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A Solution Procedure to Solve Multi objective Fractional Transportation Problem

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Abstract: In decision making process if the objective function is ratio of two linear functions and objective function is to be optimized. For example one may be interested to know the ratio of total cost to total time required for transportation. This ratio is an objective function which is fractional objective function. When there are several such fractional objectives to be optimized simultaneously then the problem becomes multi objective fractional programming problem (MOFLPP). Initially Hungarian mathematician BelaMartos constructed such type of problem and named it as hyperbolic programming problem. Same problem in general referred as Linear Fractional Programming Problem. Fractional programming problem can be converted into linear programming problem (LPP) by using variable transformation given by Charnes and Cooper. Then it can be solved by Simplex Method for Linear Programming Problem. .

In this paper we propose to solve multi objective fractional transportation problem. Initially will solve each of the transportation problem as single objective and then using Taylor series approach expand each of the problem about its optimal solution and ignoring second and higher order error terms each of the objective is converted into linear one. Then the problem reduces to MOLTPP. Evaluate each of the objectives at every optimal solution and obtain evaluation matrix. Define hyperbolic membership function using best and worst values of objective function with reference to evaluation matrix. These membership functions are fuzzy functions Compromise solution is obtained using weighted a.m. of hyperbolic membership functions and also weights quadratic mean of hyperbolic membership functions. Propose o solve problem at the end to explain the procedure.

Keywords: Multi objective, compromise solution, hyperbolic membership function.

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I. INTRODUCTION

It is well known fact that the transportation problem is special case of linear programming problem. In same way multi objective fractional transportation programming problem is special case of multi objective fractional programming problem. In this paper we will deal with multi objective fractional transportation programming problem. The problem is to transfer goods from different origins to several destinations such that many fractional objective functions attain their optimal values. Again optimizing several objectives simultaneously is not possible. Thus the golden mean is to find compromise solution. For compromise solution there are several methods which are proved to be efficient. In next section linear fractional transportation problem (LFTP) is introduced.

1. Formulation of Linear Fractional TP.(LFTP)

In general a problem is specified as under

i) There are m supply points from which goods are to be transported. Supply available at i^{th}point is at most $a_i fori = 1,2,\dots,m$
ii) There are n demand centres where good are required. Minimum requirement of j^{th} demand centre is $b_j forj = 1,2,\dots,n.$
iii) Profit matrix is $P = \|p_{ij}\| ism \times n$ matrix where p_{ij} is profit of transporting one unit from i^{th} origin to j^{th} destination.
iv) Cost matrix is $D = \|d_{ij}\| ism \times n$, matrix where d_{ij} is cost of transporting one unit from i^{th} origin to j^{th} destination
v) p_0 and d_0 be the fixed profit and cost respectively
vi) Suppose variablex_{ij} denotes number units to be transported from i^{th} origin to j^{th} destination.

The objective function is,

$$Maximize\ or\ \ MinimizeQ(x) = \frac{P(x)}{D(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}\ x_{ij}\ + p_0}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}\ x_{ij}\ + d_0} \qquad (1)$$

$Subject\ \ to\ \ constraints,$

$$\sum_{j=1}^{n} x_{ij}\ \le a_i fori = 1,2,\dots,m \qquad (2)$$

$$\sum_{i=1}^{m} x_{ij}\ \le b_j forj = 1,2,\dots,n \qquad (3)$$

$$x_{ij} \ge 0\ \ fori = 1,2,\dots,mandj = 1,2,\dots.,n \qquad (4)$$

$Q(x)$is called objective function which is fractional. Note that $P(x)$ and $D(x)$ are linear and (2) ,(3) and (4) are also linear. Hence the name of the problem is linear fractional transportation Programming problem (LFTPP).
Here $D(x) > 0$, for every $X = (x_{ij}) where X \in S$, Where S is feasible set defined by constraint (2),(3) and (4).
Lastly we assume that$a_i > 0$ and $b_j > 0$ for $i = 1,2, \dots, m$ and $j = 1,2, \dots., n$
Total supply is not less than total demand i.e. $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$ (5)
The problem is to find$X = (x_{ij}) where X \in S and$ satisfying (2-4) and having optimal value of (1).

This LFTPP has following properties,
1. The problem has a feasible solution, i.e. feasible set $S \neq \emptyset$.It means that solution set is non empty.
2. The set of feasible solutions is bounded.
3. The problem is always solvable.

