-Frechette, and K. Belitz, Verification, validation and confirmation of numerical models in the Earth sciences, Science, 263, **1994**, **641-646**. Available: http://goo.gl/H2temb.

[12]. L.O. Tedeschi, Assesment of the mathematical models, Workshop on Mathematical Model analysis and Evaluation, **2004**, **1-28**. Available: http://goo.gl/YHfDkg.

[13]. Akaike, Information theory as an extension of the maximum likelihood principle, Second International Symposium on Information Theory, Academia Kiado, Budapest, Hungary, **1973**, **267-281**. Available: http://goo.gl/ZKhp12.

[14]. T.N. Siraya, and V.A. Granovsky, Measurement Quality Characteristics in Metrology: Systematic Approach, XVII IMEKO World Congress Proceedings, Dubrovnik, Croatia, **2003**, **1057-1062**. Available: http://goo.gl/NcMJQJ.

[15]. Y. Ben-Haim, and F.M. Hemez, Robustness, fidelity and prediction looseness of models, Proc. Royal Soc. A, 468(2137), **2012**, **227-244**. Available: http://goo.gl/TBtvOx.

[16]. A.A.Scaife, Skillful long-range prediction of European and North American winters, Geophysical Research Letters, 41(7), **2014**, **2514-2519**. Available: http://goo.gl/qz2juB.

[17]. S. Kak, Information complexity of quantum gates, International Journal of Theoretical Physics, 45(5), **2006**, **933-941**. Available: http://goo.gl/iooQBt.

[18]. A. A. Guhman, Introduction in the Theory of Similarity (Moscow: High School, **1973)**, in Russian.

[19]. R.P. Poplavsky, Thermodynamics of Information Processes (Moscow: Nauka, **1981**), in Russian.

[20]. L. Brillouin**,** Science and Information Theory (New York: Dover, **2004**).

[21]. L. Brillouin, Scientific Uncertainty and Information (London: Academic Press Inc., **1964**)

[22]. B.M. Yavorskij, and A.A.Detlaf, Handbook of Physics (Moscow: Nauka, **1985**), in Russian.

[23]. L. I. Sedov, Similarity and Dimensional Methods in Mechanic (Florida: CRC Press, **1993**).

[24]. J. Collier, and M. Burch, Symmetry, Levels and Entrainment, Proceedings of the International Society for Systems Sciences, Toronto, Canada, **2000**, **1-23**. Available: http://goo.gl/Vbgt8r.

[25]. Bose, M. Wright, and G. Palmer, Uncertainty analysis of laminar aeroheating prediction for Mars entries,

[26]. NASA Ames RC, AIAA Paper 2005-4682, **2005**, **6-25**.

[27]. M. Henrion, and B. Fischhoff, Assessing uncertainty in physical constants, American Journal of Physics, 54(9), **1986**, **791-798**. Available: http://goo.gl/wYwlBu.

[28]. V.N. Matveev, and O.V. Matvejev, General Physical Uncertainty Relations as a Consequence of the Lorentz Transformations, 1-11. Available: http://goo.gl/7SoCiY.

[29]. M. Menin, Calculation of error inherent physical-mathematical model due to finite number of recorded variables, International Journal of Mathematical Models and Methods in Applied Sciences, 7, **2013**, **204-212**. Available: http://goo.gl/j7ukzX.

[30]. L. Peiwen, J.V. Lew, C. Chan, W. Karaki, J. Stephens, and J. E. O'Brien, Similarity and generalized analysis of efficiencies of thermal energy storage systems, Renewable Energy, 39, **2012**, **388-402**. Available: http://goo.gl/aN8tBJ.

[31]. K. Nishii, T. Viyasaka, H. Nakamura, Y. Kosaka, S. Yokoi, Y.N. Takayabu, H. Endo, H. Icnikawa, T. Inoue, K. Oshima, N. Sato, and Y. Tsushima. Relationship of the reproducibility of multiple variables among global climate models, Journal of the Meteorological Society of Japan, 90A, **2012**, **87-100**. Available online, http://goo.gl/gIlQES.
N.V. Fomin, B.M. Menin, V.B. Rzevskaya, and E.I. Guygo. Drum Freezers (Leningrad : Mashinostroyenie, **1986**), in Russian.

[32]. G.R. Shlykov, Theory of Measurement: equations, models, estimation accuracy (Penza: State University, **2008**), in Russian.