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The gravimetric geoid for Egypt using the far-zone topographic effects for different topographic-isostatic methods and spherical approximation

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Abstract:- It's known that all topographic reduction techniques give the same gravimetric geoid if the indirect effect has been taken into account. However, there are some requirements which should be taken into consideration when deciding to use certain gravity reduction technique to determine the gravimetric geoid. Mainly, the reduction must yield the gravity anomalies that are smooth and small, the reduction technique must correspond to a geophysical meaning and the indirect effect must be as small as possible. To achieve these requirements, the effect of topography must be taken for the whole earth. The aim of this research is to determine the gravimetric geoid for Egypt by taking the effect of topography for the whole earth. To determine the effect of topography, the area around the computation point is divided into two parts: these are near and far-zone. The near zone is calculated numerically and far-zone is calculated form spherical harmonic of topography. In this paper, three methods of gravity reduction techniques are applied. These are the Airy-Heiskanen method and the Helmert's second method of condensation as well as the generalized Helmert of condensation. The direct, primary and secondary effect of topography is taken into account in this research. The gravimetric geoids for Egypt from the three techniques are computed and compared.

Keywords:- Gravimetric Geoid for Egypt - topographic effect - Airy-Heiskanen model - Digital terrain model - Helmert´s methods of condensation.

I. INTRODUCTION

The determination of the geoid requires the movement of the topographical as well as the atmospheric masses inside the geoid to achieve harmonicity in the space outside the boundary surface. In a subsequent computation step the change of the gravitational potential by this manipulation must be considered properly. In this investigation the gravitational effects of the topographical masses are computed for determining the gravimetric geoid for Egypt. The effects of the topographic, isostatic and compensation masses on the geoid heights are evaluated as three separate contributions: the direct topographic effect on the gravity, the primary indirect topographic effect on the geoid and the secondary indirect topographic effect on the gravity in case of the gravity at the surface of the Earth. (e.g. Novàk et al. 2001; Martinec et al. 1993; Vaníček and Martinec 1994a,b). The harmonized (unknown) disturbing potential T^h can be determined for any point above the geoid by (e.g. Martinec et al. 1993),

$$T^h(r,\varphi,\lambda) = T(r,\varphi,\lambda) - \delta V(r,\varphi,\lambda)\,, \tag{1}$$

with the disturbing gravity potential of the Earth's gravitational field *T*. $\delta V(r,\varphi,\lambda)$ is the difference between the gravitational potential of the topography V^t and the gravitational potential of the compensating masses V^c (Martinec 1998),

$$\delta V(r,\varphi,\lambda) = V^t(r,\varphi,\lambda) - V^c(r,\varphi,\lambda)\,. \tag{2}$$

The spherical coordinates r, φ, λ of the computation point at the surface of the Earth refer to a geocentric coordinate system. The geoid height can be derived by Bruns's formula (Bruns 1878) from the harmonized disturbing potential as solution of the geodetic boundary value problem and a correction term on the co-geoid height, $PITE(R,\varphi,\lambda)$, the primary indirect topographical effect (Martinec 1998),

$$N(\varphi,\lambda) = \frac{T(R,\varphi,\lambda)}{\gamma_0(\varphi)} = \frac{T^h(R,\varphi,\lambda)}{\gamma_0(\varphi)} + \frac{\delta V(R,\varphi,\lambda)}{\gamma_0(\varphi)} = \frac{T^h(R,\varphi,\lambda)}{\gamma_0(\varphi)} + PITE(R,\varphi,\lambda)\,, \tag{3}$$

where $\gamma_0(\varphi)$ is the normal gravity at the reference ellipsoid and *R* the mean radius of the Earth.

The harmonized gravity disturbances δg^h can be obtained from Eq. (1) based on the gravity disturbances in the real gravity field δg by performing the first radial derivatives (after linearization and approximation) as follows (Vaníček et al. 1999; Martinec 1998):

$$\delta g^h(r,\varphi,\lambda) = -\frac{\partial T^h(r,\varphi,\lambda)}{\partial r} = -\frac{\partial T(r,\varphi,\lambda)}{\partial r} + \frac{\partial \delta V(r,\varphi,\lambda)}{\partial r} = \delta g(r,\varphi,\lambda) + DTE(r,\varphi,\lambda)\,. \tag{4}$$

The difference between the two gravity disturbances in the real and harmonized gravity field is known as the direct topographical effect on gravity, $DTE(r,\varphi,\lambda)$. Direct and primary indirect topographic-compensation effect given in Eq. **Error! Reference source not found.** and Eq. **Error! Reference source not found.**, respectively, represent the effect of the topographic and compensation masses for geoid determinations.

