Active Control and Dynamical Analysis of two Coupled Parametrically Excited Van Der Pol Oscillators

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Abstract: The dynamical behavior of two coupled parametrically excited van der pol oscillator is investigated by using perturbation method. Resonance cases were obtained, the worst one has been chosen to be discussed. The stability of the obtained numerical solution is investigated using both phase plane methods and frequency response equations. Effect of the different parameters on the system behavior is studied numerically. Comparison between the approximate solution and numerical solution is obtained.

Keywords: Vibration control, nonlinear oscillation, perturbation technique, Resonance cases, Frequency response curves.

I. INTRODUCTION

Vibrations at most time are non-desirable, humans suffered from these bad vibrations, so it must be eliminating or at least controlled. The dynamic absorber is the most common methods for reduced the vibrations. Its importance tends to as it is need low cost, and it is a simple operation at one modal frequency. El-Badawy and Nayfeh [1] adopted linear velocity feedback and cubic velocity feedback control laws. Yang, Cao and Morris [2] use Mat lab for applying numerical methods. Amer [3] investigation the coupling of two nonlinear oscillators of the main system and absorber representing ultrasonic cutting process subjected to parametric excitation force. Non-linearities necessary introduce a whole range of phenomena that are not found in linear system [4], including jump phenomena, occurrence of multiple solutions, modulation, shift in natural frequencies, the generation of combination resonances, evidence of period multiplying bifurcations and chaotic motion [5-8]. In these systems the vibrations are needed to be controlled to minimize or eliminating the hazard of damage or destruction. There are two types for vibration control, active and passive control.


Sayed and Kamel [18-19] investigated the effect of different controllers on the vibrating system and saturation control of a linear absorber to reduce vibrations due to rotor blade flapping motion. Kamel et al [20] studied the vibration suppression in ultrasonic machining described by non-linear differential equations via passive controller. Elena et al. [21] studied the formal analysis and description of the steady-state behavior of an electrostatic vibration energy harvester operating in constant-charge mode and using different types of electromechanical transducers. Orhan and Peter [22] investigate the effect of excitation and damping parameters on the super harmonic and primary resonance responses of a slider cantilever beam undergoing flapping motion.

In this paper we studied vibration control of a nonlinear system under tuned excitation force. The method of multiple scale method is applied to obtain the approximate solution of the system. Vibration method is used to reduce the amplitude of vibration at the worst resonance case. The effect of different parameters are investigated, the comparison between the numerical solution and approximation solution obtained.

II. MATHEMATICAL MODELING

The considered system is described by the equations:

\[ \ddot{X} + (\omega_1^2 - 2\epsilon \Omega f_1 \cos(\Omega t))X - \epsilon(X^2 + Y^2)X + \epsilon(\mu_1 + X^2 + aY^2)\dot{X} = -\epsilon G X^3 \]  \hspace{1cm} (1)

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\[ \ddot{Y} + (\omega_2^2 - 2\varepsilon f_2 \cos(\Omega t))Y - \varepsilon(X^2 + Y^2)Y + \varepsilon(\mu_2 + bX^2 + Y^2)Y' = -\varepsilon G Y^3 \]  \hspace{1cm} (2)

where the dots indicate differentiation with respect to \( t \), \( X \) and \( Y \) are the generalized coordinate of the plant (main system) and the controller. \( \omega_1 \) and \( \omega_2 \) are incommensurate fundamental frequencies, the parametric excitation frequencies are \( \Omega_1 \) and \( \Omega_2 \), the constants \( a \) and \( b \) are of order 1, and \( \varepsilon \) is a small parameter, \( f_1, f_2 \) are the external excitation forces, \( G_1, G_2 \) are the controller of the main system and the absorber.