2. Definitions.

i) If total demand equals to total supply, i.e.

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad (6)$$

Then LFTP is said to be balanced transportation problem.
Non negativity condition (4) and demand and supply conditions (2-3) at the most gives following bound to decision variables.
$0 \leq x_{ij} \leq \min_{i,j}\{a_i, b_j\}$ for $i = 1,2, \dots, m$ and $j = 1,2, \dots, n$.

ii) LFTP $Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij} x_{ij} + p_0}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} x_{ij} + d_0}$ (7)

is said to be in canonical form if

$$\sum_{j=1}^{n} x_{ij} = a_i for i = 1,2, \dots, m \qquad (8)$$

$$\sum_{i=1}^{m} x_{ij} = b_j for j = 1,2, \dots, \qquad (9)$$

$$x_{ij} \geq 0 \quad for i = 1,2, \dots, m and j = 1,2, \dots., n \qquad (10)$$

iii) There is exactly one equality redundant constraint in (8) and (9). When any one of the constraint in (8) and (9) are dropped, what remains is exactly $(m + n - 1)$linearly independent equations.
iv) A solution of LFTPP is said to be basic solution if it satisfies conditions (8) and (9).In basic solution there are $(m + n - 1$ non zero values of variables in basis.
v) A basic solution of the LFTPP is said to basic feasible (BFS) if all values in basis are non-negative i.e. it satisfies condition (10).
vi) The basic solution is degenerate if at least one of its basic variable equal to zero. Otherwise it is said to be non-degenerate.

3. Theorems

i) The linear fractional transportation problem is solvable if and only if itis balanced LFTPP.

The problem is balanced means $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$
ii) If all a_i and b_j in LFTPP are positive integers, then every basic solution of LFTPP is integer vector.
Hence if all $a_i and b_j$ of LFTPP are positive integers and the problem is in the canonical form then LFTPP has an optimal solution $\boldsymbol{x^*}$ with all elements as positive integers.

4. Multi Objective Linear Fractional Transportation Programming Problem.

Suppose $Q_1(x) = \frac{P_1(x)}{D_1(x)}, Q_2(x) = \frac{P_2(x)}{D_2(x)}, \dots., Q_k(x) = \frac{P_k(x)}{D_k(x)}$

are$k$ linear fractional transportation problems such that all objective functions to be optimised simultaneously subject to constraint (2) ,(3),(4) and (5) and $D_k(x) \geq 0$ for every k then the problem is called Multi Objective Linear Fractional Transportation Programming Problem (**MOLFTPP**). Where

$$Q_l(x) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^{l} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{'} x_{ij}} \quad for\ l = 1,2, \dots k$$

p_{ij}^{l} is profit of transporting one unit from i^{th} origin to j^{th} destination in case of l^{th} objective function.
d_{ij}^{l} is cost of transporting one unit from i^{th} origin to j^{th} destinationin case of l^{th} objective function.

5. Algorithm To Solve MOLFTPP

i) Consider MOLFTPP, $Maximize \{ Q_1(x), Q_2(x), \dots, Q_k(x)\}$ subject to constraint (8-10).
ii) Find optimal solution of each of the linear fractional transportation problem as single objective function subject to constraint. LINGO software can be used to find optimal solution of each of the fractional transportation problem.
iii) Suppose X_l^* is optimal solution of $Q_l(x) for l = 1,2,\dots,k$
iv) Expand objective function $Q_l(x)$ about $X_l^* = \{x_{ij}\}$ using Taylor's theorem and ignoring second and higher order terms convert $Q_l(x)$ into linear function.

Consider $Q_l(x) = \frac{P_l(x)}{D_l(x)}$ and X_l^* be the optimal solution of $Q_l(x)$, then using Taylor Series approach

differentiate $Q_l(x)$ with respect to each of the variable partially.

$$\widetilde{Q_l(x)} \cong Q_l(X_l^*) + (x_{11} - x_{11l})\left\{\frac{\partial Q_l(X_l)}{\partial x_{11}}\right\}_{at X_i^*} + (x_{12} - x_{12l})\left\{\frac{\partial Q_l(X_l)}{\partial x_{12}}\right\}_{at X_i^*} + \cdots + (x_{mn} - x_{mnl})\left\{\frac{\partial Q_l(X_l)}{\partial x_{mn}}\right\}_{at X_i^*} + O(h^2)$$

$Where O(h^2)$ Represents second and higher order terms of the expansion which we will ignore. Using this expansion each of the objective function becomes linear function. To avoid complexity of notations we write, $\widetilde{Q_l(x)} = Z_l(x) for l = 1,2,\dots,k$

The problem is to $Maximize \{Z_1(x), Z_2(x), \dots, Z_k(x)\}$ subject to (7-10) i.e.Multi Objective Linear Transportation Programming Problem. (MOLTPP).It can be solved by using fuzzy compromise approach as stated in next section.

v) Fuzzy Compromise approach for MOLTPP.

