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Decision criteria on the number of Manual and E – zone lanes to open on the Accra –Tema Motorway

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Abstract:- The dilemma of the number of toll lanes to open when toll facilities are to be established coupled with what proactive measure to take to avoid congestion of the facilities as well as what interventions could be made once congestion develops are recurrent concerns that have bedeviled the road agencies in Ghana. We investigate into the use of three main criteria as decision making tools in designing a new toll plaza and managing an existing one such as we have on the Accra – Tema Motorway. We first examine the minimum number of each lane type to open as the barest solution to provide at a plaza based on the arriving traffic and demonstrate how this criterion could be an effective toll management tool. Secondly we develop a strategy that would allow us to decide on the number of lanes to open based on the arriving traffic and the desired level of service (LOS) by using the average waiting time as a predicting measure to predict the LOS by means of (a) a regression equation or (b) a simple rule of thumb. Thirdly we explore the optimum number of lanes to open for each lane type by considering the cost of the waiting time of arrivals and the cost of operation of each lane type. In each criterion examined, we present a general solution applicable to any plaza with similar lane types such as on the Accra – Tema motorway as well as solutions specific to the toll plazas on the motorway using traffic data obtained from the Ghana Highway Authority. This paper aims to assist the Ghana Highway Authority and the Ghana Road Fund in their decisions relative to the toll plazas on the Accra – Tema Motorway and indeed in the establishment and management of new tolling facilities on newly developed roads in Ghana.

Keywords: Accra – Tema motorway, Average waiting time, Decision making tools, Level of service, Minimum number, Optimum number, Regression equation, Toll lanes, Ghana.

I. INTRODUCTION

Tolling of roads has become necessary because of the huge financial resources required to develop and maintain roads and the application of the principle that whoever benefits must pay (O'Flaherty, 1986). The collection of tolls however imposes extra cost on road users as they have to slow down and stop to pay, this increases their vehicle operating cost as well as time cost due to the extra speed change cycles often resulting in long congestion queues during peak hours. Tolling also imposes additional cost to the road subsector in setting up the appropriate infrastructure for collection and payment of tolls (Robinson et al., 1998).

The most recurrent problem in designing and operating toll plazas in Ghana is in determining the number of service points to open; initial lanes provided often have to be altered in response to increases in traffic volume after implementation. The Accra – Tema Motorway which is the first tolled road in the country is no exception to this problem. The motorway which is a four lane divided highway operated four manual lanes and an electronic toll collection lane (E – zone lane) in either direction. Peak hours were characterised with long queues with attendant delays for motorists. Previous decisions on the number of service points to provide were addressed without any prior scientific analysis. In Opoku –Boahen, Adams and Salifu (2013) we provided the initial step in addressing this problem by establishing the capacities of the lane types on the motorway.

This paper provides an objective basis for determining the number of service points to be opened at the toll plazas under the operational regime of the Accra – Tema Motorway and indeed on various tolling facilities in the country. We explore strategies how to optimally combine the manual and E - zone service points to minimise road user waiting time and toll operating costs. This research work presents an easy to use visual approach to the decision making process that could be applied by the agencies responsible for tolling highway facilities.

II. LITERATURE REVIEW

2.1 Cost associated with toll plazas

Toll plaza costs are partitioned into two categories: variable operating costs and user costs. User costs are based on consumer waiting time at the toll plaza. Variable operating costs are primarily driven by the cost of labour required in manning toll booths not utilized for electronic toll collection (Boronico and Siegel, 1998). Seongmoon (2009) argues that costs for electronic toll collection lanes involve only electricity and occasional maintenance. He argues that wages of manual lane toll collectors may vary depending on the time of day. Boronico and Siegel (1998) also states that variable costs for electronic collection lanes are negligible, involving only occasional maintenance costs.

2.1.1 User waiting time cost

Based on work reported by the world bank, Gwilliam (1997) states that it is customary to state all time values as proportions of either personal or household incomes. He further indicates that in most countries it is assumed that the value of time is directly proportional to income and recommended that the value of time be treated as increasing over time in proportion to Gross Domestic Product (GDP) per capita unless there is local evidence to the contrary. Wardman (2001) argues that household income is survey - based and can be expected to be a less reliable indicator than the GDP measure. Abrantes and Wardman (2009) establishes the elasticity of the value of time with respect to GDP per capita and found it to be highly significant.