The expression for the direct and the secondary indirect topographical effect on gravity at the surface of the Earth can be derived by applying the Stokes boundary operator to Eq. (1), which reads in spherical approximation,

$$\Delta g^h(r,\varphi,\lambda) = -\frac{\partial T^h(r,\varphi,\lambda)}{\partial r} - \frac{2}{r}T^h(r,\varphi,\lambda)\,. \tag{5}$$

Inserting the definition of T^h from Eq. (1) into Eq. **Error! Reference source not found.**, results in (Vaníček et al. 1999):

$$\begin{aligned}\Delta g^h(r,\varphi,\lambda) &= \Delta g(r,\varphi,\lambda) + \frac{\partial \delta V(r,\varphi,\lambda)}{\partial r} + \frac{2}{r}\delta V(r,\varphi,\lambda)\\ &= \Delta g(r,\varphi,\lambda) + DTE(r,\varphi,\lambda) + SITE(r,\varphi,\lambda).\end{aligned} \tag{6}$$

II. THE POTENTIAL OF TOPOGRAPHICAL MASSES AND ITS RADIAL DERIVATIVE

In the following, the geoid is approximated by a geocentric reference sphere of radius *R* (6371 km). The geocentric radii of the computation and the integration points are given by adding the orthometric heights of these points to the radius of the geocentric sphere, so that the ellipsoidal figure of the Earth is neglected (see Fig.1). The density of the topographic masses is considered to be constant throughout the paper.

The potential of the topographic masses between the topographic surface S and the sphere of radius R at the computation point *P* is given by Newton's integral in spherical coordinates as follows (Fig. 1):

$$V_t\big|_{\mathbf{r}=\mathbf{r}_P} = G\rho \iint_\sigma \int_R^{r_Q} \frac{\xi^2}{l}\bigg|_{\mathbf{r}=\mathbf{r}_P} d\xi\, d\sigma, \tag{7}$$

with

$$l := \sqrt{r^2 + \xi^2 - 2r\xi\cos\psi}\ , \tag{8}$$

the universal gravitational constant *G*, the density of the topography ρ, $r_Q = R + H_Q$, and the surface element in spherical coordinates, $d\sigma = \cos\varphi_Q d\varphi_Q d\lambda_Q$. The geocentric angle ψ is the spherical distance between the radius vectors of the computation point $\mathbf{r}_P(r,\varphi,\lambda)$ and the integration point $\mathbf{r}_Q(r_Q,\varphi_Q,\lambda_Q)$, given by:

$$\cos\psi = \sin\varphi\sin\varphi_Q + \cos\varphi\cos\varphi_Q\cos(\lambda_Q - \lambda)\,, \tag{9}$$

where φ and λ are the geodetic latitude and longitude of the computation point. Further quantities are shown in Fig. 1.

The inner integral of Eq. (7) can be given by Gradshteyn and Ryzhik (1980):

$$V_t\big|_{\mathbf{r}=\mathbf{r}_P} = \frac{1}{2}G\rho\iint_\sigma \Big[F_0(r,\psi,\xi)\big|_{\mathbf{r}=\mathbf{r}_P}\Big]_{\xi=R}^{r_Q} d\sigma\,, \tag{10}$$

with

$$F_0(r,\psi,\xi) = (\xi + 3r\cos\psi)l + r^2(3cos^2\psi - 1)\ln(l(r,\psi,\xi) + \xi - r\cos\psi) \tag{11}$$

and $\mathbf{r}_P = (R + H_P, \varphi, \lambda)$.

The integration interval in Eq. (10) can be separated into two parts, $\left[R, r_{P'}\right]$ and $\left[r_{p'},\ r_Q\right]$. The quantity $r_{P'}=R+H_{p'}$ denotes the geocentric radius of the surface point $P' \in S$, which is located under the computation point P (Fig. 1). Then the integral of Eq. (10) can be given by (Makhloof and Ilk 2008):

$$V_t\big|_{\mathbf{r}=\mathbf{r}_P} = \frac{2\pi G\rho}{r_P}\begin{Bmatrix} \frac{2}{3}(r_{P'}^3 - R^3) & \forall r_P \geq r_{P'} \\ r_P(r_{P'}^2 - R^2) & \forall r_P \leq r_{P'} \end{Bmatrix} + \frac{1}{2}G\rho\iint_\sigma\Big[r_Q\bar{l}' - r_{P'}\bar{l} + + 3r_P\cos\psi(\bar{l}' - \bar{l}) + r_P^2(3\cos^2\psi - 1)\ln\left|\frac{\bar{l}' + r_Q - r_P\cos\psi}{\bar{l} + r_{P'} - r_P\cos\psi}\right|\Big]d\sigma, \tag{12}$$

with the Euclidean distances as shown in Fig. 1 and defined by:

$$\bar{l} := \sqrt{r_P^2 + r_{P'}^2 - 2r_P r_{P'}\cos\psi}\,, \tag{13}$$

$$\bar{l}' := \sqrt{r_P^2 + r_Q^2 - 2r_P r_Q \cos\psi}\,. \tag{14}$$

The first term in Eq. (12) is due to the mass of a homogenous spherical shell of thickness $H_{P'} = r_{P'} - R$ acting on the gravitational potential at the computation point *P*. Two cases have to be distinguished: the point is located either on or outside the topography of the Earth ($r_P \geq r_{P'}$) or the point is located below or on the geoid ($r_P \leq r_{P'}$). The integral term in Eq. (12) denotes the influence of the varying terrain due to $r_Q \neq r_{P'}$ (terrain correction). In this investigation we will consider the first case ($r_P \geq r_{P'}$) as we will determine the effect of the topographic-isostatic masses at the surface of the Earth. The second case of Eq. (12) is necessary for the calculation of the primary indirect topographical effect.

