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Ács-Bölcsföldi-Birkás prime numbers

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Abstract:*After defining the set, Ács-Bölcsföldi-Birkás prime numbers will be presented from 223 to 2593 and from 200023 to 210317. How many Ács-Bölcsföldi-Birkás prime numbers are there in the interval (10^{n-1}, 10^n), where n is a positive integer number and n≥3? On the one hand, it has been counted by computer with up to 10-digits. On the other hand, the function (1) gives the approximate number of Ács-Bölcsföldi-Birkás prime numbers in the interval (10^{n-1},10^n). The function (2) gives the approximate number of Ács-Bölcsföldi-Birkás prime numbers where all digits are 3 or 7 in the interval (10^{n-1},10^n).*

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I. INTRODUCTION

The sets of special prime numbers within the set of prime numbers are well-known. For instance, left-truncatable primes (If we leave the initial digits out, the remainder will be prime.), right-truncatable primes (If we leave the last digits out, the remainder will be prime.), the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes (F_0=0, F_1=1, $F_n=F_{n-1}+F_{n-2}$), Gauss-primes (in the form 4n+3), Leyland-primes (in the form x^y+y^x, where 1≤x≤y), Pell-primes (P_0=0, P_1=1, $P_n=2P_{n-1}+P_{n-2}$), Bölcsföldi-Birkás-Ferenczi primes (all digits are prime and the number of digits is prime),.Bölcsföldi-Birkás prime numbers (all digits are prime, the number of digits is prime, the sum of digits is prime), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found a further set of special prime numbers within the set of prime numbers. It is the set of Ács-Bölcsföldi-Birkás prime numbers.

II. ÁCS-BÖLCSFÖLDI-BIRKÁS PRIME NUMBERS [3], [9], [10], [11], [12].

Definition: a positive integer number is Ács-Bölcsföldi-Birkás prime number, if
a/ the positive integer number is prime,
b/ the first digit and the last digit are prime,
c/ leaving the initial and the last digit out, the remainder will be prime.

The Ács-Bölcsföldi-Birkás prime numbers are as follows (the last digit can only be 3 or 7):
223, 227, 233, 257, 277, 337, 353, 373, 523, 557, 577, 727, 733, 757, 773, 2027, 2053, 2113, 2137, 2237, 2293, 2297, 2377, 2417, 2437, 2473, 2477, 2593,… 200023, 200033, 200117, 200237, 200293, 200297, 200437, 210097, 210193, 210317, … 2000113, 2000177, 2000293, 2000413, 2000417, etc.

T(n) is the factual frequency of Ács-Bölcsföldi-Birkás prime numbers in the interval (10^{n-1}, 10^n), where n≥3 integer. T(3)=15, T(4)=64, T(5)=326, T(6)=1972, T(7)=12634, T(8)=92833, T(9)=701262, etc. S(n) function gives the number of Ács-Bölcsföldi-Birkás prime numbers in the interval (10^{n-1},10^n).

We think that

$S(n)=0{,}069x10^{n-2}+30$, where n≥3 integer. **(1)**

The factual number of Ács-Bölcsföldi-Birkás primes and the number of Ács-Bölcsföldi-Birkás primes calculated according to function S(n) are as follows:

Number of digits	The factual number of Ács-Bölcsföldi_Birkás primes in the interval $(10^{n-1},10n^{p})$	The number of Ács- Bölcsföldi_Birkás primes calculated according to function	
n	T(n)	S(n)	T(n)/S(n)
3	15	30,69	0,49
4	64	36,9	1,73
5	326	99	3,29
6	1972	720	2,74
7	12634	6930	1,82
8	92833	69030	1,34
9	701262	690030	1,02

The Ács-Bölcsföldi-Birkás prime numbers, where all digits are 3 or 7, are as follows:
337, 373, 733, 773, 3373, 3733, … 333337, 333737, 373777, 377737, 733373, 737773, etc.

V(n) is the factual frequency of Ács-Bölcsföldi-Birkás prime numbers in the interval $(10^{n-1},10^{n})$, (all digits are 3 or 7): V(3)=4, V(4)=3, V(5)=5, V(6)=10, V(7)=16, V(8)=30, V(9)=53, V(10)=87, V(11)=185, V(12)=365, V(13)=591, V(14)=1062, V(15)=2290, V(16)=3480, V(17)=7399. .. W(n) function gives the number of Ács-Bölcsföldi-Birkás prime numbers in the interval $(10^{n-1},10^{n})$, all digits are 3 or 7. We think that $\mathbf{W(n)=0{,}22x2^{n-2}}$, where n≥3 integer.