I.T. Transport (2005) investigated travel time values in developing countries and found that the average base travel time saving values for rural travellers in Ghana in 2004 was US$ 0.18 per hour (Cedi 1,627). This base value was 64% of the rural wage rate and 52% of the household income per hour. The base value of US$ 0.18 per hour was 33 % of Ghana's GDP per capita income in 2004 when expressed in hourly terms, it is expected that this savings value would be higher when considering urban areas. The percentages put forward by I.T. Transport (2005) were consistent with Small (1992) who reviewed literature on the value of travel time and cited a range of estimates suggesting that the value of travel time is between one quarter and three quarters of gross wage. Small (1992) concludes that a reasonable average value of time for journey to work is 50 percent of the gross wage rate. Friedman and Waldfogel (1994) in their study of compliance cost of manual highway toll collection supplemented their best estimate that the value of time is half the gross wage with a conservative estimate that the value of time is only a quarter of the gross wage.

2.2 Capacity of a toll plaza and measures of performance

Having defined the parameters of the arrival and service processes; with the assumption of the General Service time distribution and a Poisson arrival process where the mean arrival rate per lane type is uniformly distributed across each lane, Seongmoon (2009) used the multiple M/G/1 queuing models to study the multi-type and multi-lane toll plaza system. He argued that the first step in designing a toll plaza is to determine the capacity N, which is the total number of lanes. N should be large enough to efficiently process transactions of all vehicles for any time period, but its upper bound may be determined by geographical restrictions. The mean arrival rate λ_i needs to be less than the mean service rate μ_i for each lane type n_i in order for the queuing system not to have infinite queues, that is

$$\frac{\lambda_i}{n_i} < \mu_i \ \ \forall i \qquad (2.1)$$

Otherwise, the queue will grow without bound and explode. Let b_i denote the minimum value of n_i satisfying the inequality (2.1). This b_i makes sure that all type *i* lanes have steady-state conditions with finite queue lengths. Although b_i guarantees a stable queuing system, toll plaza officials may impose another set of bounds for the number of type *i* lanes to open based on their strategic goals. Let the lower bound for the number of type *i* lanes to open be denoted by l_i where $l_i \geq b_i$ and the upper bound be u_i. Then, the capacity N of the toll plaza should be determined to satisfy (2.2) where K is the number of lane types.

$$\sum_{i=1}^{K} l_i \leq N \qquad (2.2)$$

If the mean arrival rates, the mean service times and the standard deviations are provided, the mean queue length L_i and the mean waiting time in the queue W_i for a type *i* lane can be obtained for the M/G/1 queuing system, using the Pollaczek–Khintchine formula as argued by Gross and Harris (1998) (as cited in Seongmoon, 2009), we can obtain L_i as

$$L_i = \frac{\lambda_i^2(\mu_i^2\sigma_i^2 + 1)}{2n_i^2\mu_i\left(\mu_i - \frac{\lambda_i}{n_i}\right)} \qquad (2.3)$$

However, the mean queue length may not be a good measure for performance comparison since service rates are different for each lane type. The mean waiting time in queue, not the queue length, impacts the perception of service quality at the toll plaza by the motorists. The mean waiting time of a vehicle in queue for a type *i* lane, obtained using Little's Law according to Gross and Harris (1998) (as cited in Seongmoon, 2009) is

$$W_i = \frac{L_i}{\frac{\lambda_i}{n_i}} = \frac{\lambda_i(\mu_i^2\sigma_i^2 + 1)}{2n_i\mu_i\left(\mu_i - \frac{\lambda_i}{n_i}\right)} \qquad (2.4)$$

and the total waiting time of all drivers at the toll plaza per unit time is

$$W = \sum_{i=1}^{K} \lambda_i W_i = \sum_{i=1}^{K} \frac{\lambda_i^2(\mu_i^2\sigma_i^2 + 1)}{2n_i\mu_i\left(\mu_i - \frac{\lambda_i}{n_i}\right)} \quad (2.5)$$