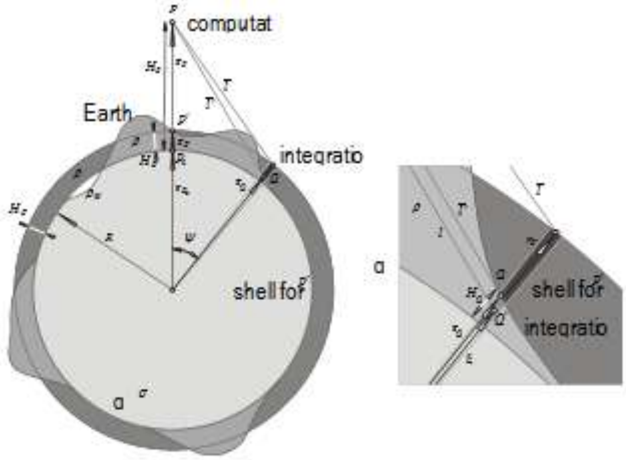


Fig. 1 Geometry of the topography in spherical approximation

The first derivatives of the potential of the topographic masses with respect to *z* can be calculated using the following formula:

$$\frac{\partial V_t}{\partial z} = \frac{\partial V_t}{\partial r} \tag{15}$$

$$\frac{\partial V_t}{\partial r}\bigg|_{\mathbf{r}=\mathbf{r}_P} = -\frac{4\pi G\rho}{3r^2}(r_{p'}^3 - R^3)\bigg|_{\mathbf{r}=\mathbf{r}_P} + G\rho\iint_\sigma\Big[F_1(r,\psi,\xi\big|_{\mathbf{r}=\mathbf{r}_P})\Big]_{\xi=r_{P'}}^{r_Q}d\sigma\,, \tag{16}$$

with the function,

$$F_1(r,\psi,\xi) = \Big(\Big[(\xi^2 + 3r^2)\cos\psi + \xi r(1 - 6\cos^2\psi)\Big]/l(r,\psi,\xi) + + r(3\cos^2\psi - 1)\ln\left|\xi - r\cos\psi + l(r,\psi,\xi)\right|\Big). \tag{17}$$

The integration domain is divided into two integration sub-domains $\sigma = \sigma_0 \cup \sigma_{Fz}$ where σ_0 the near-zone is and σ_{Fz} is the far-zone. Then second term in Eq. (16) can be written as the sum of the two parts as follow

$$G\rho \iint_{\sigma} \left[F_1(r,\psi,\xi|_{\mathbf{r}=\mathbf{r}_P}) \right]_{\xi=r_{P'}}^{r_Q} d\sigma = G\rho \iint_{\sigma_0} [F_1(r,\psi,\xi] d\sigma_0 + G\rho \iint_{\sigma_{Fz}} [F_1(r,\psi,\xi] d\sigma_{Fz} \qquad (18)$$

The first term can be calculated till to small geocentric angle ψ by numerical integration. The second term can be calculated using spherical harmonic expansion (Makhloof and Ilk 2008):

2.1 Direct topographic effect (DTE)

The direct topographical effect DTE on gravity was given by Martinec and Vaníček (1994a) in spherical approximation. *DTE* can be obtained from the difference between the first derivatives of the potential of the topographic and isostatic masses as follows (see Fig. 2). Then, the *DTE* for the Airy-Heiskanen model can be given by:

$$\begin{aligned} DTE^{ter}\Big|_{\mathbf{r}=\mathbf{r}_P} &= G\rho \iint_{\sigma} \frac{\partial}{\partial r} \int_{\xi=R}^{R+H_Q} l^{-1}(r,\psi,\xi)\Big|_{\mathbf{r}=\mathbf{r}_P} \xi^2 d\xi d\sigma - G\Delta\rho \iint_{\sigma} \frac{\partial}{\partial r} \int_{\xi=R-T-t_Q}^{R-T} l^{-1}(r_P,\psi,\xi)\Big|_{\mathbf{r}=\mathbf{r}_P} \xi^2 d\xi d\sigma \\ &= A^b + G\rho \iint_{\sigma} \frac{\partial}{\partial r} \int_{\xi=R+H_{P'}}^{R+H_Q} l^{-1}(r,\psi,\xi)\Big|_{\mathbf{r}=\mathbf{r}_P} \xi^2 d\xi d\sigma - \\ &- A^{bi} + G\Delta\rho \iint_{\sigma} \frac{\partial}{\partial r} \int_{\xi=R-T-t_P}^{R-T-t_Q} l^{-1}(r_P,\psi,\xi)\Big|_{\mathbf{r}=\mathbf{r}_P} \xi^2 d\xi d\sigma, \end{aligned} \qquad (19)$$

where t_P is the root under the computation point and t_Q is the thickness of the variable root (see Fig. 2). A^b and A^{bi} are the effects of the topographic and the isostatic Bouguer shells. Therefore, these terms are identical if the mass conservation principle is applied. In this case both terms can be skipped and the difference between remaining two terms is called the direct terrain effect DTE^{ter} on gravity. The first term on the right-hand side of Eq. (19) has been discussed already (Eq. (18)). The second term is determined following the same procedures as used in the first term of Eq. (18).

