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Hybrid Projective Synchronization and Chaos Control

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Abstract:- In this paper we study the hybrid projective synchronization and chaos control by using Adaptive feedback controllers based on Lyapunov's direct method Especially, the controller can be simplified into a single scalar one to achieve complete synchronization. Numerical simulations are demonstrated to verify the effectiveness of the control strategies
K**eywords :-** Adaptive control method, Hybrid projective synchronization

I. INTRODUCTION

Since chaotic attractors were found by Lorentz in 1963, many chaotic systems have been constructed, such as the Lorentz system, Chen system, and Lü system [1–8]. Now these days, the study of chaotic systems has attracted more and more attention. since the pioneering work of Ott, Grebogi, and Yorke and the seminal work of Pecora and Carroll, which are simultaneously reported in 1990, Chaos control and synchronization of chaotic systems have been interesting research fields. In many fields, such as secure communication, neural networks, optimization of nonlinear system performance, ecological systems, modeling brain activity, the synchronization such as complete synchronization [9, 10], phase synchronization [11, 12], partial synchronization [13], generalized synchronization [14], projective synchronization [15– 17], and so forth have been applied.

Recently, hybrid projective synchronization (HPS) was proposed. It can be considered as an extension of projective synchronization because complete synchronization and anti-synchronization are both its special cases. It is worthy of study because the response signals can be any proportional to the drive signals by adjusting the factors and it can be used to extend binary digital to variety M-nary digital communications for achieving fast communication. However, the controllers based on different control methods in the existing literatures, such as nonlinear feedback control [18– 20], active control [21– 23], adaptive control [24– 26], and so forth, are mostly vectorial and they are difficult to be put into practice. So, the controllers which are simple, efficient, and easy to implement are required to be designed for both chaos control and HPS between two chaotic systems.

II. HYBRID PROJECTIVE SYNCHRONIZATION BY ADAPTIVE FEEDBACK CONTROL LAW

For two dynamical systems

$\dot{x} = f(x)$ (1)

$\dot{y} = g(y) + u(x,y)$ (2)

where $x = (x_1, x_2, x_3, x_4)^T$ $y = (y_1, y_2, y_3, y_4)^T \in R^n$ are state variables of the drive system (1) and the response system (2), respectively, and $f: R^n \to R^n$ are nonlinear vectorial functions $u(x,y) = (u_1(x,y), u_2(x,y), u_3(x,y), \ldots \ldots, u_n(x,y))^T$ is the nonlinear control vector. If there exists a nonzero constant matrix $\propto = diag(\propto_1, \propto_2, \propto_3, \propto_4)$ such that , namely, $\lim_{t\to\infty} y_i - x_i = 0$ $i = 1,2,3 \ldots$, then the response system and the drive system are said to be in HPS. In particular, the drive-response systems achieve complete synchronization when all values of are equal to 1 and the two chaotic systems are said to be in antisynchronization when all values of are equal to −1.

In this section, we study the hybrid projection synchronization of two identical chaotic systems. The response system corresponding to the drive system (1.1) is defined as follows:

$\dot{y}_1 = \propto (y_2 - y_1) + u_1$
$\dot{y}_2 = b y_1 - c y_2 - y_1 y_3 + u_2$
$\dot{y}_3 = y_1^2 - d y_3 + u_3$
$\dot{y}_4 = -y_1 y_3 - \delta y_4 + u_4$ (3)

Where $(u_1, u_2, u_3, u_4)^T$ is the nonlinear control vector.System (1) and (2) are in HPS as long as

$\lim_{t\to\infty} y_i - x_i = 0$ $i = 1,2,3 \ldots$ (4)

Define the state error vector as

$e_i = y_i - \alpha_i x_i$
$e_1 = y_1 - \alpha_1 x_1$
$e_2 = y_2 - \alpha_2 x_2$
$e_3 = y_3 - \alpha_3 x_3$ (5)
$e_4 = y_4 - \alpha_4 x_4$

And the error vector

$\dot{e}_1 = \dot{y}_1 - \alpha_i \dot{x}_1$
$\dot{e}_2 = \dot{y}_2 - \alpha_1 \dot{x}_2$
$\dot{e}_3 = \dot{y}_3 - \alpha_3 \dot{x}_3$ (6)
$\dot{e}_4 = \dot{y}_4 - \alpha_4 \dot{x}_4$