2.3 The operating and user- waiting cost minimization model

Seongmoon (2009) states that after capacity N of a plaza is determined, the next step is to figure out how to optimally operate the toll plaza. Operations of the toll plaza involve two major costs – operating costs and user-waiting costs, but with different weights associated with peak and off-peak times. Seongmoon (2009) further states that from discussions with toll plaza officials he found that the main goals of operating a toll plaza may not be the same between peak and off-peak times. During peak hours maximization of welfare benefits may be more important than savings of operating costs, and, hence, higher weights are given to user-waiting costs. On the other hand, during off-peak hours, such as around midnight, congestion rarely occurs, service standards may be well-satisfied even with partial capacity operation, and labour costs of toll collectors are higher, compared with day-time labour costs. Thus, higher weights are given to operating costs during off-peak times. Considering these issues, Seongmoon (2009) developed the following optimization model in order to find the optimal dynamic lane configuration over time.

Let C_i denote the rate of the operating cost of a type i lane and d be the rate of the value of time to the driver waiting in the queue. Then, the objective function (2.6) represents the sum of the total operating costs for all lanes used and the total user-waiting costs incurred by all drivers in the queues per unit time. The objective is to find the optimal lane configuration (n_1^* , n_2^* , n_3^*, ... n_k^*), which is the number of lanes to open for each payment type in order to minimise the total costs, Z, which is the sum of operating and user-waiting costs, by satisfying the following several constraints.

Minimise

$$Z = \sum_{i=1}^{K}(C_i n_i + d\lambda_i W_i), \quad (2.6)$$

Subject to

$$\frac{\lambda_i}{n_i} < \mu_i \;\; \forall i, \quad (2.7)$$

$$n_i \geq l_i \;\; \forall i, \quad (2.8)$$

$$n_i \leq u_i \;\; \forall i, \quad (2.9)$$

$$\sum_{i=1}^{k} n_i \begin{cases} = N \quad (peakhours), \\ \leq N \quad (off-peakhours), \end{cases} \quad (2.10)$$

$$W_i = \frac{\lambda_i(\mu_i^2\sigma_i^2 + 1)}{2n_i\mu_i\left(\mu_i - \frac{\lambda_i}{n_i}\right)} \;\; \forall i, \quad (2.11)$$

$$W_i \leq S_i \;\; \forall i \;\; (off-peakhours), \quad (2.12)$$

$$n_i = integer \;\; \forall i \, , \quad (2.13)$$

First, the number of toll lanes to open is restricted such that the arrival rate to each lane should be less than the service rate of each lane in order to be stable without an unlimited queue as in a constraint (2.7). Next, constraints (2.8) and (2.9) show that the variables representing the number of lanes to open for each collection type may have lower and upper bounds, that is, l_i and u_i. A constraint (2.10) picks an equality during peak times for full capacity operation and takes on an inequality during off-peak times allowing partial capacity operation. Waiting time in the queue for a type i lane is computed in (2.11) by using the M/G/1 queuing process. A constraint (2.12) is only included in the above optimization model during off-peak times in order to guarantee that the predetermined service standards for wait times are met even with partial capacity operation.. Obviously, the number of lanes to open should be an integer as in (2.13).

III. METHODOLOGY

3.1 Data collection

Data collection was carried out to ascertain the cost of operating the manual and E -zone lanes on the Accra – Tema motorway per hour as well as the mean arrival rates to the toll lanes per hour.

3.1.1 Manual lane operating cost

Open ended interviews were conducted with manual lane toll operators at the toll booths as well as a principal toll officer from the Ghana Road Fund Secretariat to ascertain the monthly salary or wage of a manual lane toll operator and whether there were differences in wages for day and night hours.

3.1.2 E - zone lane operating cost

The electrical specifications of the components of the E- zone lane which were the roadside antennae and personal computer were requested from the engineers at the toll plaza. The specifications were used to compute the electrical energy consumption of the lane per hour, the energy consumption was then converted into monetary terms using the Public Utilities Regulation Commission (PURC) electricity tariff rates for non residential use of June 2010.

3.1.3 Mean arrival rate per lane type

Twenty four hour traffic count data conducted over a period of seven days on the motorway in both directions from the 13th to the 20th of April 2010 was obtained from the Ghana Highway Authority (GHA). The data was used to estimate the mean arrival rate to both the Accra and Tema toll plazas per hour.

