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A Two-Stage Robust Optimization Model for Liner Shipping Capacity Deployment and Transportation under Demand Uncertainty

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Abstract:*The global economy is undergoing a complex and dynamic transition from post-pandemic recovery to a new phase of normalized development, with globalization continuing to deepen. As a vital part of the global economic system, maritime transportation plays a key role in linking international markets. Container liner shipping, in particular, is crucial to global supply chains but faces challenges in capacity deployment due to demand uncertainty. This study develops a two-stage robust optimization model to maximize total profit, incorporating spot demand volatility and contract fulfillment constraints. A GA-C&CG (Genetic Algorithm–Column-and-Constraint Generation) algorithm is proposed for solution. Sensitivity analysis indicates that higher uncertainty budgets lead to more conservative decisions and lower profits, while greater market volatility also reduces profitability. Increasing contract fulfillment rates slightly impacts profit but enhances customer loyalty. Higher freight rates encourage the use of larger vessels, though high uncertainty prompts more cautious deployment.*

Keywords:*Uncertain demand;Liner shipping capacity scheduling; Robust optimization; GA-C&CG algorithm*

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I. INTRODUCTION

The current global economic landscape is increasingly complex and dynamic, as the world economy gradually recovers from the impact of the COVID-19 pandemic and transitions toward normalized development. Meanwhile, the process of globalization continues to advance. Container liner shipping plays a pivotal role in global maritime trade, handling approximately 80% of the world's trade volume. It provides reliable transportation services through fixed routes and scheduled sailings. However, fluctuations in the global economy, demand uncertainty, and geopolitical risks have posed significant challenges to capacity deployment. Therefore, optimizing liner shipping capacity deployment strategies is essential for enhancing the operational performance of liner shipping companies.

This study investigates the container liner shipping capacity deployment problem under demand uncertainty, with a particular focus on incorporating the fulfillment rate[1] requirements of contract transportation demand. Considering multiple shipping routes and heterogeneous vessel types under weekly service frequency and demand scenarios, the capacity deployment process is divided into a strategic level and a tactical level. In the first stage, vessel deployment plans are formulated to adapt to fluctuating demand and enable flexible capacity configuration. In the second stage, based on the initial deployment plan, capacity is allocated in response to realized market demand to ensure service reliability. To this end, we develop a two-stage robust optimization model and propose a corresponding deployment strategy to address the challenges posed by uncertain demand in liner shipping operations.

II. LITERATUREREVIEW

Both domestic and international scholars have conducted extensive research on liner shipping capacity deployment under uncertainty. In existing studies, the uncertainties in container liner capacity planning primarily stem from demand fluctuations, freight rates and transportation costs, vessel and slot sharing, as well as port operations and route selections. To address these challenges, robust optimization, stochastic programming, and dynamic programming are commonly employed to enhance the stability and adaptability of deployment strategies. Wang[2] investigate the container slot allocation problem under demand uncertainty, incorporating empty container repositioning and freight pricing considerations. They formulate a two-stage stochastic nonlinear integer programming model and propose a sample average approximation approach based on Lagrangian relaxation and dual decomposition techniques. The proposed method effectively addresses the challenges arising from uncertain demand and empty container repositioning. Chen[3] address the joint fleet deployment problem of liner shipping alliances under demand uncertainty using a robust optimization approach. By analyzing the vessel pool and key factors influencing collaborative deployment among alliance members, they develop an effective joint fleet deployment plan that enhances operational resilience in uncertain environments. Lai [4] develop a two-stage robust optimization model to optimize fleet deployment and revenue management in liner shipping networks under demand uncertainty. Xiang[5] develop a two-stage robust model to optimize fleet deployment and empty container repositioning, aiming to minimize operational costs. Wang[6] propose a two-stage robust model to optimize fleet planning under uncertain demand and freight rates, enhancing decision robustness in liner shipping services. Existing studies mainly focus on fleet deployment, empty container repositioning, and revenue management under uncertainty, providing theoretical support and decision-making references for optimizing liner shipping capacity deployment. However, liner shipping companies need to balance long-term contract customer loyalty with the high potential revenue from spot market demand, as well as make sequential deployment decisions for different vessel types. The joint optimization of contract demand fulfillment rates and demand uncertainty remains an area requiring further research. Monemi R N[7] unified framework is proposed for liner shipping, jointly optimizing network design, fleet deployment, and empty container repositioning. Service routes are treated as endogenous decisions, and a Benders decomposition-based method is used to solve the integrated problem. Guangmei L[8] developed a game-theoretic model to investigate how capacity allocation and pricing strategies can enhance the resilience of the shipping supply chain under limited capacity.