We can solve equations (1)&(2) analytically using multiple time scale perturbation technique as:

\[ X(t, \varepsilon) = x_1(T_1, T_1) + \varepsilon x_1(T_2, T_1) + \varepsilon^2 x_1(T_2, T_1) + O(\varepsilon^3) \]  \hspace{1cm} (3)

\[ Y(t, \varepsilon) = y_1(T_1, T_1) + \varepsilon y_1(T_2, T_1) + \varepsilon^2 y_1(T_2, T_1) + O(\varepsilon^3) \]  \hspace{1cm} (4)

where \( T_1 = 2\pi \) represents a fast time scale characterizing motions with the natural and excitation frequencies, and \( T_1 = \varepsilon t \) represents a slow time scales characterizing modulation and phases of both modes of vibration. The times derivatives transform are reacted in terms of the new time scales as:

\[ \frac{dx}{dt} = D_1 + \ldots \ldots \ldots \]  \hspace{1cm} (5)

\[ \frac{d^2x}{dt^2} = D_2^2 + 2\varepsilon D_2 D_1 + \varepsilon^2 D_1^2 + \ldots \ldots \ldots \]  \hspace{1cm} (6)

From equations (3) to (6) we have:

\[ \dot{X}(t; \varepsilon) = \sum_{n=0}^{2} \varepsilon^n (D_1 + \varepsilon D_1) x_n + O(\varepsilon^3) \]  \hspace{1cm} (7)

\[ \dot{X}(t; \varepsilon) = \sum_{n=0}^{2} \varepsilon^n (D_1^2 + 2\varepsilon D_2 D_1 + \varepsilon^2 D_1^2) x_n + O(\varepsilon^3) \]  \hspace{1cm} (8)

\[ \dot{Y}(t; \varepsilon) = \sum_{n=0}^{2} \varepsilon^n (D_1 + \varepsilon D_1) y_n + O(\varepsilon^3) \]  \hspace{1cm} (9)

\[ \dot{Y}(t; \varepsilon) = \sum_{n=0}^{2} \varepsilon^n (D_1^2 + 2\varepsilon D_2 D_1 + \varepsilon^2 D_1^2) y_n + O(\varepsilon^3) \]  \hspace{1cm} (10)

Substituting from equations (3), (4) and (7)-(10) into equations (1) and (2) and equating the same power of \( \varepsilon \) we have:

\[ O(\varepsilon^3) : (D_1^2 + \omega_1^2) x_1 = 0 \]  \hspace{1cm} (11)

\[ (D_1^2 + \omega_1^2) y_1 = 0 \]  \hspace{1cm} (12)

\[ O(\varepsilon) : (D_1^2 + \omega_1^2) x_1 = -2D_1D_1 x_1 + 2f_1 x_1 \cos(\Omega t) + x_1^3 + x_1 y_1^2 - \mu_1(D_1 x_1) \]  
\[ - x_1^2 (D_1 x_1) - ay_1^2 (D_1 x_1) + G_1(D_1 x_1)^3 \]  \hspace{1cm} (13)

\[ (D_1^2 + \omega_2^2) y_1 = -2D_1D_1 y_1 + 2f_2 y_1 \cos(\Omega t) + y_1^3 + x_1^2 y_1 - \mu_2(D_1 y_1) \]  
\[ - y_1^2 (D_1 y_1) - bx_1^2 (D_1 y_1) + G_1(D_1 y_1)^3 \]  \hspace{1cm} (14)

\[ O(\varepsilon^2) : (D_1^2 + \omega_1^2) x_2 = -D_1^2 x_1 - 2D_1D_1 y_1 + 2x_1 f_1 \cos(\Omega t) + 3x_1^2 x_1 + 2x_1 y_1 y_1 \]  
\[ + x_1^2 y_1^2 - \mu_1(D_1 x_1) - \mu_2(D_1 x_1) - x_1^2 (D_1 x_1) - x_1^2 (D_1 x_1) \]  
\[ - 2x_1 x_1(D_1 x_1) - ay_1^2 (D_1 x_1) - ay_1^2 (D_1 x_1) - 2 ay_1 y_1(D_1 x_1) \]  
\[ - 3G_1(D_1 x_1)^2 (D_1 x_1) - 3G_1(D_1 x_1)^2 (D_1 x_1) \]  \hspace{1cm} (15)