Consider MOLTPP

$Maximize \{Z_1(x), Z_2(x), \dots, Z_k(x)\}$ ---------------- (11)

Subject to (7-10) and $X \in S$, where S is set of feasible solutions.

Solution to (11) is often conflicting as several objectives cannot be optimized simultaneously. To find compromise solutions first solve each of the objective function as marginal or single objective function. Here we have converted Multi-objective Linear Fractional Transportations Programming problem to Multi-objective Linear Programming problem using Taylor series approach. Thus each of the MOLTPP can be solved using any one of the techniques of LPP. Once we solve each of the problems individually then evaluate each of the objective function at optimal solutions of all the objectives. Suppose X_l^* is optimal solution of l^{th} objective function. Find values of each objective at optimal solution of l^{th} objective function for all $l = 1,2,\dots,k$.Thus we have evaluation matrix of order $(k \times k)$ as given below.

$$\begin{bmatrix} Z_1(x_1^*) & Z_1(x_2^*) \dots\dots\dots & Z_1(x_k^*) \\ & . & \\ Z_2(x_1^*) & Z_2(x_2^*) \dots\dots\dots & Z_2(x_k^*) \\ & \vdots & \\ Z_k(x_1^*) & Z_k(x_2^*) \dots\dots\dots & Z_k(x_k^*) \end{bmatrix}$$

6. Fuzzy Programming Approach For Compromise Solution Of MOLTPP

Define Hyperbolic Membership function for each of the linearized objective function as under,.

Consider multi objective linear transportation problem.

$Maximize\ Z(x) = [Z_1(x), Z_2(x), \dots, Z_k(x)]'$

$$i.e. Maximize\ Z_k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij}, \quad k = 1,2,\dots,K$$

$Subject\ to$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,\dots,m, \sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\dots,n$$

$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ And $x_{ij} \geq 0$ $for\ all\ i\ and\ j$

Subject to $x \in \boldsymbol{X}$

Where $\boldsymbol{X}$ set of feasible solutions.

Solve each of the objectives as single objective function and then find evaluation matrix. Find best possible value of each of the objective and worst possible value for each of the objective. Define these values as under,

$U_k = Max\ Z_k(x)\ , k = 1,2,\dots,K\ , L_k = Min\ Z_k(x)\ , k = 1,2,\dots,K$

Hyperbolic membership function for k^{th} objective function is defined as under, $\varphi_k : X \to [0,1]$. As given below,

$$(\varphi_k(x)) = \begin{cases} 1 & if\ Z_k(x) \leq L_k \\ \frac{1}{2}\tanh\left(\left(\frac{U_k + L_k}{2} - Z_k(x)\right)\alpha_k\right) + \frac{1}{2} & if\ L_k \leq Z_k(x) \leq U_k \\ 0 & if\ Z_k(x) \geq U_k \end{cases}$$

Where $\alpha_k = \frac{6}{U_k - L_k}$

7. Compromise Solution of MLOFTPP

a) Aggregation operator as weighted arithmetic mean of hyperbolic membership functions. Thus the MOLFPP becomes single objective nonlinear transportation problem.

$Maximize\ \ \mu(x) = \sum_{i=1}^{k} \varphi_i\, w_i$

$Subject\ to\ given\ set\ of\ constraints$

b) Aggregation operator as weighted quadratic mean of hyperbolic membership functions. Thus the MOLFPP becomes single objective nonlinear transportation problem.

$$Maximize\ \mu\,(x) = \left(\sum_{i=1}^{k} \varphi_i^2\, w_i\right)^{1/2} = \sqrt{\sum_{i=1}^{k} \varphi_i^2\, w_i}$$

$$Subject\ to\ given\ set\ of\ constraints$$

Thus to find compromise solution is nothing but single objective nonlinear programming problem. In this method objective function is nonlinear and constraints are linear. Hence the problem is solved by LINGO techniques of nonlinear optimization.