III. PROBLEM DESCRIPTION AND NOTATION

3.1 Linear static analysis or equivalent static analysis

The liner shipping capacity deployment and transportation problem studied in this chapter is formulated as a two-stage robust optimization model. In the first stage, a shipping company operating R routes and owning vessels of K types-each with limited availability from set A-must select a deployment plan under demand uncertainty. The deployment plan determines the number, sequence, speed, and service frequency of each vessel type on each route. Once the deployment is fixed, vessel speed and sailing frequency are also fixed, and the deployment scheme can be uniquely represented by a corresponding speed set.

Container transport demand generally falls into two categories: contract demand, which arises from long-term agreements between shippers and carriers-typically stable and high in volume but lower in price; and spot demand, which is short-term, volatile, and priced higher. The carrier must balance profitability with customer satisfaction, particularly for contract clients, by introducing a contract fulfillment rate to quantify service quality. Given that most demand information is unavailable at the deployment stage, a robust optimization approach is employed to guard against the worst-case demand realizations. In the second stage, based on the chosen deployment, the carrier allocates vessel capacity between contract and spot demands, aiming to maximize revenue while maintaining service levels.

The following assumptions are made:

- Transshipment of containers is not considered in the model;
- The transportation of empty containers is not included;
- All containers transported are assumed to be standard 20-foot TEUs;
- A complete round voyage is defined as a vessel departing from and returning to the home port;
- A minimum level of contract demand must be satisfied for each origin-destination (OD) pair.

3.2Notation

This paper develops a liner shipping capacity scheduling model considering uncertain demand from the perspective of a liner shipping company. The model parameters are described in Table 1:

Table 1 Parameters and explanations

Parameters	Parameter description
$U_c = \{1, 2, \cdots, \Omega_c\}$	The set of scenarios for contract transportation demand.
$U_s = \{1, 2, \cdots, \Omega_s\}$	The set of scenarios for spot transportation demand.
$\mathrm{K} = \{1, 2, \cdots k\}$	The set of available vessel types.
$R = \{1, 2, \cdots r\}$	The set of shipping routes operated by the liner shipping company.
$L_{m,od} = \{1, 2, \ldots, l\}$	The set of voyage legs served under the vessel deployment plan.
$P_{o,d,w}$	The set of routing paths from origin port o to destination port d in week w of the planning horizon.
$q^{c}{}_{o,d,w}$	The volume of contractual transport demand from origin port o to destination port d in week w of the planning horizon.
$q^{s}{}_{o,d,w}$	The volume of spot transport demand from origin port o to destination port d in week w of the planning horizon.
$\bar{q}^{s}_{o,d,w}$	The nominal value of spot transport demand from origin port o to destination port d in week w of the planning horizon.
$\Delta q^{s}{}_{o,d,w}$	The deviation of spot transport demand from origin port o to destination port d in week w of the planning horizon.
$c_{\omega}^{c,wod}$	The unit penalty cost for unmet contractual demand.
$c_{\omega}^{s,wod}$	The unit penalty cost for unmet spot demand.
Γ	The uncertainty budget.
$D(\theta)^c$	The unmet contractual demand under demand scenario.
$D(\theta)^s$	The unmet spot demand under θ demand scenario.
V_k	The capacity of a type-k vessel.
κ_k	The total number of type-k vessels.
$c_{f,k}^{m}$	The fuel cost associated with deploying a type-k vessel under deployment plan m.
$c_{o,k}^{m}$	The fixed operating cost of deploying a type-k vessel under deployment plan m.
$c\square$	The unit transportation cost of shipping containers via path p.
γ_{od}^{c}	The unit freight rate for transporting contractual demand from the origin port to the destination port.
γ_{od}^{s}	The unit freight rate for transporting spot demand from the origin port to the destination port.
∂	The fulfillment ratio requirement for contractual demand.
$\mu_{k,m}$	The number of type-k vessels deployed under deployment plan m.
$Z_{m,r}$	A binary decision variable indicating whether deployment plan m is selected for route r.
$x_{k,m,i}$	A binary decision variable indicating whether a type-k vessel is assigned to the *i-th* departure in the deployment plan.
$y_{p,w,o,d}^{c}$	The number of contractual containers transported via path p.
$y_{p,w,o,d}^{s}$	The number of spot containers transported via path p.