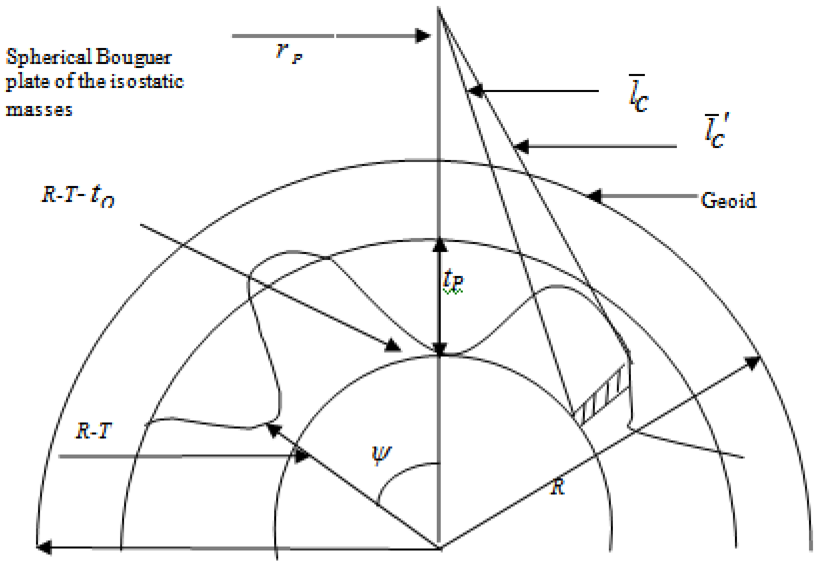


Fig. 2: Geometry of the isostatic masses in case of Airy-Heiskanen model

In *Helmert's second method of condensation*, the topographical masses are condensed on the geoid as a single layer (Heiskanen and Moritz 1967). In this model, the single-layer density k_Q is a function of the density and height of the topographic masses. Using the mass conservation principle, the local single-layer density can be given in spherical approximation as follows (Wichiencharoen 1982; Martinec 1998):

$$k_Q R^2 d\sigma = \rho \iint_\sigma \int_R^{R+H_Q} \xi^2 d\xi d\sigma \,. \tag{20}$$

The potential of the condensed masses at the computation point Q can be given by (Heiskanen and Mortiz, 1967):

$$V_c(P) = G \iint_\sigma \frac{k'}{l_c} . R_c{}^2 d\sigma', \tag{21}$$

where $l_c = \sqrt{r_Q^2 + R_c{}^2 - 2 r_Q R_c \cos\psi}$

with the density of the surface layer k' and the radius of the (approximate) condensation sphere R_c. In case of *generalized Helmert's condensation method* it holds $R_c = R - D$, and in case of Helmert's second condensation method $R_c = R$ (see Fig. 3). Then, the DTE of this technique is given by:

$$\begin{aligned} DTE(r,\varphi,\lambda)\big|_{\mathbf{r}=\mathbf{r}_P} &= \left.\frac{\partial \delta V}{\partial r}\right|_{\mathbf{r}=\mathbf{r}_P} \\ &= G\rho \iint_\sigma \frac{\partial}{\partial r} \int_{\xi=R}^{R+H_Q} l^{-1}(r,\psi,\xi)\big|_{\mathbf{r}=\mathbf{r}_P} \xi^2 d\xi d\sigma - G\rho \iint_\sigma \left(\int_{\xi=R}^{R+H_Q} \xi^2 d\xi \right) \frac{\partial}{\partial r} l^{-1}(r,\psi,R)\big|_{\mathbf{r}=\mathbf{r}_P} d\sigma. \end{aligned} \tag{22}$$

The contribution of the integral of Eq. (22) can be developed in the following form (Novàk et al. 2001):

$$\begin{aligned} DTE\big|_{\mathbf{r}=\mathbf{r}_P} &= A^b + G\rho \iint_\sigma \frac{\partial}{\partial r} \int_{\xi=R+H_{P'}}^{R+H_Q} l^{-1}(r,\psi,\xi)\, \xi^2 d\xi \big|_{\mathbf{r}=\mathbf{r}_P} d\sigma - \\ &\quad - A^{cb} - G\rho \iint_\sigma \int_{\xi=R+H_{P'}}^{R+H_Q} \xi^2 d\xi \frac{\partial}{\partial r} l^{-1}(r,\psi,R)\big|_{\mathbf{r}=\mathbf{r}_P} d\sigma, \end{aligned} \tag{23}$$

where A^b and A^{cb} are the effects of the topographic Bouguer shell and the corresponding condensed Bouguer layer. The terms A^b and A^{cb} are equal if the mass conservation principle is applied (Wichiencharoen 1982). In this case, both terms can be skipped. The difference between the other two terms is called the direct terrain effect DTE^{ter} in the following, which accounts for the entire *DTE* on gravity.