\[ (D_1^2 + \omega_2^2) y_2 = -D_1^2 y_1 - 2D_1D_1 y_2 + 2y f_2 \cos(\Omega t) + 2x_1 y_1 x_1 + x_1^2 y_1 \]  
\[ + 3y_1^2 y_1 - \mu_1(D_1 y_1) - \mu_2(D_1 y_1) - bx_1^2 (D_1 y_1) - b x_1^2 (D_1 y_1) \]  
\[ - 2bx_1(D_1 y_1) - y_1^2 (D_1 y_1) - y_1^2 (D_1 y_1) - 2 y_1 y_1(D_1 y_1) \]  
\[ - 3G_1(D_1 y_1)^2 (D_1 y_1) - 3G_1(D_1 y_1)^2 (D_1 y_1) \]
The general solution for equations (11) & (12) can be written in the form:

\[ x(T, T') = A(T, T') e^{i\alpha T} + a(T, T') e^{-i\alpha T}, \]
\[ y(T, T') = B(T, T') e^{i\alpha T} + b(T, T') e^{-i\alpha T}, \]

where A and B are unknown complex functions, which can be determined by imposing the solvability conditions at the next approximation order by eliminating the secular terms, and solving resulting equation gives:

\[ x_1 = H e^{3i\alpha T} + H e^{(\alpha + \Omega) T} + H e^{(\alpha - \Omega) T} + H e^{i(\alpha + 2\Omega) T} + H e^{i(\alpha - 2\Omega) T} + H e^{i(\alpha + 4\Omega) T} + H e^{i(\alpha - 4\Omega) T} + H e^{i(\alpha + 6\Omega) T} + H e^{i(\alpha - 6\Omega) T} + H e^{i(\alpha + 8\Omega) T} + H e^{i(\alpha - 8\Omega) T} + H e^{i(\alpha + 10\Omega) T} + H e^{i(\alpha - 10\Omega) T} + \cdots \]  
\[ y_1 = H e^{3i\alpha T} + H e^{(\alpha + \Omega) T} + H e^{(\alpha - \Omega) T} + H e^{i(\alpha + 2\Omega) T} + H e^{i(\alpha - 2\Omega) T} + H e^{i(\alpha + 4\Omega) T} + H e^{i(\alpha - 4\Omega) T} + H e^{i(\alpha + 6\Omega) T} + H e^{i(\alpha - 6\Omega) T} + H e^{i(\alpha + 8\Omega) T} + H e^{i(\alpha - 8\Omega) T} + H e^{i(\alpha + 10\Omega) T} + H e^{i(\alpha - 10\Omega) T} + \cdots \]

where \( H_n, n = 1 \ldots 10 \) are complex function in \( T_1 \) and \( \text{cc} \) denotes the complex conjugate terms. Substituting equations (17), (18), (19), and (20) into equations (15) and (16) the following are obtained, after eliminating the secular term and solve it:

\[ x_2 = H e^{3i\alpha T} + H e^{(\alpha + \Omega) T} + H e^{(\alpha - \Omega) T} + H e^{i(\alpha + 2\Omega) T} + H e^{i(\alpha - 2\Omega) T} + H e^{i(\alpha + 4\Omega) T} + H e^{i(\alpha - 4\Omega) T} + H e^{i(\alpha + 6\Omega) T} + H e^{i(\alpha - 6\Omega) T} + H e^{i(\alpha + 8\Omega) T} + H e^{i(\alpha - 8\Omega) T} + H e^{i(\alpha + 10\Omega) T} + H e^{i(\alpha - 10\Omega) T} + \cdots \]  
\[ y_2 = H e^{3i\alpha T} + H e^{(\alpha + \Omega) T} + H e^{(\alpha - \Omega) T} + H e^{i(\alpha + 2\Omega) T} + H e^{i(\alpha - 2\Omega) T} + H e^{i(\alpha + 4\Omega) T} + H e^{i(\alpha - 4\Omega) T} + H e^{i(\alpha + 6\Omega) T} + H e^{i(\alpha - 6\Omega) T} + H e^{i(\alpha + 8\Omega) T} + H e^{i(\alpha - 8\Omega) T} + H e^{i(\alpha + 10\Omega) T} + H e^{i(\alpha - 10\Omega) T} + \cdots \]