IV. MODEL FORMULATIONAND SOLUTION

4.1 Model Formulation

(1) Vessel Operation Mode

According to Lai, vessel deployment depends on route distance, sailing speed, port turnaround time, and round trip duration. Weekly service requires vessels to call at the same port every 7 days. The planning horizon is typically 13 or 26 weeks. If a type-k vessel is assigned to sailing j with round trip time t□ , it will operate on sailings j, j + t□ , ..., assuming immediate turnaround.

For route r, with average port time ro□$_r$□ = 20h and speed 22 knots, the minimum sailing time is vo□$_r$□ = d_r / 22. To maintain weekly frequency, the minimum number of vessels required is:

$$T_r = [\frac{\sum_1^{I_r} ro_{trn} + vo_{trn}}{168}] \tag{1}$$

where 168 is the number of hours in a week.

For deployment plan , the number of round trips each vessel completes on route r is:

$$RT_{rm} = \frac{w}{N_m} \tag{2}$$

where W is the total number of weeks in the planning horizon, and N□ is the number of vessels required in plan m.

(2) Fuel Cost

The fuel cost incurred by a type-k vessel sailing on route r under deployment plan m is distance-dependent and given by:

$$c_{f,k}^m = \rho * F_k(d_r, vo_{trn}) \tag{3}$$

Where:

$$F_k(d_r, vo_{trn}) = vo_{trn} * A_k * (\frac{d_r}{vo_{trn}})^{B_k} \tag{4}$$

Here, ρ = 500 is the unit fuel cost, and A□ , B□ are vessel-specific fuel consumption coefficients.

Hence, the following model is proposed to address the liner shipping capacity scheduling problem:

$$\max(Q(x,\rho) - \sum_{k \in K} \sum_{m \in M_k} (c_{f,k}^m + c_{o,k}^m)\mu_{k,m}) \tag{5}$$

$$s.t \quad \mu_{\mathrm{k}} = \sum_{i \in \{1,2 \cdots \mathrm{N}_m\}} x_{k,m,i} \quad \forall k \in K_m, m \in M \tag{6}$$

$$\mu_{\mathrm{k}} \le \kappa_{k,m} \quad \forall k \in K_m, m \in M \tag{7}$$

$$\sum_{k \in K_m} x_{k,m,i} = z_{mr} \quad \forall \mathrm{r} \in \mathrm{R}, \mathrm{m} \in M_{\mathrm{r}}, \ \mathrm{i} \in \{1, \cdots \mathrm{N}_m\} \tag{8}$$

$$\sum_{m \in M_r} z_{mr} = 1 \quad \forall \mathrm{r} \in R \tag{9}$$

$$x_{k,m,i} = \begin{cases} 1, & if \ type-kvessel \ assigned \ to \ departure \ i \\ 0, & otherwise \end{cases} \tag{10}$$

$$z_{m,r} = \begin{cases} 1, & if \ routerselects \ plan \ m \\ 0, & otherwise \end{cases} \tag{11}$$

$$\mu_{mr} = \{0,1,2,\cdots,\mu_{mr}\} \tag{12}$$

$$x_{kmi} \in \{0,1\} \tag{13}$$

$$z_{mr} \in \{0,1\} \tag{14}$$

Second Stage：

$$Q(x,\Gamma) = \min_{q\in\Omega} R(x,q_c,q_s) \tag{15}$$

With respect to spot transport demand:

$$U^s(\Gamma) = \left\{ q^s_{o,d,w} \in \mathbb{R} \mid q^s_{o,d,w} = \bar{q}^s_{o,d,w} + f^s_{o,d,w}\hat{q}^s_{o,d,w}, (o,d)\in D, w\in W_{o,d}, f^s \in F^s(\Gamma) \right\} \tag{16}$$

$$F^s(\Gamma) = \{ f^s \in \mathbb{R} \mid \sum_{(o,d)\in D}\sum_{w\in W_{o,d}} \left| f^s_{o,d,w} \right| \le \Gamma, -1 \le f^s_{o,d,w} \le 1, (o,d)\in D, w\in W_{o,d} \} \tag{17}$$