In the case of *generalized Helmert of condensation*, the topographic masses are condensed on an internal surface parallel to the geoid, at a depth *D* below the geoid. If *D* is not equal to 21 km, this method is called *generalized Helmert's method of condensation* (Heck. 2003). The direct terrain effect after removing the effects of the topographical Bouguer shell and the condensed Bouguer layer can be given as follows (see Fig.3**Error! Reference source not found.**):

$$\begin{aligned} DTE^{ter}\big|_{\mathbf{r}=\mathbf{r}_P} &= G\rho \iint_\sigma \frac{\partial}{\partial r} \int_{\xi=R+H_{P'}}^{R+H_Q} l^{-1}(r_P,\psi,\xi)\, \xi^2 d\xi \big|_{\mathbf{r}=\mathbf{r}_P} d\sigma - \\ &\quad - G\rho \iint_\sigma \int_{\xi=R+H_{P'}}^{R+H_Q} \xi^2 d\xi \frac{\partial}{\partial r} l^{-1}(r,\psi,R-D)\big|_{\mathbf{r}=\mathbf{r}_P} d\sigma, \end{aligned} \tag{24}$$

where again both of the two integrals are computed as shown in Eq. (19)

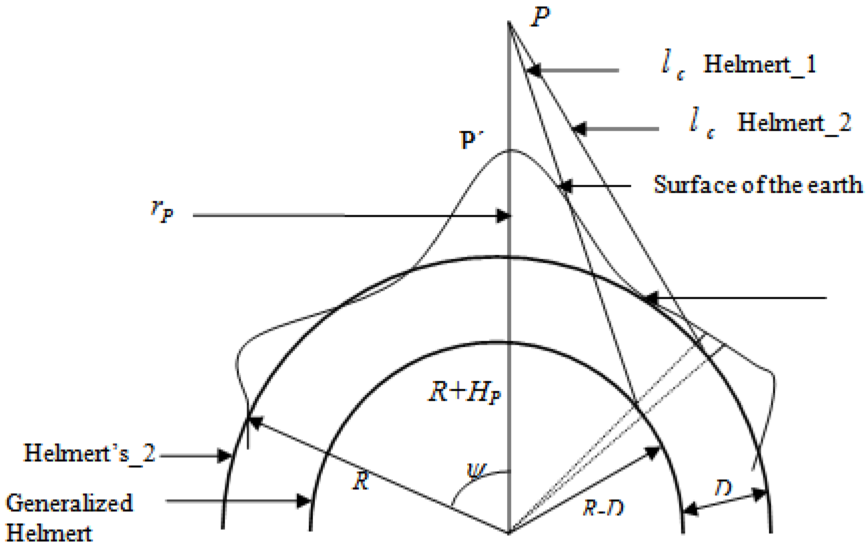


Fig. 3: Geometry of Helmert's condensation methods

2.2 The primary indirect topographical effect on geoid heights

The primary indirect topographical effect PITE on the geoid is given by Bruns's formula (Bruns 1878; Eq. **Error! Reference source not found.**) as follows:

$$PITE = PITE^{shell} + PITE^{ter} := \frac{\delta V}{\gamma_0} = \frac{\delta V^{shell}}{\gamma_0} + \frac{\delta V^{ter}}{\gamma_0}. \tag{25}$$

The potential of the Bouguer shell of the topographical masses and the potential of the Bouguer layer of the condensed masses is given by:

$$\frac{\delta V^{shell}}{\gamma_0} = \frac{2\pi G\rho_{cr}\left[(\mathrm{R}+H_{p'})^2 - R^2\right]}{\gamma_0} - \frac{4\pi G\rho_{cr}\left[(R+H_{P'})^3 - R^3\right]}{3(\mathrm{R}+\mathrm{H}_{P'})\gamma_0}. \tag{26}$$

Then, the primary indirect terrain effect $PITE^{ter}$ on the geoid in case of Helmert's second method of condensation and generalized Helmert method of condensation is given by (Martinec and Vaniček 1994b),

$$\begin{aligned} PITE^{ter}\Big|_{\mathbf{r}=\mathbf{r}_{P_0}} &= \frac{\partial V^{ter}}{\gamma_0} = \frac{G}{\gamma_0}\rho\iint_{\sigma}\int_{\xi=R+H_{P'}}^{R+H_Q} l^{-1}(R,\psi,\xi)\,\xi^2 d\xi\Big|_{\mathbf{r}=\mathbf{r}_{P_0}} d\sigma - \\ &-\frac{G}{\gamma_0}\rho\iint_{\sigma}\int_{\xi=R+H_{P'}}^{R+H_Q} l^{-1}(R,\psi,R-D)\,\xi^2 d\xi\Big|_{\mathbf{r}=\mathbf{r}_{P_0}} d\sigma. \end{aligned} \tag{27}$$

The first integral is calculated according to Eq. (12) and the second term is numerically integrated.