II. CONCLUSION

Above method dives efficient solutions of multi objective fractional transportation problem using fuzzy programming approach.

Example: Consider following two objective fractional transportation problem,

$$Maximize\quad Q_1(x) = \frac{10x_{11} + 14x_{12} + 8x_{13} + 12x_{14} + 8x_{21} + 12x_{22} + 14x_{23} + 8x_{24} + 9x_{31} + 6x_{32} + 15x_{33} + 9x_{34}}{15x_{11} + 12x_{12} + 16x_{13} + 8x_{14} + 10x_{21} + 6x_{22} + 13x_{23} + 12x_{24} + 13x_{31} + 15x_{32} + 12x_{33} + 10x_{34}}$$

$$Maximize\quad Q_2(x) = \frac{14x_{11} + 9x_{12} + 11x_{13} + 9x_{14} + 12x_{21} + 9x_{22} + 6x_{23} + 15x_{24} + 6x_{31} + 9x_{32} + 12x_{33} + 10x_{34}}{12x_{11} + 14x_{12} + 7x_{13} + 17x_{14} + 6x_{21} + 11x_{22} + 13x_{23} + 10x_{24} + 9x_{31} + 15x_{32} + 12x_{33} + 16x_{34}}$$

$Subjectto$

$x_{11} + x_{12} + x_{13} + x_{14} = 15, \quad x_{21} + x_{22} + x_{23} + x_{24} = 25$

$x_{31} + x_{32} + x_{33} + x_{34} = 20, \quad x_{11} + x_{21} + x_{31} = 15$

$x_{12} + x_{22} + x_{32} = 25\ , \quad x_{13} + x_{23} + x_{33} = 5\ , \ x_{14} + x_{24} + x_{34} = 15$

$,\ x_{ij} \geq 0\ fori = 1,2,3\ andj = 1,2,3,4$

a) **Step1**: A LINGO code to solve $Q_1(x)$ is as under,

```
MAX
= (10*X11+14*X12+8*X13+12*X14+8*X21+12*X22+14*X23+8*X24+
9*X31+6*X32+15*X33+9*X34)/(15*X11+12*X12+16*X13+8*X14+10*X21+6*X22+13*X23+12*X24+13*X31+15*X3
2+12*X33+10*X34);
X11+X12+X13+X14=15; X21+X22+X23+X24=25;
X31+X32+X33+X34=20; X11+X21+X31=15; X12+X22+X32=25;
X13+X23+X33=5;X14+X24+X34=15;
X11>=0; X12>=0;X13>=0;X14>=0;X21>=0; X22>=0;X23>=0;X24>=0;
X31>=0;X32>=0;X33>=0;X34>=0;
```

Optimal Solution of $Q_1(x)$ is,

$X_1^* = \{x_{14} = 15, x_{22} = 25\ , x_{31} = 15, , x_{33} = 5, \}\ andQ_1(X_1^*) = 1.314286$

Similarly Optimal Solution of $Q_2(x)$ is,

$X_2^* = \{x_{11} = 5, x_{12} = 5, x_{13} = 5, x_{21} = 10, x_{24} = 15, x_{32} = 20\}\ andQ_2(X_2^*) = 1.02996$

b) Following MATLAB program finds coefficients of x_{ij}'s to convert it into linear function using Taylor Series.

```
% A program to find coefficients of linear objective function.
P1 = [10;14;8;12;8;12;14;8;9;6;15;9];D1 = [15;12;16;8;10;6;13;12;13;15;12;10];
X1 = [0;0;0;15;0;25;0;0;15;0;5;0];
C1P = P1'*(D1'*X1);C1D = D1'*(P1'*X1);Z1 = (C1P-C1D)/((D1'*X1)^2);
P2 = [14;9;11;9;12;9;6;15;6;9;12;10];D2 = [12;14;7;17;6;11;13;10;9;15;14;16];
X2 = [5;5;5;0;10;0;0;15;0;20;0;0];
C2P = P2'*(D2'*X2);C2D = D2'*(P2'*X2);Z2 = (C2P -C2D)/((D2'*X2)^2);
U1 = (P1'*X1) /(D1'*X1); L1 = (P1'*X2)/(D1'*X2);
U2 = (P2'*X2)/(D2'*X2); L2 = (P2'*X1)/(D2'*X1);
```

c) **LinearObjective Functions**.