III. STABILITY ANALYSIS

After studying the different resonance numerically to see the worst resonance; one of the worst cases has been chosen to study the system stability. The selected resonance case \( \Omega = \omega_1 \) and \( \omega_2 = \omega_1 \). In this case we introduce the detuning parameter \( \sigma \) according to:

\[ \Omega = \omega_1 + \epsilon \sigma_1 \text{ and } \omega_2 = \omega_1 + \epsilon \sigma_2 \]  

where \( \sigma_1 \) and \( \sigma_2 \) are called detuning parameters. Also for stability investigation, the analysis is limited to the second approximation. So our solution is only depend on \( T_2 \) and \( T_1 \). Substituting equation (23) into equations (13) and (14) and eliminating the leads to the solvability conditions:

\[ -2i \omega_2 B + 2A^2 B - i (\omega_1, \mu_{AB}) - i (\omega_2 A - 2i \omega_2 AB + 3i G_1 A)^2 + f e^{i\sigma_1} = 0 \]  
\[ -2i (\omega_1 B + 3A^2 B + 2A A B - 2i \omega_2 A B - 2i \omega_2 A A B + 3i G_1 A)^2 + f e^{i\sigma_1} = 0 \]

To analyze the solution of equations (5.30) and (5.31), it is convenient to express A and B in the polar form as:

\[ A(T_1) = \frac{1}{2} a_1(T_1) e^{i\beta_1(T_1)} \quad \text{and} \quad B(T_1) = \frac{1}{2} b_1(T_1) e^{i\beta_1(T_1)} \]

where A, B, and \( \beta_1, \beta_2 \) are the steady state amplitudes and phases of the motions respectively. Inserting equation (26) into equations (24) and (25) and equating real and imaginary parts, we obtain:
\begin{equation}
\alpha' = -\frac{1}{2} \left[ \mu_i + b_i^2 \right] a_i - \frac{1}{8} \left[ 1 + 3G_i \omega_i^2 \right] a_i^3 - f_i \frac{a_i \sin \phi_i}{2\omega_i} \tag{27}
\end{equation}

\begin{equation}
\frac{1}{2} \alpha_i (\sigma_1 - \phi_i') = -\frac{3}{4\omega_i} a_i^3 - \left[ \frac{1}{2\omega_i} b_i \right] a_i - f_i \frac{a_i \cos \phi_i}{2\omega_i} \tag{28}
\end{equation}

\begin{equation}
b_i' = -\left[ \mu_i + \frac{1}{4} a_i^2 b_i \right] b_i + \left[ -\frac{1}{8} \left[ 1 + 3G_i \omega_i^2 \right] b_i^3 - \frac{f_i}{2\omega_i} b_i \sin \phi_i \right] \tag{29}
\end{equation}

\begin{equation}
\frac{1}{2} b_i (\sigma_2 - \phi_i') = -\frac{3}{4\omega_i} b_i^3 - \left[ \frac{1}{2\omega_i} a_i \right] b_i - f_i \frac{b_i \cos \phi_i}{2\omega_i} \tag{30}
\end{equation}

where \( \phi_i = \sigma_i' - 2\beta_i \) and \( \phi_i' = \phi_i' = 0 \) and the periodic solution at the fixed points corresponding to equations (27)-(30) is given by:

\begin{equation}
\frac{1}{4} \left[ 1 + 3G_i \omega_i^2 \right] a_i^2 + \left[ \mu_i + \frac{1}{2} a_i b_i \right] = -f_i \frac{a_i \sin \phi_i}{\omega_i} \tag{31}
\end{equation}

\begin{equation}
\sigma_1 + \frac{3}{4\omega_i} a_i^2 + \frac{1}{2\omega_i} b_i^2 = -f_i \frac{a_i \cos \phi_i}{\omega_i} \tag{32}
\end{equation}

\begin{equation}
\frac{1}{8} \left[ -1 + 3G_i \omega_i^2 \right] b_i^2 + \left[ \mu_i + \frac{1}{4} a_i^2 b_i \right] = -f_i \frac{b_i \sin \phi_i}{\omega_i} \tag{33}
\end{equation}

\begin{equation}
\sigma_2 + \frac{3}{4\omega_i} b_i^2 + \frac{1}{2\omega_i} a_i^2 = -f_i \frac{b_i \cos \phi_i}{\omega_i} \tag{34}
\end{equation}