In the second stage：

$$LP \;\; R(x,q) = \max \sum_{(o,d)\in L}\sum_{w\in W}\sum_{p\in P_{o,d,w}} [(\gamma^c_{o,d} - c_p) y^c_{p,w,o,d} + (\gamma^s_{o,d} - c_p) y^s_{p,w,o,d} - \delta] \tag{18}$$

$$c_p = c_{p1} + c_{p2} \tag{19}$$

$$\delta = c^{c,wod}_{\omega} \bar{q}^c_{p,w,o,d} + c^{s,wod}_{\omega} \bar{q}^s_{p,w,o,d} \tag{20}$$

$$s.t. \sum_{(o,d)\in L}\sum_{w\in W_{od}}\sum_{p\in P_{o,d,w}} B^{l,\tau,m}_{p,o,w,d} y_{p,o,d,w} \le \sum_{k\in K_m} V_{k,m,j} \tag{21}$$

$$\sum_{p\in P_{o,d,w}} y^c_{p,o,d,w} \le q^c_{o,d,w} \tag{22}$$

$$\sum_{p\in P_{o,d,w}} y^m_{p,o,d,w} \le q^m_{o,d,w} \tag{23}$$

$$\sum\nolimits_{p\in P} y^c_{p,o,d,w} \ge \alpha q^c_{o,d,w} \tag{24}$$

$$B^{l,\tau,m}_{p,o,w,d} = \begin{cases} 1 & , \; if \;\; transport \;\; containers \;\; via \;\; path \;\; p \\ 0 & , \;\; otherwise \end{cases} \tag{25}$$

$$y_{p,o,d,w} \ge 0 \tag{26}$$

Eq. (5): Maximize profit; Eq. (6): Total deployed type-k vessels equals sum across plans; Eq. (7): Deployment ⩽ available fleet per vessel type; Eq. (8): One vessel type per sequence position; Eq. (9): One deployment plan per route; Eq. (10): Total deployed vessels ⩽ fleet size; Eq. (11)-(14): Variable constraint; Eq. (15): Worst-case profit in the second stage; Eq. (16)-(17): Generation of uncertain spot demand; Eq. (18): Second-stage objective—includes freight rates, transport cost and penalty; Eq. (19): Unit transport cost; Eq. (20): Penalty for unmet demand; Eq. (21): Shipment per sailing ⩽ vessel capacity; Eq. (22)-(23): Actual fulfilled demand ⩽ total demand; Eq. (24): Minimum contract demand fulfillment ratio; Eq. (25): Binary variable—vessel uses path p; Eq. (26): Non-negativity constraints.

4.1 Algorithm Design

Considering the two-stage robust optimization framework, a GA-C&CG algorithm is designed to solve the model. The original problem is decomposed into a master problem (first stage) and a sub problem (second stage). Let $\lambda^{c,1}_{o,d,w} \ge 0$, $\lambda^{s,1}_{o,d,w} \ge 0$, $\lambda^{c,1}_{o,d,w} \ge 0$, $\lambda^{cap} \ge 0$ be the dual variables of constraints (17)–(20), respectively. The dual of the sub problem is then formulated as:

$$\begin{aligned} &\min_{f,\lambda} \sum_{(o,d)\in D}\sum_{w\in W_{od}} (\lambda^{s,1}_{o,d,w} + c^{s,wod}_w) f^s_{odw}\hat{q}^s_{odw} + \sum_{o,d,w} (\lambda^{c,1}_{odw}\bar{q}^c_{odw} + \lambda^{s,1}_{odw}\bar{q}^s_{odw}) + \\ &\sum_{o,d,w} (c^{c,wod}_w \bar{q}^c_{odw} + c^{s,wod}_w \bar{q}^s_{odw}) \end{aligned} \tag{27}$$

s.t.

