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Comparison between two mathematical models for problems in monitoring and inspection of arcs

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Abstract:- The problems involving routing arcs have great applicability to real problems and can be addressed in different ways depending on their constraints. They consist in determining routes for vehicles so arcs in a graph. This paper aims to present and compare the results of two mathematical models for solving a problem that belongs to the class of Periodic Capacitated Arc Routing Problem (PCARP). It is an issue that has gained more attention in the last decade, it is necessary to determine a route to one or multiple vehicles respecting their capacities, considering a discrete time horizon in a way that the demands of each arc are served.

Keywords:- Arcs, Capacitated, Monitoring, PCARP, Periodic, Routing

I. INTRODUCTION

Researchers have solved many logistical problems such as facilities location, inventory management and vehicle routing. They have taken a sequence of resolution involving strategic, tactical and operational decisions levels [1]. One of these problems is Arc Routing Problem (ARP), which aims to determine the lowest cost to traverse a set of arcs in a graph, and may be subject to constraints. This class of problems can be applied in a number of practical contexts such as garbage collection, mail delivery, snow removal, inspection of power lines, monitoring and maintenance of railroads and highways.

When contextualizing these problems, some variations are obtained, one of them is the Periodic Capacitated Arc Routing Problem (PCARP), and it comes to vehicles with limited capacity that must traverse, from well-defined time horizon, a route to cover the demand of each arc, which is established in the form of frequency. Depending on how the problem is handled, a number of complications for its resolution may occur.

The aim of this paper is to compare two models that are placed in the context of monitoring and inspection of railroads or highways. At each time, some roads need to be inspected in order to avoid possible accidents and to ensure the continuous flow of vehicles.

Corberán and Prins [2] presents in their work an overview of the recent results in the literature until the date of the publication about Arc Routing Problems. This problems arising in the Bridges of Königsberg Problem and that try to determine one or more routes that cover all (or partially) links (arcs or edges) of a graph, satisfying some constraints with the lowest cost possible.

One of the main problems encountered in literature is the Capacitated Arc Routing Problem (CARP). It is characterized by having a non-negative demand associated with each arc of the graph. Since each vehicle of a fleet has a certain capacity, these vehicles must traverse the arcs collecting or delivering certain demands without exceeding it, this problem is proposed by Golden and Wong in 1981[3].

CARP is widely used to solve the problem of urban waste collection as an operational-level decision, daily taken. However, in some problems it is necessary to make tactical decision, which involved a longer time horizon with multiple periods, subject to constraints involving certain frequencies. Taking these into account, PCARP was proposed by Lacomme, Prins and Ramdane-Chérif [4], who presents this problem, its variants and a Genetic Algorithm to solve it. The first mathematical modeling problem proposed only occurred in 2004 in the work of Chu, Labadi and Prins [5], where it was also proposed preliminary lower bounds and a metaheuristic called Scatter Search. Kansou and Yassine [6] combined an algorithm of ant colony with an insertion heuristic to solve the PCARP, achieving robust results and fast performance.

In general, the PCARP is a natural extension of the CARP, where the problem is solved for multiple periods instead of being solved for only one day [7]. Solving PCARP implies in simultaneously determining the decisions in tactical and operational level during a time horizon. Unfortunately, in many applications of this problem, there are some complications, such as: the demands can be float in accordance with the days or spaced in constraints between service days [8].

One problem is suggested in Section II in order to contextualize a problem that will be solved by mathematical models proposed in Section III, as the models achieve different goals, an adaptation for the same problem is explained in Section IV, followed by the result in V and finally the conclusions in Section VI.

II. PROBLEM SUGGESTED

An example of a problem that can be solved with PCARP is the problem in Figure 1, which shows a railroad network in which some traverses should be visited once every 16 days, or once every 24 days to be inspected in order to avoid possible accidents.

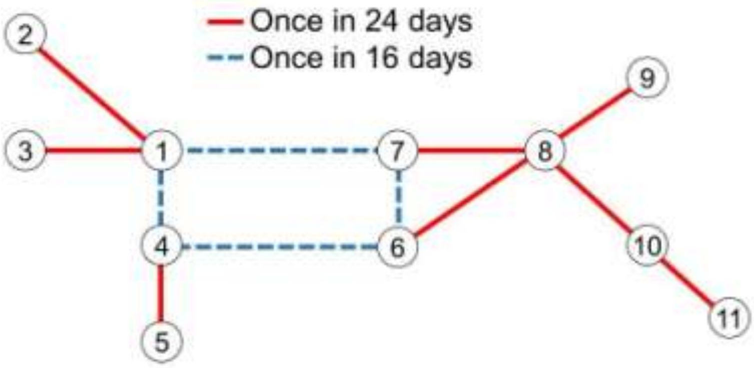

Fig. 1 – Representation of a problem of inspection in a railroad network

The distance from a node to another connected by a single arc in Figure 1 is measured in time and is equivalent to 1 day of service or crossing. For example, it takes up to one day to leave node 1 and move to node 7, working or just passing through. A feature of this proposed problem is that the displacement of vehicles, in general, is slow, and does not need to return to a specific node at the end of the day (also known as depot).