Squaring equations (5.41) and (5.42) and summation, yields:

\begin{equation}
\left[ \frac{1}{4} \left[ 1 + 3G_i \omega_i^2 \right] a_i^2 + \left[ \mu_i - \frac{2}{3} a_i \left( \sigma_2 + \frac{1}{2\omega_i} a_i^2 + \frac{f_i}{\omega_i} \cos \phi_i \right) \right] \right]^2 + \left[ \sigma_1 + \frac{3}{4\omega_i} a_i^2 - \frac{2\omega_i}{3\omega_i} \left( \sigma_2 + \frac{1}{2\omega_i} a_i^2 + \frac{f_i}{\omega_i} \cos \phi_i \right) \right]^2 = f_i^2 \frac{1}{\omega_i^2} \tag{35}
\end{equation}

Similarly, from equations (33) and (34), we get:

\begin{equation}
\left[ \frac{1}{8} \left[ -1 + 3G_i \omega_i^2 \right] b_i^2 + \left[ \mu_i + \frac{1}{4} a_i^2 b_i \right] \right]^2 + \left[ \sigma_2 + \frac{3}{4\omega_i} b_i^2 + \frac{1}{2\omega_i} a_i^2 \right]^2 = f_i^2 \frac{2}{4\omega_i^2} \tag{36}
\end{equation}

From equations (35) and (36), we have the following cases:

**Case1:** \( a = b = 0 \) (the trivial solution).

**Case2:** \( a \neq 0, b = 0 \), in this case, the frequency response equation (35) is given by:

\begin{equation}
\left[ \frac{1}{4} \left[ 1 + 3G_i \omega_i^2 \right] a_i^2 \right]^2 + \left[ \sigma_1 + \frac{3}{4\omega_i} a_i^2 \right]^2 = f_i^2 \frac{1}{\omega_i^2} \tag{37}
\end{equation}

After that we have:

\begin{equation}
\left[ \frac{3a_i^2}{2\omega_i} \right] \sigma_1 + \frac{1}{16} \left[ 1 + 6G_i \omega_i^2 + 9G_i^2 \omega_i^4 \right] - \frac{16f_i}{\omega_i^2 a_i^2} + \frac{9}{\omega_i^2} = 0 \tag{38}
\end{equation}
The solution of algebraic equation (38) has two roots, given by:

$$\sigma_i = \frac{1}{2} \left[ \frac{3a_i^2}{2\omega_i} \pm \frac{1}{2} \left[ 1 + 6G_i \omega_i^2 + 9G_i^2 \omega_i^4 - \frac{16f_{1i}}{\omega_i^2 a_i^4} + \frac{18}{\omega_i^2} \right] \right] \quad (39)$$

**Case3:** $a \neq 0, b \neq 0$, in this case the frequency response equations (35) and (36) are given by:

$$\left[ \frac{1}{4} (1 + 3G_i \omega_i^2) a_i^2 + \left( \mu_i - \frac{2}{3} a\omega_i \left( \sigma_2 + \frac{1}{2\omega_2} a_i^2 + f_2 \cos \varphi_2 \right) \right) \right]^2$$

$$+ \left[ \sigma_1 + \frac{3}{4 \omega_i} a_i^2 - \frac{2\omega_2}{3 \omega_i} \left( \sigma_2 + \frac{1}{2 \omega_2} a_i^2 + f_2 \cos \varphi_2 \right) \right]^2 = \frac{f_2^2}{\omega_i^2} \quad (40)$$

### 2.1 Linear Solution:

To study the stability of the linear solution of the obtained fixed points, let us consider $A$ and $B$ in the polar forms:

$$A \left(T_1\right) = \frac{1}{2} \left( p_1 - iq_1 \right) e^{i\delta T_1}, \quad B \left(T_1\right) = \frac{1}{2} \left( p_2 - iq_2 \right) e^{i\delta T_1}$$  

(41)

where $p_1, q_1, p_2$ and $q_2$ are real functions in $T_1$.