Where $\alpha = diag(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ and $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are different ,desired scaling factors for HPS. The error dynamical system between the system (1) and (3) can be written as

$\dot{e}_1 = -\alpha_1(x_2 - x_1) + \alpha(y_2 - y_1) + u_1$
$\dot{e}_1 = \alpha e_2 + \alpha\alpha_2 x_2 - \alpha e_1 - \alpha\alpha_1 x_2) + u_1$] [( ]
$\dot{e}_2 = [by_1 - cy_2 - y_1 y_3 + u_2 - \alpha_2 \ \ \Upsilon - \alpha) x_1 - x_1 x_3 + \Upsilon x_2$
$\dot{e}_2 = x_1 x_3(\alpha_2 - \alpha_1\alpha_3) + x_1(b\,\alpha_1 + \alpha\alpha_2 - \Upsilon\,\alpha_2) - x_2(\Upsilon\,\alpha_2 + c\,\alpha_2) + u_2 + be_1 - ce_2 - e_1 e_3 - e_1\,\alpha_3\,x_3 - \alpha_1\,x_1 e_3$
$\dot{e}_3 = y_1^{\,2} - dy_3 + u_3 - \alpha_3(-\beta x_3 - \delta x_4 + x_1 x_2)$
$\dot{e}_3 = e_1^{\,2} + 2e_1\,\alpha_1\,x_1 - de_3 - d\,\alpha_3\,x_3 + u_3 + \alpha_3\,\beta x_3 + \delta\,\alpha_3\,x_4 - \alpha_3\,x_1 x_2 + \alpha_1^{\,2}\,x_1^{\,2}$
$\dot{e}_4 = -y_1 y_3 - \delta y_4 + u_4 - \alpha_4(-dx_4 + fx_3 + x_1 x_2)$
$\dot{e}_4 = -e_1 e_3 - e_1\,\alpha_3\,x_3 - \alpha_1\,x_1 e_3 - \alpha_1\alpha_3\,x_1 x_3 - \delta e_4 - \delta\,\alpha_4\,x_4 + u_4 + \alpha_4\,dx_4 - \alpha_4\,fx_3 - \alpha_4\,x_1 x_2$ (7) The state errors satisfy

$\lim_{t\to\infty} e_{1(t)} = 0$, $\lim_{t\to\infty} e_{2(t)} = 0$, $\lim_{t\to\infty} e_{3(t)} = 0$, $\lim_{t\to\infty} e_{4(t)} = 0$ (8)

Then the global and asymptotical stability of the system (7) means that system (1) and (3) are in HPS. Now choose the controller as

$u_1 = \alpha\alpha_1\,x_2 - \alpha\alpha_2\,x_2$
$u_1 = x_2(\alpha\alpha_1 - \alpha\alpha_2)$
$u_2 = -x_1 x_3(\alpha_2 - \alpha_1\alpha_3) - x_1(b\,\alpha_1 + \alpha\alpha_2 - \Upsilon\,\alpha_2) + x_2\,\alpha_2\,(\Upsilon + c)$
$u_3 = -[-d\,\alpha_3\,x_3 + \alpha_3\,\beta x_3 + \delta\,\alpha_3\,x_4 - \alpha_3\,x_1 x_2 - \alpha_1^{\,2}\,x_1^{\,2}]$
$u_4 = -\alpha_1\alpha_3\,x_1 x_3 + \delta\,\alpha_4\,x_4 - \alpha_4\,dx_4 + \alpha_4\,fx_3 + \alpha_4\,x_1 x_2$ (9) Designed a controller as

u_i = -Kx_i , u_j=0, $j \neq i$

Where I, j,=1,2,3,………….

$u_4 = -\alpha_1\alpha_3\,x_1 x_3 + \gamma\,\alpha_4 x_4 - \alpha_4\,x_4 - \alpha_4\,dx_4 + \alpha_4\,fx_3 + \alpha_4\,x_1 x_2$ -Ke_4

Where Kis the feedback gain and is adapted according to following update law

K = $K_2 e_1^2$ K(0) = 0, K_2 >0 (10)

Consider a Lyapunov function

V_1(t)=$\frac{1}{2}$[e_1^2 e_2^2 e_3^2 e_4^2] +$\frac{1}{2K_2}[k - k^*]^2$ (11)

It is clear that V_1 (t) is a positive definite function.By applying the controller (9) to (7)