$Maximize\{Z_1(x)\ \ and\ \ Z_2(x)\}\ , Where,$

$$Z_1(x) = 1.3143 - 0.185x_{11} - 0.0034x_{12} - 0.0248x_{13} + 0.0028x_{14} - 0.0098x_{21} + 0.0078x_{22} - 0.0059x_{23} - 0.0148x_{24} - 0.0154x_{31} - 0.0261\,x_{32} - 0.0015x_{33} - 0.00079x_{34}$$

$$Z_2(x) = 1.0296 + 0.0024x_{11} - 0.0080x_{12} + 0.0056x_{13} - 0.0126x_{14} + 0.0086x_{21} - 0.0034x_{22} - 0.0109x_{23} + 0.0070x_{24} - 0.0048x_{31} - 0.0095\,x_{32} - 0.0036x_{33} - 0.0096x_{34}$$

$Subject\ \ to \qquad x_{11} + x_{12} + x_{13} + x_{14} = 15, x_{21} + x_{22} + x_{23} + x_{24} = 25$

$x_{31} + x_{32} + x_{33} + x_{34} = 20, x_{11} + x_{21} + x_{31} = 15, \ \ x_{12} + x_{22} + x_{32} = 25\ ,$

$x_{13} + x_{23} + x_{33} = 5\ , x_{14} + x_{24} + x_{34} = 15,$

$,x_{ij} \geq 0\ for\ \ i = 1,2,3\ and\ \ j = 1,2,3,4$

c) **Evaluation Matrix:**

$$\begin{bmatrix} \boldsymbol{Z_1(X_1^*)} & \boldsymbol{Z_1(X_2^*)} \\ \boldsymbol{Z_2(X_1^*)} & \boldsymbol{Z_2(X_2^*)} \end{bmatrix} = \begin{bmatrix} 1.3143 & 0.6038 \\ 0.6939 & 1.0296 \end{bmatrix}$$

Thus $U_1 = 1.3143\,, L_1 = 0.6038\,, U_2 = 1.0296\,, L_2 = 0.6939$

d) Hyperbolic Membership Function:

$$\varphi_k(x) = \frac{1}{2}\tanh\left(\left(\frac{U_k + L_k}{2} - Z_k(x)\right)\alpha_k\right) + \frac{1}{2}$$

Where $\alpha_k = \frac{6}{U_k - L_k}$, for $k=1,2$

$\frac{U_1+L_1}{2} = 0.9590$, $\alpha_1 = \frac{6}{U_1-L_1} = 8.44475$ $\frac{U_2+L_2}{2} = 0.86175$, $\alpha_2 = \frac{6}{U_2-L_2} = 17.8731$

$$\varphi_1(x) = \frac{1}{2}\tanh\left(\left(\frac{U_1+L_1}{2} - Z_1(x)\right)\alpha_1\right) + \frac{1}{2} = \frac{1}{2}\tanh\left(0.9590 - 8.4475\, Z_1(x)\right) + \frac{1}{2}$$

$$\varphi_2(x) = \frac{1}{2}\tanh\left(\left(\frac{U_2+L_2}{2} - Z_2(x)\right)\alpha_2\right) + \frac{1}{2} = \frac{1}{2}\tanh\left(0.86175 - 17.8731\, Z_2(x)\right) + \frac{1}{2}$$

e) Compromise Solution

i) Weighted A.M. mean of hyperbolic membership functions.

$Maximize\ \ \mu(x) = \sum_{i=1}^{k} \varphi_i\, w_i$ *Subject to given set of constraints*

```
MAX
=(0.5*@TANH(0.9590 -8.44475*(1.3143-0.185*X11-0.0034*X12
-0.0248*X13+0.0028*X14-0.0098*X21+0.0078*X22-0.0059*X23
-0.0148*X24-0.0154*X31-0.0261*X32-0.0015*X33-0.0079*X34))+0.5)*0.1+
(0.5*@TANH(0.86175-17.8731*(1.0296+0.0024*X11-0.0080*X12+0.0056*X13-
.0126*X14+0.0086*X21-0.0034*X22-0.0109*X23+0.007*X24-0.0048*X31
-0.0095*X32-0.0036*X33-0.0096*X34))+0.5)*0.9;
! ENTER DEMAND AND SUPPLY CONSTRAINTS;
X11+X12+X13+X14=15;X21+X22+X23+X24=25;X31+X32+X33+X34=20; X11+X21+X31=15;
X12+X22+X32=25;X13+X23+X33=5; X14+X24+X34=15;
! ENTER NON NEGATIVITY CONSTRAINTS;
X11>=0; X12>=0; X13>=0; X14>=0;X21>=0; X22>=0; X23>=0; X24>=0;
X31>=0; X32>=0; X33>=0; X34>=0;
@GIN(X11);@GIN(X12);@GIN(X13);@GIN(X14);
@GIN(X21);@GIN(X22);@GIN(X23);@GIN(X24);
@GIN(X31);@GIN(X32);@GIN(X33);@GIN(X34);
```