(2)

The V(n) factual number of Ács-Bölcsföldi-Birkás primes and the number of Ács-Bölcsföldi-Birkás primes calculated according to function W(n) (where all digits are 3 or 7) are as follows:

	The V(n) factual number in the interval $(10^{n-1},10^{n})$	The number calculated according to function $W(n)=0{,}22x2^{n-2}$	
n	V(n)	W(n)	V(n)/W(n)
3	4	0,44	9,09
4	3	0,88	3,41
5	5	1,76	2,84
6	10	3,52	2,84
7	16	7,04	2,27
8	30	14,08	2,13
9	53	28,16	1,88
10	87	56,32	1,54
11	185	112,64	1,64
12	365	225,28	1,62
13	591	450,56	1,31
14	1062	901,12	1,18
15	2290	1802,24	1,27
16	3480	3604,48	0,97
17	7399	7208,96	1,03

III. THE NUMBER OF THE ELEMENTS OF THE SET OF ÁCS-BÖLCSFÖLDI-BIRKÁS PRIME NUMBERS [3], [9],[10], [11], [12].

Let's take the set of Mills' prime numbers!

Definition: The number $m=[M \text{ ad } 3^{n}]$ is a prime number, where M=1,306377883863080690468614492602 is the Mills' constant, and n=1,2,3,… is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following: m=2, 11, 1361, 2521008887,…

The connection n→m is the following:1→2, 2→11, 3→1361, 4→2521008887,… The Mills' prime number $m=[M \text{ ad } 3^{n}]$ corresponds with the interval $(10^{m-1},10^{m})$ and vice versa. For instance:2→(10, 10^{2}),

$11 \rightarrow (10^{10}, 10^{11})$, $1361 \rightarrow (10^{1360}, 10^{1361})$, etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the intervals $(10^{m-1}, 10^{m})$ that contain at least one Mills' prime number is infinite. The number of Ács-Bölcsföldi-Birkás primes in the interval $(10^{m-1}, 10^{m})$ is $S(m)=0{,}069x10^{m-2}+30$. The number of Ács-Bölcsföldi-Birkás prime numbers is probably infinite: $\lim_{n\to\infty} T(n)=\infty$ is probably where $n\geq 3$ integer.

IV. THREE-PARTS PRIME NUMBERS

Definition: a positive integer number is three-parts prime number, if
a/ the positive integer number is prime,
b/ the number of digits is 3k, where k≥1 integer,
c/ dividing it into three equally long parts, every part is a prime.

The three-parts prime numbers are as follows (the last digit can only be 3 or 7):
: 223, 227, 233, 257, 277, 337, 353, 373, 523, 557, 577, 727, 733, 757, 773,
110237, 110261, 110273, 110311, 110323, 110359, 110503, 110543, 110567, 110573, 110597, 110711, 110729, 110731, 110753, 110771, 111103, 111119, 111143, 111317, 111323, 111337, 111341, 111347, 111373,…
101002019, 101002037, 101002067, 101002073, 101002103, 101002127, 101002199, 101002241, etc.

H(n) is the factual frequency of three-parts prime numbers in the interval $(10^{n-1}, 10^{n})$ where $n\geq 3$ integer. H(3)=15, H(6)=2540, H(9)=564329 , etc.
F(n) function gives the number of three-parts prime numbers in the interval $(10^{n-1}, 10^{n})$, where $n\geq 3$ integer. We think that

$$F(n)=2{,}015x6^{n-2} \quad \text{where } n\geq 3 \text{ integer.} \tag{3}$$

The factual number of three-parts prime numbers and the number of three-parts prime numbers calculated according to function F(n) are as follows:

Number of digits n	The factual number three-parts primes in the interval $(10^{n-1}, 10^{n})$ H(n)	The number of three-parts primes according to function $F(n)=2{,}015x6^{n-2}$ F(n)	H(n)/F(n)
3	15	12,09	1,24
6	2540	2611,44	0,97
9	564329	564071,04	1,00
12		121839344,60	
15		$2{,}63173x10^{10}$	
18		$9{,}47423x10^{11}$	

The three-parts prime numbers, where all digits are 3 or 7, are as follows:
337, 373, 733, 773, …
373773733, 733773773, 773373373, 773373773, 773733773 etc.
P(n)= is the factual frequency of three-parts prime numbers in the interval $(10^{n-1}, 10^{n})$ where all digits are 3 or 7.
Q(n) function gives the number of three- parts prime numbers in the interval $(10^{n-1}, 10^{n})$ where all digits are 3 or 7.

$$Q(n)=1{,}424^{n-2} \quad \text{where } n=3k \text{ and } k\geq 1 \text{ integer} \tag{4}$$