In the case of the *Airy-Heiskanen model*, the primary indirect terrain effect for topographic-compensation masses is given by the formula:

$$\begin{aligned} PITE^{ter}\Big|_{\mathbf{r}=\mathbf{r}_{P_0}} &= \frac{G}{\gamma_0}\rho\iint_{\sigma}\int_{\xi=R+H_{P'}}^{R+H_Q} l^{-1}(R,\psi,\xi)\Big|_{\mathbf{r}=\mathbf{r}_{P_0}} \xi^2 d\xi d\sigma - \\ &+\frac{G}{\gamma_0}\Delta\rho\iint_{\sigma}\int_{\xi=R-T-t_P}^{R-T-t_Q} l^{-1}(R,\psi,\xi)\Big|_{\mathbf{r}=\mathbf{r}_{P_0}} \xi^2 d\xi d\sigma. \end{aligned} \tag{28}$$

Also, Eq.(28) ca be manipulated in the same way as Eq, (27).

2.3 The secondary indirect topographical effect on gravity

For the *Stokes-Helmert* problem, the spherical form of the secondary indirect topographical effect *SITE* on gravity was formulated by Vaníček et al. (1999) as a re-scaled value (the scale is equal to 2/*r*) of the residual topographical potential evaluated at a radius equal to the radius of the Earth's surface. The different topographic-compensation models will be discussed in the following paragraphs.

In *Helmert's second method of condensation and generalized Helmert method of condensation*, the difference between the gravitational potential of the topographical shell and its condensed counterpart is zero in case of mass conservation. The *SITE* can be computed using the following expression (Vaníček et al. 1999):

$$\begin{aligned} SITE^{ter}\Big|_{\mathbf{r}=\mathbf{r}_{P'}} &= \frac{2G}{r_{P'}}\rho\iint_{\sigma}\int_{\xi=R+H_{P'}}^{R+H_Q} l^{-1}(r,\psi,\xi)\Big|_{\mathbf{r}=\mathbf{r}_{P'}}\xi^2 d\xi d\sigma - \\ &-\frac{2G}{r_{P'}}\rho\iint_{\sigma}\int_{\xi=R+H_{P'}}^{R+H_Q} l^{-1}(r,\psi,R-D)\Big|_{\mathbf{r}=\mathbf{r}_{P'}}\xi^2 d\xi d\sigma. \end{aligned} \tag{29}$$

Again, Eq.(29) ca be manipulated in the same way as Eq, (27).

In case of the *Airy-Heiskanen model*, the *SITE* is calculated according to (Makhloof and Ilk 2008):

$$\begin{aligned} SITE^{ter}\Big|_{\mathbf{r}=\mathbf{r}_{P'}} &= \frac{2G}{r_{P'}}\rho\iint_{\sigma}\int_{\xi=R+H_{P'}}^{R+H_Q} l^{-1}(r,\psi,\xi)\Big|_{\mathbf{r}=\mathbf{r}_{P'}}\xi^2 d\xi d\sigma + \\ &+\frac{2G}{r_{P'}}\Delta\rho\iint_{\sigma}\int_{\xi=R-T-t_P}^{R-T-t_Q} l^{-1}(r,\psi,\xi)\Big|_{\mathbf{r}=\mathbf{r}_{P'}}\xi^2 d\xi d\sigma. \end{aligned} \tag{30}$$

Finally, Eq.(30) ca be manipulated in the same way as Eq, (27).

III. NUMERICAL ANALYSIS

3.1 Gravity data

The gravity data available for this investigation consists of 935 gravity station irregularly distributed on the Egyptian territory. All coordinates are referred to GRS80. These data can be classified into three groups: about 180 gravity stations observed by Egyptian Survey Authority (ESA) and these data are observed along the first order levelling lines, a bout 77 first order gravity stations forming the National Gravity Standardization Base Net (NGSBN77) and the rest of the stations are observed by General Petroleum Company (GPC).

3.2 Height data

The available height model for Egypt, used in this study has been produced by Abd-Elmotaal (2010). The grid spacing in latitude and longitude directions for this DTM are (12"*12"). This DHM extended from 22-40 in latitude and from 19-35 in longitude. For calculating the spherical harmonic expansion of topography a global DTM such as GEBCO (General Bathymetric Chart of the Oceans) with one arc-minute resolution (http://www.ngdc.noaa.gov/mgg/gebco) is used.