$$\gamma_{od}^{c} - c_p + c_w^{c,wod} \le \lambda_{odw}^{c,1} + \lambda_{odw}^{c,2} + \lambda^{cap} \tag{28}$$

$$\gamma_{od}^{s} - c_p + c_w^{s,wod} \le \lambda_{odw}^{s,1} + \lambda^{cap} \tag{29}$$

$$f_{odw}^{s} = f_{odw}^{+} - f_{odw}^{-} \tag{30}$$

$$\sum_{o,d,w} (f_{odw}^{+} + f_{odw}^{-}) \le \Gamma \tag{31}$$

$$f_{odw}^{+}, f_{odw}^{-} \in [0,1] \tag{32}$$

$$\lambda_{odw}^{c,1}, \lambda_{odw}^{s,1}, \lambda_{odw}^{c,2}, \lambda^{cap} \ge 0 \tag{33}$$

The procedure of the algorithm is as follows:

Step 1: Initialize the lower bound LB = $-\infty$ and upper bound UB = $+\infty$and the scenario set.Generate an initial deployment plan(x^0,z^0).as the starting point of the first stage. Set the iteration counter t=0,Solve the subproblem to identify the worst-case scenario within the polyhedral uncertainty set.

Step 2: Solve the master problem $\max_{x,z,\wp} \wp\text{-}\sum_{k,m}(c_{f,k}^{m} + c_{p,k}^{m})\mu_{k,m}$ to obtain the current deployment plan (x^t , z^t), and the corresponding lower bound profit $\wp^t$, let $LB = \wp^t$;

Step 3: Solve the sub problem based on the deployment plan (x^t , z^t), and identify the worst-case scenario f_{t+1}, with its corresponding profit. Let $UB = \min(UB, worst\_profit(x_t, z_t))$;

Step 4: Check for convergence, if $UB - LB \le \varepsilon$ terminate and output the optimal deployment and profit; otherwise, use GA to generate a new deployment plan and return to Step 2, If （x^t,z^t）=（x^{t+1},z^{t+1}）a new deployment plan is generated with the assistance of the GA, and the procedure returns to Step 2.

V. NUMERICAL EXAMPLE ANALYSIS

5.1 Numerical Example

In order to validate the effectiveness of the proposed model and algorithm, numerical experiments are carried out. The planning horizon is defined as 13 weeks, corresponding to one fiscal quarter. A shipping network is constructed based on two CMA CGM-operated liner services. Route 1 covers the ports of Tanger Med, Casablanca, Agadir, Rotterdam, Hamburg, Antwerp, and Le Havre; Route 2 includes Bremerhaven, Rotterdam, Antwerp, Montreal, and Halifax. Each vessel departs from the initial port of its assigned route, sequentially visits all ports along the route, and returns to the origin port upon completing the loop. Let D denote the set of distances (in nautical miles) between all origin-destination (o-d) port pairs for the two selected routes. The set of distances (in nautical miles) between consecutive ports on Route 1:

$$D(1) = \{215, 445, 2005, 2295, 2655, 2895, 230, 1790, 2080, 2440, 2680, 1560, 1850, 2210, 2450, 290, 650, 890, 360, 600, 240, 2895\}$$

The set of distances (in nautical miles) between consecutive ports on Route 2:

$$D(2) = \{230, 330, 3600, 4400, 100, 3370, 4170, 3270, 4070, 800, 4400\}$$

The vessel-related parameters are shown in Table2:

Table 2 The vessel-related parameters.

Vessel Type	Vessel Capacity	Number of Vessels Held	Fixed Operating Cost (USD)	Bk	Ak	Maximum Operating Speed (knots)
k=1	1500	5	55800	1	7	22
k=2	3000	5	82400	1	10	22
k=3	5000	5	122500	1	15	22
k=4	10000	5	183000	1	25	22
k=5	13000	5	252,000	1	35	22
k=6	18000	5	286700	1	40	22

The unit penalty cost for unmet contractual demand is defined as $c_w^{c,o,d} = \psi * \bar{\eta}_{w,o,d}$, where $\bar{\eta}_{w,o,d}$ denotes the average spot-to-contract freight rate ratio, and ψ is set to 0.2, The unit opportunity cost for unmet spot demand is given by $c_w^{s,o,d} = \gamma_{o,d}^s$ .The contract freight rate is assumed proportional to the distance between origin and destination, denoted do,d, Let $\gamma_{o,d}^c$ be the unit contract freight rate (USD per nautical mile per TEU), so the total contract freight rate is $\gamma_{od}^c = d_{od} * \gamma_{od}^c$, Let $\gamma_{od}^c = 3$ .Spot freight rates are higher; the parameter $\eta_{w,o,d}$ represents the ratio between spot and contract freight rates, and thus $\gamma_{od}^s = \eta_{w,o,d} * \gamma_{od}^c$, To capture temporal fluctuations, $\eta_{w,o,d}$ is assumed to follow a uniform distribution over [2, 4].