The P(n) factual numbers of three-parts prime numbers and the number of three-parts prime numbers calculated according the function Q(n) (where all digits are 3 or 7) are as follows:

Number of digits n	The factual number in the interval $(10^{n-1}, 10^n)$ P(n)	The number calculated according to function $Q(n)=1,424^{n-2}$ Q(n)	P(n)/Q(n)
3	4	1,404	2,81
6	0	4,11	0
9	5	11,87	0,42
12	0	33,33	0
15	11	98,99	0,11
18	80	285,86	0,28
21	210	825,45	0,25
24	1293	2383,53	0,54
27	6884	6882,56	1,00

V. FOUR- PARTS PRIME NUMBERS

Definition: a positive integer number is four-parts prime number, if
a/ the positive integer number is prime,
b/ the number of digits is 4k, where k≥1 integer.
c/ dividing it into four equally long parts, every part is a prime.

The four-parts prime numbers are as follows (the last digit can only be 3 or 7):
2237,2273, 2333, 2357, 2377, 22557, 2757, 2777, 3253, 3257, 3323, 3373, 3527, 3533,3557, 3727, 3733, 5227, 5233, 5237,
5273, 5323, 5333, 5527, 5557, 5573, 5737, 7237, 7253, 7333, 7523, 7537, 7573, 7577, 7723, 7727, 7753, 7757, 11020213, 11020241, 11020253, 11020271, 11020297, 11020313, 11020397, 11020507, etc.
K(n) is the factual frequency of four-parts prime numbers in he interval $(10^{n-1}, 10^n)$, where $n\geq4$ integer.
K(4)=38 , K(8)=41336.
L(n) is the factual frequency of four-parts prime numbers in the interval $(10^{n-1}, 10^n)$, where all digits are 3 or 7.
L(4)=3 , L(8)=3 , L(12)=31, L(16)=5, L(20)=0, L(24)=420, L(28)=1923.
M(n) funktion gives the number of four-parts prime numbers in the interval $(10^{n-1}, 10^n)$, where all digits are 3 or 7.
We think that

$$M(n)=1,3355^{n-2} \quad \text{where } n\geq4 \text{ integer.} \tag{4}$$

The L(n) factuel number of four-parts prime numbers and the number of four-parts prime numbers calculated according to funktion M(n) (where all digits are 3 or 7) are as follows:

Number of digits n	The L(n) factual number int he interval $(10^{n-1}? 10^n)$ L(n)	The number calculated according to funktion $M(n)=1,3355^{n-2}$ M(n)	L(n)/M(n)
4	3	1,78	1,69
8	3	5,67	0,53
12	31	18,05	1,72
16	5	57,41	0,09
20	0	182,64	0
24	420	580,99	0,72
28	1923	1848,17	1,04

Five-parts, six-parts etc. primes can be defined and examined in a similar way.

VI. CONCLUSION

Countless different sets of special prime numbers have been known. We have found the following set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

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REFERENCES:

[1]. http://oeis.org/A019546
[2]. Freud, Robert – Gyarmati, Edit: Number theory (in Hungarian), Budapest, 2000
[3]. http://ac.inf.elte.hu → VOLUMES → VOLUME 44 (2015)→ VOLLPRIMZAHLENMENGE→FULL TEXT
[4]. http://primes.utm.edu/largest.html
[5]. http://mathworld.wolfram.com/SmarandacheSequences.html
[6]. Dubner, H.:"Fw:(Prime Numbers) Rekord Primes All Prime digits" Februar 17. 2002
[7]. http://listserv.nodak.edu/scripts/wa.exe?A2=ind0202&L=nmbrthry&P=1697
[8]. Harman, Glyn: Counting Primes whose Sum of Digits is Prime.
[9]. Journal of Integer Sequences (2012. , Vol. 15, 12.2.2.)
[10]. ANNALES Universitatis Scientiarum Budapestiensis de Rolando Eötvös Nominate Sectio Computatorica,2015, pp 221-226
[11]. International Journal of Mathematics and Statistics Invention, February 2017: http://www.ijmsi.org/Papers/Volume.5.Issue.2/B05020407.pdf
[12]. Bölcsföldi-Birkás-Ferenczi prime numbers (full prime numbers) International Organisation of Scientific Research, April 2017 http://www.iosrjournal.org/iosr-jm/pages/v13(2)Version-4.html Bölcsföldi-Birkás prime numbers
[13]. DIGITEL OBJECT IDENTIFIER NUMBER (DOI), May 2017
[14]. http://dx.doi.org or www.doi.org
[15]. Article DOI is: 10.9790/5728-1302043841
[16]. Bölcsföldi-Birkás prime numbers

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