Substituting equation (41) into the linear parts of equations (24) and (25) and equating the imaginary and real parts, we have the following cases:

**Case1:** ($a \neq 0, b = 0$)

$$p_1' + \frac{1}{2} \mu_i p_1 - \frac{1}{2} \left( -\sigma_1 + \frac{f_1}{\omega_i} \right) q_1 = 0 \quad (42)$$

$$q_1' - \frac{1}{2} \left( \sigma_1 - \frac{f_1}{\omega_i} \right) p_1 + \frac{1}{2} \mu_i q_1 = 0 \quad (43)$$

The stability of the linear solution is obtained from the zero characteristic equation:

$$\begin{vmatrix}
-\frac{1}{2} \mu_i - \lambda & \frac{1}{2} \left( -\sigma_1 + \frac{f_1}{2\omega_i} \right) \\
\frac{1}{2} \left( \sigma_1 + \frac{f_1}{2\omega_i} \right) & -\frac{1}{2} \mu_i - \lambda
\end{vmatrix} = 0 \quad (44)$$

Then, we have that:

$$\lambda_{1,2} = \frac{1}{2} \left[ \mu_i \pm \sqrt{\frac{f_1^2}{\omega_i^2} - \sigma_1} \right] \quad (45)$$

**Case2:** ($a \neq 0, b \neq 0$)

$$p_1' + \frac{1}{2} \mu_i p_1 - \frac{1}{2} \left( -\sigma_1 + \frac{f_1}{\omega_i} \right) q_1 = 0 \quad (46)$$

$$q_1' - \frac{1}{2} \left( \sigma_1 - \frac{f_1}{\omega_i} \right) p_1 + \frac{1}{2} \mu_i q_1 = 0 \quad (47)$$
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\[ p_2' + \mu_2 p_2 + \frac{1}{2} \left( -\sigma_2 + \frac{f_2}{\omega_2} \right) q_2 = 0 \] (48)

\[ q_2' - \frac{1}{2} \left( \sigma_2 + \frac{f_2}{\omega_2} \right) p_2 - \mu q_2 = 0 \] (49)

Equations (46)-(49) can be written in the matrix form:

\[
\begin{bmatrix}
    p_1' \\
    q_1' \\
    p_2' \\
    q_2'
\end{bmatrix} =
\begin{bmatrix}
    -\frac{1}{2} \mu_1 & \frac{1}{2} \left[ -\sigma_1 + \frac{f_1}{\omega_1} \right] & 0 & 0 \\
    -\frac{1}{2} \mu_1 & -\frac{1}{2} \mu_1 & 0 & 0 \\
    0 & 0 & -\mu_2 & \frac{1}{2} \left[ -\sigma_2 + \frac{f_2}{\omega_2} \right] \\
    0 & 0 & \frac{1}{2} \left[ \sigma_1 + \frac{f_2}{\omega_2} \right] & -\mu_2
\end{bmatrix}
\begin{bmatrix}
    p_1 \\
    q_1 \\
    p_2 \\
    q_2
\end{bmatrix}
\] (50)

The stability of a particular fixed point with respect to perturbation proportional to \(\exp(\lambda T_1)\) is determined by zeros of characteristic equation:

\[
\begin{vmatrix}
    -\frac{1}{2} \mu_1 - \lambda & \frac{1}{2} \left[ -\sigma_1 + \frac{f_1}{\omega_1} \right] & 0 & 0 \\
    \frac{1}{2} \left[ \sigma_1 + \frac{f_1}{\omega_1} \right] & -\frac{1}{2} \mu_1 - \lambda & 0 & 0 \\
    0 & 0 & -\mu_2 - \lambda & \frac{1}{2} \left[ -\sigma_2 + \frac{f_2}{\omega_2} \right] \\
    0 & 0 & \frac{1}{2} \left[ \sigma_1 + \frac{f_2}{\omega_2} \right] & -\mu_2 - \lambda
\end{vmatrix} = 0
\] (51)