The error dynamics

is $\dot{e}_1 = \alpha\,e_2 - \alpha\,e_1$
$\dot{e}_2$ = be_1 – ce_2 – $e_1 e_3$ – $e_1\,\alpha_3\,x_{3} - \alpha_1 x_1 e_3$
$\dot{e}_3 = 2e_1\,\alpha_1\,x_1 - de_3 e_1^2 + e_1^2$
$\dot{e}_4 = -e_1 e_3 - e_1\,\alpha_3\,x_3 - \alpha_1\,x_1 e_3 - \delta e_4 - ke_4$
$\dot{e}_4 = -e_1 e_3 - e_1\,\alpha_3\,x_3 - \alpha_1\,x_1 e_3 - e_4(\delta + k)$ (12)
$\dot{v}_{1(t)} = [e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + \frac{1}{K_2}[k - k^*]\dot{k}]$

$\dot{e}_1 = -\alpha\,e_1$
$\dot{e}_2 = -ce_2$
$\dot{e}_3 = -de_3$
$\dot{e}_4 = -(c + \delta)e_4$
$\dot{v}_{1(t)} = [-\alpha\,e_1(e_1) - ce_2(e_2) - de_3(e_3) - (\delta + k)e_4 e_4 + \frac{1}{K_2}[k - k^*]\dot{k}]$
K = $k_2 e_1^2$

$\dot{v_{1(t)}} = [-e_1^2(-k+k^*+\propto) - ce_2^2 - de_3^2 - (\delta+k)e_4^2] + \frac{1}{K_r}[k-k^*](k_2e_1^2)]$

$\dot{v_{1(t)}} = [-e_1^2(-k+k^*+\propto) - ce_2^2 - de_3^2 - (\delta+k)e_4^2]$

$\dot{v_{1(t)}} = -e^T pe$ (13)

$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}$

$P = \begin{bmatrix} k^* - k - \propto & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & 0 & (\delta+k) \end{bmatrix}$ (14)

The symmetric matrix should be positive definite when satisfies the following conditions

$(-k+k^*-\propto) > 0$

$c(-k+k^*-\propto) > 0$

$d[(c)(-k+k^*-\propto)] > 0$ (15)

$(\delta+k)(d)(c)\,[(-k+k^*-\propto)] > 0$

Since $a > 0, b > 0, c > 0, d > 0, k > 0$,the symmetric matrix p is positive when $k^* > k$, then $\dot{v_{1(t)}}$ is a negative semidefinite.Based on Barbalat's Lemma $\dot{v_{1(t)}} \to 0$ as $t \to \infty$.It follows that the error variables becomes zero as t tends to infinity ,

$\lim_{t\to\infty} |\, y_i - x_i \,| = 0$

$i = 1,2,3 \ldots$ this means that the two chaotic system(1) and (3) are in HPS under the controller (9).

III. NUMERICAL SIMULATIONS

To verify and demonstrate the effectiveness of the proposed adaptive control laws for HPS, we will display the numerical simulation results. Let the system parameters be $a = 5, b = 10, c = 3.8, d =$ 1 and , the initial states of the drive system and the response system are $y_1 = 1, y_2 = 1, y_3 = 1, y_4 = 1$ and derive system conditions $x_1(0) = 1, x_2(0) = 1, x_3(0) = 1, x_4(0) = 1$, respectively, Fourth-order Runge-Kutta method is used to solve the system with time step size 0.001.

IV. FIGURES

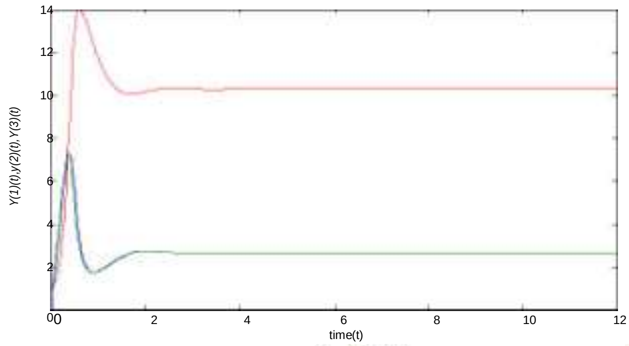

Fig.1 State trajectories of the response system of y_1, y_2, y_3 with the values of $u_1 = 0, u_2 = 11, u_3 = 3$