3.3 Geoid computations

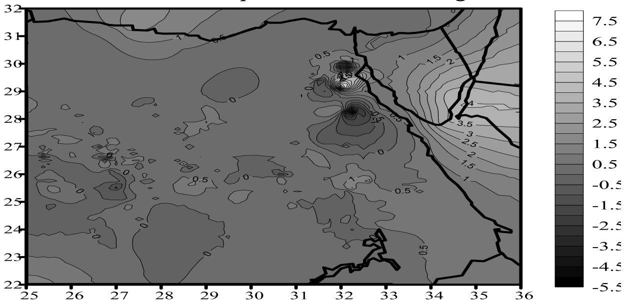
In this section, the direct terrain effects are computed for the three method of gravity reductions. The near- zone effect is computed using numerical integration for a cap size of $\psi \le 3^0$. The rest of topography is computed using spherical harmonic expansion. The summation of the two parts gives the total direct terrain effect which is shown in Fig. 4. The secondary indirect terrain effect has been manipulated in the same way as DTE and shown in Fig. 5. Then, the reduced gravity anomalies in the framework of the remove-restore technique is computed by

$$\Delta g = \Delta g_F + DTE + SITE - \Delta g_{\mathrm{Re}\,f}. \tag{31}$$

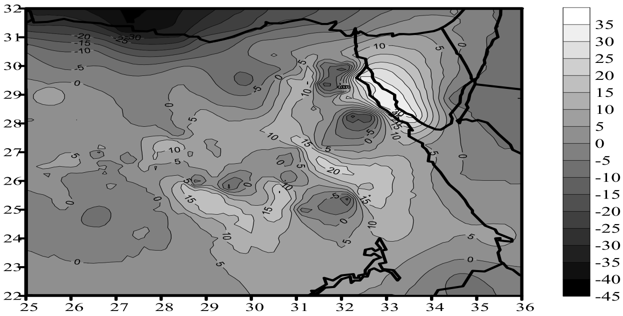
where $\Delta g_{\mathrm{Re}\,f}$ is the effect of the reference ellipsoid on gravity anomalies. Thus the final computed geoid is given by:

$$N = N_{\Delta g} + PITE + N_{\text{Re}\,f}\,. \tag{32}$$

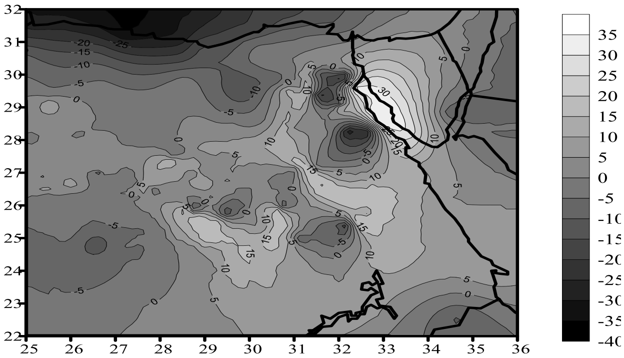
Finally, the gravimetric geoid for different techniques are shown in Fig.6.



Helmert's second method of condensation
(Max=7.83, Min=-8.99, Mean=0.31, Std=0.24)

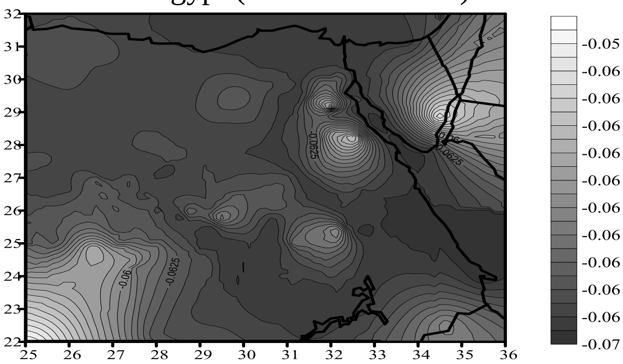


Generalized Helmert's method of condensation (condensation depth=32 km)
(Max=32.85, Min=-39.29,Mean=5.21, Std=10.32)

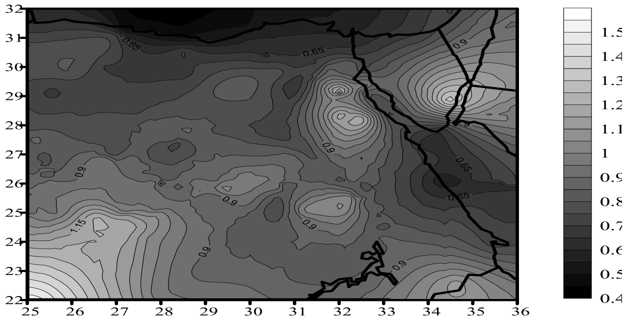


Airy-Heiskanen model (Max=32.79, Min=-35.79,
Mean=4.1, Std=10.46)