Contract demand is stable and predictable, with weekly accumulated demand for each (o-d) pair following a normal distribution with mean 800 TEU and standard deviation 200 TEU. Spot demand is volatile and hard to predict, modeled using a budgeted uncertainty set. The nominal spot demand for each (o-d) pair follows a normal distribution with mean 1000 TEU and standard deviation 300 TEU. Maximum deviation bounds individual fluctuations, while the uncertainty budget Γ = {0, 15, 30, 45, 60, 66} controls overall uncertainty level, reflecting the decision-maker's risk aversion. Let $\partial \in [0,1]$ denote the required service level for contract demand.

5.1 Simulation Results Analysis

Given a medium-level setting with uncertainty budget Γ = 30, market volatility ξ = 0.5, and contract demand fulfillment rate α = 0.5, the planning horizon spans 13 weeks. The algorithm runs for 1000 iterations with a convergence threshold of 1×10^{-5} . The optimal vessel deployment sequence for Route 1 is:2 → 2 → 2 → 4 → 5 → 6 → 1 → 4 → 5 → 5 → 2 → 4 → 3; for Route 2:1 → 6 → 2 → 2 → 6 → 1 → 1 → 4 → 3 → 2 → 6 → 4 → 5. The total profit over the 13-week horizon is 2.25×10^8 USD.

(1) Sensitivity Analysis of Uncertainty Budget

To further investigate the impact of decision-makers'risk aversion and market volatility on the two-stage liner capacity deployment and transportation problem, we examine the variation in total profit under different uncertainty budgets Γ ∈ {0, 15, 30, 45, 60, 66} and market volatility levels ξ ∈ {0.1, 0.3, 0.5, 0.7}, while fixing the contract demand fulfillment rate at α = 0.5. The solution algorithm developed in the previous section is applied to solve each scenario. The resulting changes in total profit with respect to uncertainty budgets under different volatility levels are illustrated in the figure 1 below.