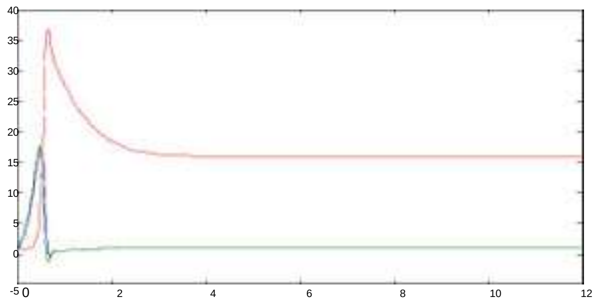

Fig.2 State trajectories of the response system of y_1, y_2, y_4 with the values of $u_1 = 0, u_2 = 11, u_4 = 3$

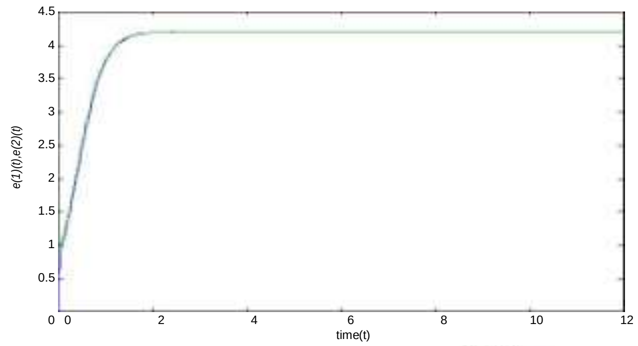

Fig 3. Error dynamics between $e_1(t), e_2(t)$

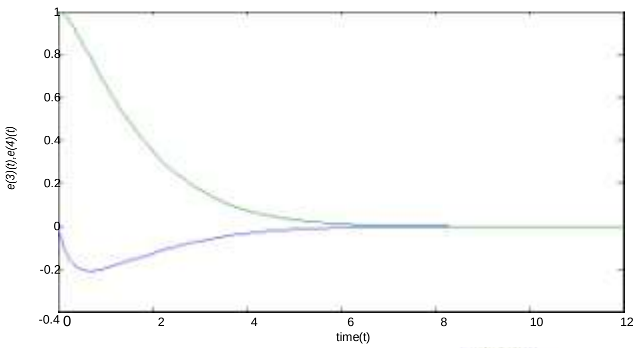

Fig 4. Error dynamics between $e_3(t), e_4(t)$

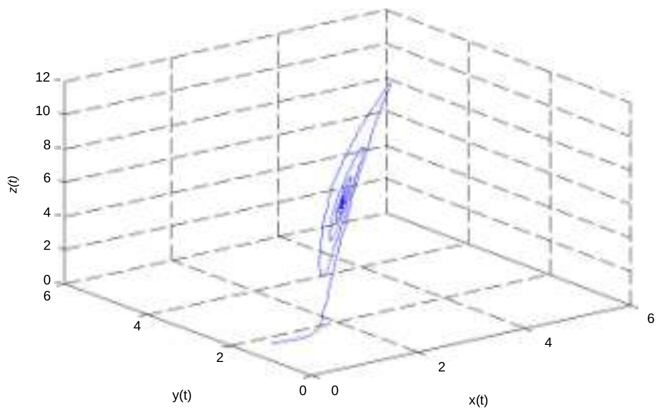

Fig 5. Chaotic attractor of response system

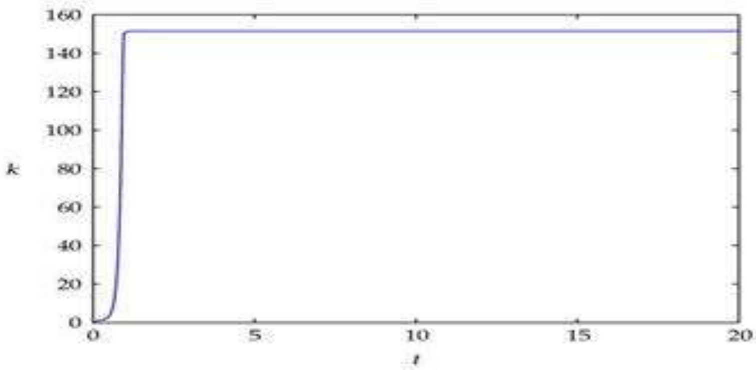

Fig. 6: Adaptive feedback control gain *k* with time history.

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