For different values of weights we have following solutions.

	X1*	**(0.2,0.6)**	**(0.3,0.7)**	**0.6,0.4)**	**(0.8,0.2)**	**(0.9,0.1)**	**X2***
X11	0	0	15	15	11	14	5
X12	0	0	0	0	0	0	5
X13	0	1	0	0	4	0	5
X14	15	14	0	0	0	1	0
X21	0	0	0	0	0	1	10
X23	25	23	5	6	10	5	0
X33	0	2	5	4	0	5	0
X24	0	0	15	15	15	14	15
X31	15	15	0	0	4	0	0
X32	0	2	20	19	15	20	20
X33	5	2	0	1	1	0	0
X34	0	1	0	0	0	0	0
Q1	**1.3143**	1.20871	0.65	0.66709	0.68997	0.65992	0.60377
Q2	0.70345	0.6831	0.92	0.93423	0.98701	0.90812	**1.0296**

ii) Weighted Quadratic Mean of hyperbolic membership functions.

$$Maximize\ \mu\,(x) = \left(\sum_{i=1}^{k} \varphi_i^2\, w_i\right)^{1/2} = \sqrt{\sum_{i=1}^{k} \varphi_i^2\, w_i}$$

Subject to given set of constraints

```
MAX
=@SQRT(((0.5*@TANH(0.9590 -8.44475*(1.3143-0.185*X11-0.0034*X12
-0.0248*X13+0.0028*X14-0.0098*X21+0.0078*X22-0.0059*X23
-0.0148*X24-0.0154*X31-0.0261*X32-0.0015*X33-0.0079*X34))+0.5)^2)*0.1+
((0.5*@TANH(0.86175-17.8731*(1.0296+0.0024*X11-0.0080*X12+0.0056*X13-
.0126*X14+0.0086*X21-0.0034*X22-0.0109*X23+0.007*X24-0.0048*X31
-0.0095*X32-0.0036*X33-0.0096*X34))+0.5)^2)*0.9);
! ENTER DEMAND AND SUPPLY CONSTRAINTS;
X11+X12+X13+X14=15;X21+X22+X23+X24=25;X31+X32+X33+X34=20;
 X11+X21+X31=15; X12+X22+X32=25;X13+X23+X33=5; X14+X24+X34=15;
! ENTER NON NEGATIVITY CONSTRAINTS;
X11>=0; X12>=0; X13>=0; X14>=0;X21>=0; X22>=0; X23>=0; X24>=0;
X31>=0; X32>=0; X33>=0; X34>=0;
@GIN(X11);@GIN(X12);@GIN(X13);@GIN(X14);
@GIN(X21);@GIN(X22);@GIN(X23);@GIN(X24);
```

@GIN(X31);@GIN(X32);@GIN(X33);@GIN(X34);

For different values of weights we have following solutions.

	X1*	**(0.1,0.9)**	**(0.2,0.8)**	**(0.4,0.6)**	**(0.6,0.4)**	**(0.9,0.1)**	**X2***
X11	0	0	15	12	15	14	5
X12	0	0	0	0	0	0	5
X13	0	2	0	3	0	0	5
X14	15	13	0	0	0	1	0
X21	0	0	0	0	0	1	10
X23	25	21	6	19	5	5	0
X33	0	2	4	2	5	5	0
X24	0	2	15	4	15	14	15
X31	15	15	0	3	0	0	0
X32	0	4	19	6	20	20	20
X33	5	1	1	0	0	0	0
X34	0	0	0	11	0	0	0
Q1	**1.3143**	1.1088	0.6671	0.9069	0.6500	0.6599	0.60377
Q2	0.70345	0.7065	0.9342	0.8540	0.9200	0.9081	**1.0296**

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