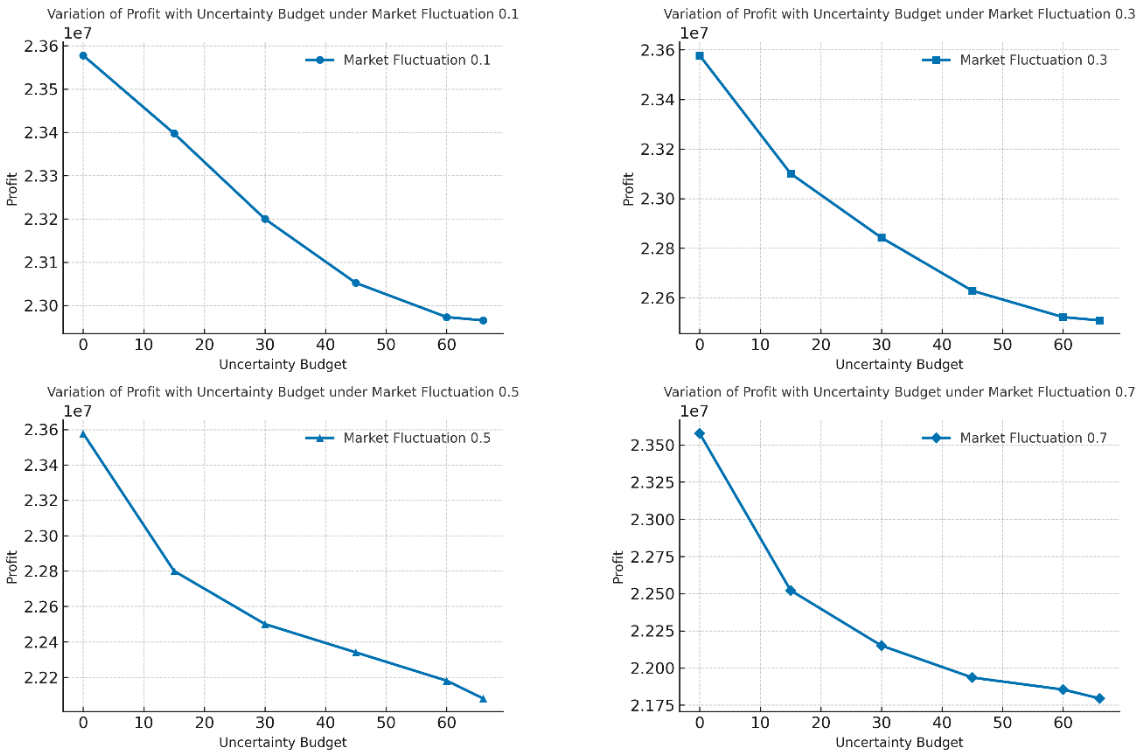

Fig. 1. Total profit variation with uncertain budget under market fluctuations

As the uncertainty budget increases, the total profit over the planning horizon gradually decreases. When the budget is zero, the model becomes deterministic, reflecting minimal risk aversion and maximum economic efficiency, as no resources are reserved for future uncertainties. As the budget grows, more capacity is allocated to hedge against risk, reducing the resources available for actual transport and thus lowering profits.

In robust optimization, a larger uncertainty budget enhances system robustness. At low budget levels, the system is highly sensitive to demand fluctuations, and robustness improves rapidly. At higher levels, sensitivity decreases, and the decline in profit slows, achieving a balance between robustness and economic performance.

(2) Sensitivity Analysis of Market Volatility

To examine the impact of market volatility on liner capacity planning, total profit is evaluated under uncertainty budgets $\Gamma = \{0, 15, 30, 45, 60, 66\}$ with a contract fulfillment rate of 0.5 and volatility levels $\xi = \{0.1, 0.5, 0.7\}$, using the algorithm from the previous section. When $\Gamma = 0$, the model is deterministic, and total profit remains unchanged across volatility levels. The profit trends are shown in Figure 2.

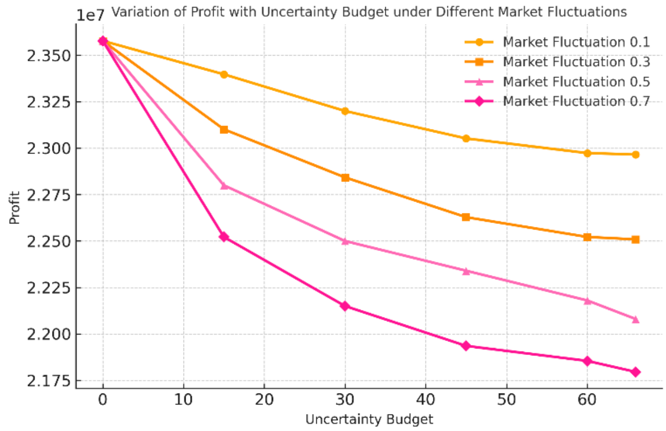

Fig. 2. Profit variation trend under different market fluctuations

As market volatility increases, the total profit over the 13-week planning horizon shows a decreasing trend. Horizontally, higher levels of volatility lead to a gradual and more pronounced decline in profit. Vertically, under fixed volatility levels, increasing the uncertainty budget enhances model robustness, reducing the negative impact of uncertainty on profit. This reflects improved solution stability, where profit loss incurred to hedge against disruptions tends to stabilize after a certain threshold.

(3) Sensitivity Analysis of Contract Demand Fulfillment Rate

To further examine the impact of contract demand fulfillment rate on the liner capacity deployment and transportation problem, total profit is evaluated as the fulfillment rate varies from 0 to 1, under an uncertainty budget of $\Gamma = 30$ and market volatility level $\xi = 0.5$. The solution is obtained using the algorithm proposed in the previous section. The results are shown in Figure 3.