After extract we obtain that:

\[ \lambda^4 + r_1 \lambda^3 + r_2 \lambda^2 + r_3 \lambda + r_4 = 0 \] (52)

According to Routh-Hurwitz criterion, the above linear solution is stable if the following are satisfied:

\[ r_1 > 0, r_2 - r_3 > 0, r_3 (r_2 - r_3) - r_1^2 r_4 > 0, r_4 > 0. \] (53)

2.2 Non-Linear Solution:

To determine the stability of the fixed points, one lets:

\[ a = a_0 + a_1, b = b_0 + b_1, \varphi_m = \varphi_{m0} + \varphi_{m1} (m = 1, 2) \] (54)

where \(a, b\) and \(\varphi_{m0}\) are solutions of equations (27)-(30) and \(a_1, b_1, \varphi_{m1}\) are perturbations which are assumed to be small comparing to \(a_0, b_0, \varphi_{m0}\). Substituting equation (54) in to equations (27)-(30) and keeping only the linear terms in \(a_{11}, b_{11}, \varphi_{m1}\) we obtain:

1. For the case \((a \neq 0, b = 0)\), we have:
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\[
a_{11}' = \left[ -\frac{1}{2} \mu_1 - \frac{3}{8} a_{10}^2 - \frac{9}{8} \omega_1^2 G a_{10}^2 + \frac{1}{2 \omega_1} f_1 \sin \varphi_{10} \right] a_{11} + \left[ \frac{1}{2 \omega_2} f_1 a_{10} \cos \varphi_{10} \right] \varphi_{11}
\]

\[
\varphi_{11}' = \left[ \frac{\sigma_1}{a_{10}} + \frac{9}{4 \omega_1} a_{10} + \frac{1}{\omega_2 a_{10}} f_1 \cos \varphi_{11} - \frac{1}{\omega_1} f_1 \sin \varphi_{11} \right] \varphi_{11}
\]

The stability of a given fixed point to a disturbance proportional to \( \exp(\lambda t) \) is determined by the roots of:

\[
\begin{bmatrix}
-\frac{1}{2} \mu_1 - \frac{3}{8} a_{10}^2 - \frac{9}{8} \omega_1^2 G a_{10}^2 + \frac{1}{2 \omega_1} f_1 \sin \varphi_{10} & \frac{1}{2 \omega_1} f_1 a_{10} \cos \varphi_{10} \\
\frac{\sigma_1}{a_{10}} + \frac{9}{4 \omega_1} a_{10} + \frac{1}{\omega_2 a_{10}} f_1 \cos \varphi_{11} & -\frac{1}{\omega_1} f_1 \sin \varphi_{11}
\end{bmatrix}
= 0
\]

Consequently, a non-trivial solution is stable if and only if the real parts of both eigenvalues of the coefficient matrix (57) are less than zero.

2. For the practical solution \((a \neq 0, b \neq 0)\), we have:

\[
a_{11}' = \left[ -\frac{1}{2} \mu_1 - \frac{3}{8} a_{10}^2 - \frac{9}{8} \omega_1^2 G a_{10}^2 + \frac{1}{2 \omega_1} f_1 \sin \varphi_{10} \right] a_{11} + \left[ \frac{1}{2 \omega_2} f_1 a_{10} \cos \varphi_{10} \right] \varphi_{11}
\]

\[
\varphi_{11}' = \left[ \frac{\sigma_1}{a_{10}} + \frac{9}{4 \omega_1} a_{10} + \frac{1}{\omega_2 a_{10}} f_1 \cos \varphi_{11} - \frac{1}{\omega_1} f_1 \sin \varphi_{11} \right] \varphi_{11}
\]

\[
b_{11}' = \left[ -\frac{1}{2} \mu_2 - \frac{3}{8} b_{10}^2 - \frac{9}{8} \omega_2^2 G b_{10}^2 + \frac{1}{2 \omega_2} f_2 \sin \varphi_{20} \right] b_{11} + \left[ \frac{1}{2 \omega_1} f_1 b_{10} \cos \varphi_{20} \right] \varphi_{21}
\]

\[
\varphi_{21}' = \left[ \frac{\sigma_2}{b_{10}} + \frac{9}{4 \omega_2} b_{10} + \frac{1}{\omega_1 b_{10}} f_2 \cos \varphi_{20} - \frac{1}{\omega_2} f_2 \sin \varphi_{20} \right] \varphi_{21}
\]