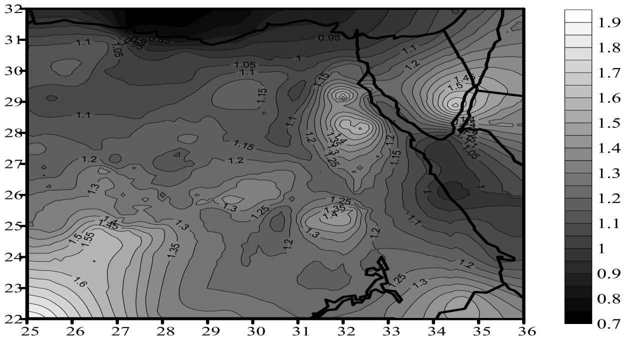
Fig.4: Direct terrain effect on gravity for different topographic-compensation models for Egypt (all units in mGal)



Helmert's second method of condensation
(Max=-.0064, Min=-.053, Mean=-0.063, Std=0.001)

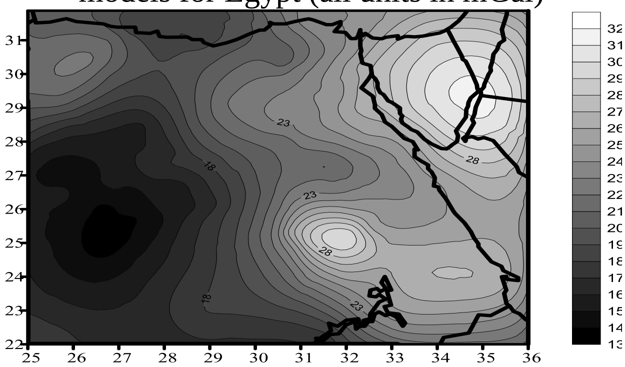


Generalized Helmert's method of condensation (condensation depth=32 km)
(Max=1.92, Min=0.72, Mean=1.18, Std=0.128)

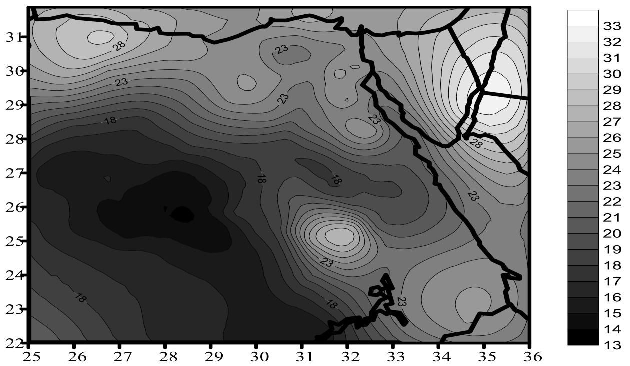


Airy-Heiskanen model (Max=1.56, Min=0.44, Mean=0.88, Std=0.122)

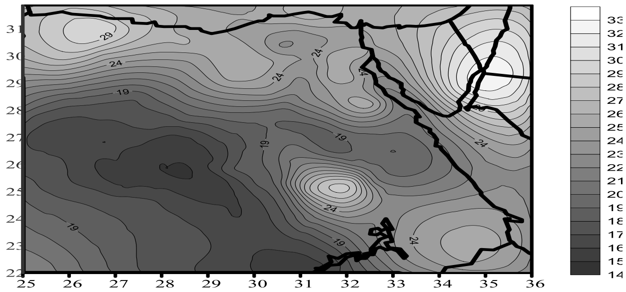
Fig.5: Secondary indirect terrain effect on gravity for different topographic-compensation models for Egypt (all units in mGal)



Helmert's second method of condensation
(Max=31.45, Min=13.57, Mean=21.60, Std=4.56)



Generalized Helmert's method of condensation (condensation depth=32 km)
(Max=32.74, Min=13.78, Mean=21.48, Std=4.50)



Airy-Heiskanen model (Max=32.74, Min=14.70, Mean=21.97, Std=4.24)

Fig.6: Gravimetric geoid for Egypt (all units in m)

IV. CONCLUSIONS

In this study, the direct topographic, the secondary indirect topographic and the primary indirect topographic effects for three gravity reduction techniques for Egypt are computed. The effect of the far-zone topography is transformed into the spectral domain following Molodenskij's approach. The formulae for the far-zone effects show a sufficient accuracy for spherical caps larger than $\psi_0 = 3^o$. The rest of the topography around the computation point are computed numerically. Then, the gravimetric geoid for Egypt has been computed for three different gravity reduction techniques. The results indicated that, the geoid calculated from Generalized Helmert model of condensation is approximately the same as the geoid computed from Helmert's first method of condensation. Also, there are differences between the geoid calculated from Airy-Heiskanen model and those calculated from Helmert's methods of condensation. The results indicated that there some differences between gravemtric geoids for Egypt computed in spite of taking the far-zone effects of topography. The reasons for these differences are most probably happen as a results of interpolation errors and using constant density for the crust and muntle. Finally, some investigations including the use of the adapted reference field technique with variable density contrast (Abd-Elmotaal and kühtreiber (2003) is required as there are some differences between geoid calculated in this investigation and the geoid calculated in this paper.

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