The time horizon is defined based on the arc of greater periodicity. Periodicity is understood by the interval of time in which an arc should be serviced at least once. In the example of Figure 1, there is a time horizon of 24 days, i.e., within the period of 24 days the arcs need to have their demands satisfied. That implies that the arcs at intervals of 16 days should have two traverses during these 24 days, so that the interval between one passage and another is of 15 days at most, or even, that in 16 days the section is traversed at least one time.

As it is a cyclical problem, at the end of the 24 days, the car should be at the same node where it started its course on day 1, which is equivalent to saying that what should happen on 25^{th} day is exactly what happens on the first day. Arcs at lower periodicity should take into account the sequence of days after the end of the first 24 days. For example, if the car passed the arc 3-7 during the days 3 and 9 it would have cover the demand of the first 24 days, but as the day 3 would be equivalent to day 27, after closing the first cycle it would not attend the next one.

III. MATHEMATICAL MODELS

Monroy, Amaya and Langevin (2013) [7] work with the problem of monitoring road network that is performed periodically. The streets or highways are divided into classes according to the need for vigilance and the hierarchy of roads, these classes are the basis to determine the demands (number of passes) of each arc along the time horizon. The objective is to assign a set of routes, one for each car on each day, satisfying the frequency of each class of roads in each sub period of time without exceeding the capacity of the vehicle.

The model proposed by Monroy, Amaya and Langevin is for directed graphs, and the notations used to formulate the problem are the following:

Ω_c: set of arcs of class

c T: set of sets Ω_c

e_c: set of sub-periods (sub-horizons) associated to arcs of class c $O(n)$: set of arcs leaving node $n \in N$

$I(n)$: set of arcs entering node $n \in N$ S: sub-set of arcs

$N(S)$: set of nodes incident to the arcs of

S V: set of routes (shifts)

w_j: set of routes corresponding to sub-period

j fi: number of required services in sub-

period j p_a: weight of arc a

On its instances, it uses a time horizon of 14 days and 3 types of classes. Class 1 is formed by arcs that demands at least one serving passage on days 1 to 5, 6 to 6, 8 to 12 and 13 to 14; class 2 has demands between days 1 to 7 and 8 to 14; and class 3 is formed by arcs that require passage from day 1 until day 14. Figure 2 below illustrates this situation:

	Class of arcs	Monday – Friday (1)	Saturday – Sunday(1)	Monday – Friday (2)	Saturday – Sunday (2)
Sub-periods	CLASS 1	w_1	w_2	w_3	w_4
	CLASS 2	w_5		w_6	
	CLASS 3	$w_7 = V$			

Fig. 2 – Sub-time horizons associated to types of arcs. Source: Monroy *et al*. [7]

Its formulation consists in defining k routs that service the demands; the decision variables used arc notations and are as follows:

x_a^k assumes the value 1 if the arc a is served by route k or 0 otherwise

v_a^k number of times that the arc a is traversed by the route k (without being served) The problem is modeled as follows:

$$\max z \quad = \sum_{k \in V} \sum_{a \in R} p_a x_a^k \tag{1}$$

Subject to:

$$\sum_{a \in O(n)} (x_a^k + y_a^k) - \sum_{a \in I(n)} (x_a^k + y_a^k) \ = 0 \ \forall k \in V, n \in N \tag{2}$$

$$\sum_{a \in R} (c_a x_a^k) + \sum_{a \in R} (t_a y_a^k) \leq Q \ \forall k \in V \tag{3}$$

$$M \sum_{a \in I(N(S))} (x_a^k + y_a^k) \geq \sum_{a \in O(N(S))} x_a^k + y_a^k \ \forall S \subset A, 1 \notin N(S), k \in V \tag{4}$$

$$\sum_{k \in w_j} x_a^k \geq f_j \quad \forall j \in e_c, a \in \Omega_c, c = 1, \dots, |T| \tag{5}$$

$$x_a^k \in \{0,1\} \ \forall a \in R, k \in V \tag{6}$$

$$y_a^k \in Z^+ \ \forall a \in A, k \in V \tag{7}$$

Priority arcs need more treatments during the time interval, so the objective function (1) maximizes the number of services taking into account the arc of each class and the respective weight. Conservation of flow of vehicles is assured by (2). Each route that will be taken by a car is bounded by a capacity (3), and the total capacity is the sum of the costs of crossing t_a and serving the arc c_a. The connectivity constraints are given in (4). The relationship between the numbers of services for each period of and class of the arcs is given in the equation (5). And the restrictions (6) and (7) define the variables.