The stability of a particular fixed point with respect to perturbations proportional to \( \exp(\lambda t) \) depends on the real parts of the roots of the matrix. Thus, a fixed point given by equations (58)-(61) is asymptotically stable if and only if the real parts of all roots of the matrix are negative.

IV. NUMERICAL RESULTS:

The nonlinear dynamical system is solved numerically using Rung-Kutta fourth order method by using Maple 16 software. At non resonance case (basic case) as shown in Fig.1, we can see that the steady state amplitude without controller is about 0.08 and with controller is about 0.01.

3.1) Resonance cases

Sub harmonic resonance, \( \Omega_1 = 2 \omega_1 \), for the main system the phase plane is a limit cycle and its steady state amplitude which is the largest without controller is about 0.9 and about 0.17, which appears in Fig.(3). In Fig (4), Sub harmonic resonance \( \Omega_2 = 2 \omega_2 \), for the main system the phase plane is a limit cycle and its steady state amplitude without controller is about 0.0004 and about 0.02.

3.2) Effect of control

The effect of controller appears at Fig.(5) and Fig.(6), in Fig.(5), the amplitude of the main system at Sub harmonic resonance: when \( \Omega_1 = 2 \omega_1 \), is decreasing to 0.05. Similarly for the Fig.(6), the amplitude of the main system Sub harmonic resonance: when \( \Omega_2 = 2 \omega_2 \), decreasing to 0.01.

3.3) Effect of parameters
For the damping coefficient $\mu_1$, Fig(7) (a) shows that the steady state amplitude of the main system is monotonic increasing function. For the parameter $\omega_1$, Fig.(7) (b) shows that the steady state amplitude of the main system is monotonic decreasing function. But that the steady state amplitude of the main system is monotonic increasing function of the excitation forces $f_1, f_2$, Fig (7) (c),(f) shows.

3.4) Frequency response curves

The frequency response in the second case $(a \neq 0, b = 0)$ which represented by equation (37) is a nonlinear algebraic equation solved numerically of the amplitude against the detuning parameter $\sigma_1$. Fig.(8)(a) shows that the steady state amplitude is monotonic decreasing function on the natural frequency. Fig.(8)(b) shows that the steady state amplitude is a monotonic increasing function in the non-linear parameter $\mu_1$. The excitation force $f_1$ of the main system is a monotonic increasing function at the steady state amplitude which appeared on the Fig.(8)(c). The steady state amplitude is a monotonic decreasing function on the gain $G_1$.
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Fig. (7), Effects of different parameters.
Fig. (8): Response curves at $(a \neq 0, b = 0)$.

Fig. (9): Response curves at $(a \neq 0, b \neq 0)$.

4) Comparison between approximation solution and numerical solution

In this subsection we compare between the numerical solutions (which we obtained by using Maple 16 software) and the approximation solution (which we obtained by using equations (58) to (61)). All this comparison done in the resonance sub-harmonic case which we choose to be the worst we obtained that there is a good agreement between the two solutions.
The dynamical behavior of two coupled parametrically excited van der pol oscillator is investigated by using perturbation method. Resonance cases were obtained, the worst one has been chosen to be discussed which is $f_\Omega = 2\alpha_i$. Hence the stability of the system and controller is studied using the frequency response functions from the above study, the following results are concluded:

1. The steady state of the system without controller is about 0.08 which is considering basic case.
2. The damping coefficient $\mu_i$ is monotonic decreasing function.
3. The natural frequency $\omega_i$ is monotonic decreasing function.
4. The forces $f_1, f_2$ are monotonic increasing functions.
5. The numerical solution has a good agreement with the approximation solution.

V. CONCLUSION

REFERENCES:


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