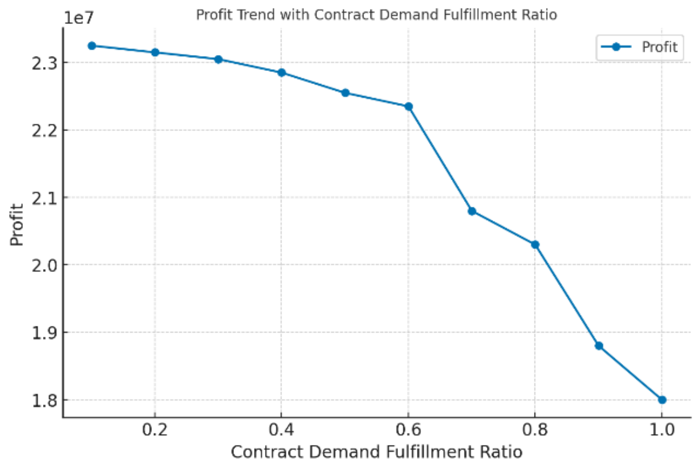

Fig. 3. Profit variation trend with contract demand fulfillment rate

Transportation profit decreases as the required service level increases. However, when the contract fulfillment rate is within the range of [0.1, 0.6], the reduction in profit is minimal. This indicates that a relatively high service level can be achieved with only a small sacrifice in profit within this range.

(4) Sensitivity Analysis of the Ratio Between Spot Freight Rate and Contract Freight Rate

To further investigate the impact of the ratio between spot and contract freight rates on the two-stage liner capacity deployment and transportation problem, total vessel deployment is analyzed under an uncertainty budget of $\Gamma = 15$ and a contract fulfillment rate of $\alpha = 0.5$. The ratio is varied from 2 to 4, and the deployment distribution of different vessel types is shown in Figure 4.

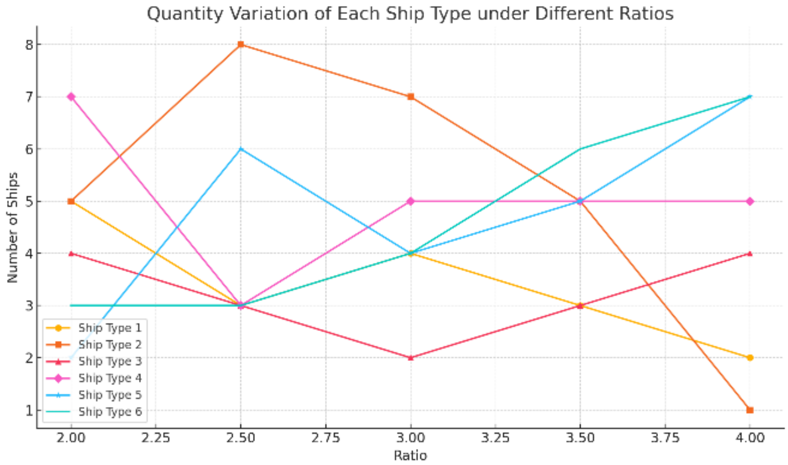

Fig. 4. Variation in deployment of vessel types under different ratios

The results show that as the spot freight rate increases, the fleet size gradually expands. Shipping companies tend to deploy larger vessels to accommodate more transport demand. This indicates that a favorable market environment encourages carriers to allocate larger ships to capture greater market share.

As the uncertainty budget increases from 15 to 45 and 60, the vessel deployment results are shown in Table 3. The results indicate that with higher uncertainty budgets, shipping companies adopt more conservative strategies, anticipating greater deviations below expected demand levels. Consequently, their decisions tend to favor deploying a larger number of smaller-capacity vessels.

Table 3 Fleet Deployment Plan

Γ	ratio	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6
45	4	5	6	3	5	3	4
60	4	5	6	3	4	6	2

VI. Conclusion

This chapter begins with a description of the two-stage liner shipping capacity deployment and transportation problem. Based on the characteristics of the problem, a two-stage robust optimization model is formulated with the objective of maximizing total profit over the planning period. A hybrid Genetic Algorithm and Column-and-Constraint Generation (GA-C&CG) method is then designed to solve the model. Subsequently, simulation experiments are conducted based on real-world operations of shipping companies to validate the effectiveness of the proposed model and solution approach. Finally, a sensitivity analysis is performed on three key factors: uncertainty budget, contract demand fulfillment ratio, and the ratio of spot to contract freight rates. The results show that the uncertainty budget not only reflects demand fluctuations but also represents a trade-off between economic efficiency and robustness. Moderate increases in the contract demand fulfillment ratio have limited impact on profits, suggesting that maintaining a higher fulfillment rate can help retain long-term customers. Moreover, as the ratio of spot to contract freight rates increases, shipping companies tend to deploy larger vessels to accommodate more spot demand while ensuring contract commitments, thereby enhancing overall profitability.

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