A model proposed specifically to solve the suggested problem considers an undirected graph $G= (X,E)$ with n nodes, so, $X=\{x_1, x_2, ..., x_n\}$, and m edges with $E=\{e_1, e_2, ..., e_m\}$ which have to be traversed by nk cars defined by $K = \{1,2, ...,nk\}$. Each arc e is formed by a pair of nodes, and it is called $x_{ij} = (x_i, x_j)$ or $[i,j]$ and it is associated to a cost c_{ij}.

The demands of periodicity of each arc are expressed in the maximum number of period in which the arc must be attended at least once $MP(x_{ij})$. There is still the time horizon H, formed by np periods $H= \{1,2, ...,np\}$, where each period is symbolized by the letter p. the model is based on Binary Linear Programming, so, to solve the problem were created three binary variables listed below:

x_{ijkp} takes the value 1 if the arc of car k moves from node i to node j in period p

p_{ijp} takes value 1 if the arc x_{ij} does not respect the periodicity in the period p

f_{ikp} takes value 1 if the car k is stopped at the node i on day p

The variables p_{ijp} and f_{ikp} were created so that the problem is feasible. Each variable p_{ijp} has associated with it a cost PU_{ij}, which is a punishment if the periodicity of an arc is not achieve. This allows that in the proposed model there is a delay in attending to some arcs, since the amount should be the lowest possible. The variable f_{ikp} allows that the car is given a day off on a given day. The problem is modeled as follows, with the objective function given by the equation:

$$\min Z \quad = \sum_{[i,j] \in E} \sum_{k=1}^{nk} \sum_{p=1}^{np} c_{ij} * x_{ijkp} + \sum_{[i,j] \in E} \sum_{p=1}^{np} PU_{ij} * p_{ijp} \tag{8}$$

Subject to:

$$\sum_{[i,j]\in E} x_{ijkp} + f_{jkp} - \sum_{[i,j]\in E} x_{jik,p+1} - f_{jk,p+1} = 0 \quad \forall j \in X, k \in K, p \in H \tag{9}$$

$$\sum_{[i,j]\in E} x_{ijkp} + \sum_{[i,j]\in E} x_{jikp} + \sum_{i=1}^{n} f_{ikp} = 1 \quad \forall\, p \in H, k \in K \tag{10}$$

$$\sum_{k=1}^{nk} \begin{pmatrix} x_{ijkp} + x_{jikp} + x_{ijk,p+1} + x_{jik,p+1} + \dots \\ + x_{ijk,p+MP(x_{ij})-1} + x_{jik,p+MP(x_{ij})-1} \end{pmatrix} + p_{ijp} \geq 1 \quad \forall\, [i,j] \in E, p \in H \tag{11}$$

$$x_{ijkp}, p_{ijp}, f_{ikp} \in \{0,1\} \quad \forall\, [i,j] \in E, k \in K, p \in H \tag{12}$$

Constraint (9) ensures the daily flow of cars, allowing the day off, and, as cycles will be formed at the end of period *np*, the car must return to the same location as it was on day 1, i.e., the period *np* + 1 is equal to day 1, and so on. That goes for the whole time horizon and for all the constraints. Constraint (10) ensures that all cars will have some designation for the day *p*, moreover, it implicitly ensures that the capacity of each car is not exceeded.

The constraint (11), is the most complex. It refers to the frequency that must be treated in each traverse. In constraint (11) it is possible that p_{ijp} assumes 1, i.e., an arc can be delayed in one day to the cost of being punished with a cost PU_{ij}. However, if the demand is only one passage throughout the time horizon, the constraint can be simplified to just one day *p*, *p* = 1, which covers the entire time horizon and the variable p_{ijp} can be ruled out, as was done in the proposed problem. Finally, equation (12) requires that all variables are binary.

IV. MODEL ADAPTATION

The two approached modeling cater different scopes, for the comparison between the models, it was made an adjustment to the models proposed by Monroy, Amaya and Langevin [7], where:

- It was considered a time horizon of 24 days, which were subdivided into 2 groups based on the days of each periodicity. So it was obtained the first group comprising days 1 to 16, and the other group from days 17 to 24;
- The graph was considered undirected;
- The objective function was defined to minimize the cost of travel;
- Since the model proposed by Monroy *et al*. needs a depot, node 1 was chosen because it is connected to arcs with greater needs (1-7, 1-4).
- Two classes of arcs were defined. The first class, Class 1, needs 2 passes in the time horizon w_1, w_2, which are formed respectively by days 1 to 16 and 17 to 24. Class 2 requires only on pass in the interval w_3 that encompasses the entire time horizon. Figure 3 illustrates the following classes:

	Days 1 - 16	Days 17 - 24
CLASS 1	w_1	w_2
CLASS 2	w_3	

Fig. 3 – Sub-time horizons associated to types of arcs for adaptation

- It was necessary to create two routes, the first with a capacity of 16 attendances, which included the first 16 days, and another with a capacity of 8 attendances that included days 17 to 24, which in the future will form a single route that must satisfy the periodicities approximately.
- Constraints that prevent sub cycles were simplified only considering the subsets of arcs that ensured that the depot was always included in the routes. In the work proposed by Monroy, Amaya and Langevin a similar simplification was made, stating that if the route presented sub cycles, then restrictions were added in order to avoid them.

V. RESULT

After the adjustments, the model of Monroy *et al*. came to an exact solution, which presented the following routes: 1 - 7 - 6 - 8 - 9 - 8 - 10 - 11 - 10 - 8 - 6 - 4 - 1 - 2 - 1 - 3 - 1 and 1 - 7 - 8 - 7 - 6 - 4 - 5 - 4 - 1, the first route encompassing the first 16 days and the next the following 8 days. In general it is equivalent to saying that throughout the 24 days, the route obtained was 1 - 7 - 6 - 8 - 9 - 8 - 10 - 11 - 10 - 8 - 6 - 4 - 1 - 2 - 1 - 3 - 1- 7 - 8 - 7 - 6 - 4 - 5 - 4 - 1.

It is observed that the solution is interesting, but would not satisfy the resolution of the problem, because when evaluating the arcs that have the need of more than 1 pass over the time horizon, a failure occurs, and it can be noticed that:

- Arc 1-7 is served on days 1 and 17, within the limit of the periodicity because the arc is traversed exactly 16 days after the first pass. Considering the feature of being a cyclical problem, it would be traversed again on day 25, 8 days after the last passage;
- Arc 6-4 is served on days 11 and 21, and in the future 35, also within the periodicities;
- Arc 1-4 is met on days 12 and 24, and in the future 36, also within the periodicities;
- Arc 7-6 is served on days 1 and 20, which would not comply with the periodicity because between days 2 and 19 it is not traversed, it would have a new service on the 25th day but it overstepped in the first time.

Indeed, in the specific model if it is not possible to satisfy the arcs within the periodicity there would occur a punishment in arc 7-6 for delays and the solution would be accepted. However, the particular model presented the following optimal route to the same problem: 6 - 4 - 1 - 7 - 8 - 10 - 11 - 10 - 8 - 7 - 6 - 4 - 5 - 4 - 1 - 2 - 1 - 3 - 1 - 7 - 6 - 8 - 9 - 8 - 6 where no arc would be late. Table 1 lists the two solutions with emphasis on the critical arcs that require more than one pass over the time horizon.

Table 1 – Solution presented when solving the models

Day	Model of Monroy *et al.* (2013)			Especific model	
	Start Node	Finish Node	Route	Start Node	Finish Node
1	1	7	1	6	4
2	7	6	1	4	1
3	6	8	1	1	7
4	8	9	1	7	8
5	9	8	1	8	10
6	8	10	1	10	11
7	10	11	1	11	10
8	11	10	1	10	8
9	10	8	1	8	7
10	8	6	1	7	6
11	6	4	1	6	4
12	4	1	1	4	5
13	1	2	1	5	4
14	2	1	1	4	1
15	1	3	1	1	2
16	3	1	1	2	1
17	1	7	2	1	3
18	7	8	2	3	1
19	8	7	2	1	7
20	7	6	2	7	6
21	6	4	2	6	8
22	4	5	2	8	9
23	5	4	2	9	8
24	4	1	2	8	6

VI. CONCLUSION

The fact that the specific model does not contain any delay, shows that the objects were achieved in a more effective way than using an adaptation of the proposed model in [7], another considerable factor is that the model of Monroy *et al*. had a faster computationally solution for this experiment. When solving models in software CPLEX 12.4 on a computer with Intel Core i7 2 GHz of processing and 4 Gb of RAM memory and 64-bits operating system, the model of Monroy *et al*. was solved in less than 1 second (it is important to highlight the simplification made to avoid sub cycles, and that no sub cycle was formed on the first try) while the specific model takes less than 30 seconds to get a solution with the optimal value, but this value is only confirmed after 330 seconds. Considering that this is a solution for a long-term planning, these values become irrelevant when compared to 24 days.

Routing problems in arcs cover a field of endeavor still not so exploited, and have great applicability to real contexts, capable of being adapted as required. It is very relevant to planning of services that are not restricted to just one day and that cover a longer time horizon, good results can be achieved and accidents can be prevented as is the case in this study.

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