3.2 Analysis of data

An abridged version of Seongmoon's (2009) non-linear integer programming model in excel format was used to obtain the unconstrained optimal number of lanes to open for each lane type, in order to minimise Z, which is the operating and user-waiting time costs for various traffic volumes.

Minimise

$$Z = \sum_{i=1}^{K} (C_i n_i + d\lambda_i W_i), \quad (2.6)$$

Subject to

$$\frac{\lambda_i}{n_i} < \mu_i \quad \forall i, \quad (2.7)$$

$$W_i = \frac{\lambda_i(\mu_i^2 \sigma_i^2 + 1)}{2n_i\mu_i\left(\mu_i - \frac{\lambda_i}{n_i}\right)} \quad \forall i, \quad (2.11)$$

$$n_i = integer \quad \forall i\,, \quad (2.13)$$

Where C_i is the operating cost of a type i lane, d is the value of time of a motorist in queue and W_i is the average waiting time of a vehicle in a type *i* lane.

IV. RESULTS AND DISCUSSION

4.1 Operating cost of manual and E – zone lanes on the Accra – Tema motorway

Interviews conducted with personnel of the Ghana Road Fund Secretariat as well as the manual lane toll booth attendants indicated that the wages of a manual lane operator was US$ 121.4 (GH¢ 170) per month and is fixed for both day and night hours. Assuming full time work hours of 56 hours per week, then the operating cost of the manual lane was US$ 0.54 (GH¢ 0.76) per hour.

The electrical specifications of the components of the E - zone lane as obtained from engineers at the toll plazas on the motorway is given in Table 1. The operating cost of the E – zone lane was computed using the electrical energy requirements of its components.

Table 1 Electrical specifications of the components of the E – zone lane on the Accra - Tema motorway

Component	**Specification**
	***AC* input *voltage* / frequency / current**
Roadside antenna (radio device)	100 to 240 VAC / 50 or 60 Hz / 0.7A
Central processing unit (CPU)	100 to 240 VAC/ 50 or 60 Hz / 4A
Monitor	100 to 240 VAC / 50 or 60 Hz + 3 Hz / 2.0A

Ghana uses electricity of 230 volts; this means that the E – zone lane consumes 1541 watts of energy in an hour. Ghana uses a two part system in costing electricity. Table 2 gives the costs of 1000 and 2000 watts of electricity respectively as at June 2010 in Ghana, and thus the cost of operating the E – zone lane per hour was US$ 2.38 (GH¢ 3.33).

Table 2 Unit cost of electricity for non residential use (PURC, June 2010 tariff reckoner)

Unit		**Cost (Ghana cedi)**
1 unit	(1000 watt hour)	3.17
2 unit	(2000 watt hour)	3.47

4.2 User waiting time cost

Ghana's GDP per capita in 2009 was US$ 1500 (The Central Intelligence Agency World Factbook website, 2010). This was the latest estimate of GDP per capita at the time of data collection and was used as an estimate of the personal income of a driver arriving at the toll booth. Assuming full time work of 40 hours per week for 50 weeks per year, then the average hourly income was US$ 0.75 (GH¢ 1.05). A conservative estimate of two quarters of this value US$ 0.38 (GH¢ 0.53) was picked as the hourly rate of value of time for an urban commuter driver, this was in accordance with the recommendations of Small (1992) in his study of urban transportation economics. It is worth stating here that US$ 0.36 (GH¢ 0.5) was and still is the minimum amount of money that an arriving car would have to pay on the spot in order to use any toll facility in Ghana and that all vehicles other than cars actually pay two to three times more (Toll plazas are usually 50km apart on the same highway). To reflect the goal of maximization of welfare benefits during peak hours, the hourly rate of waiting time cost during peak times was assigned 1.5 times cost during off-peak times in accordance with the recommendations of Seongmoon (2009). The hourly rate of user cost during peak times becomes US$ 0.56 (GH¢ 0.79) and that for off peak times is US$ 0.38 (GH¢ 0.53).

4.3 Toll plaza lane requirements

Three criteria were considered as decision making tools to decide on the total number of lanes to open at the toll plazas, at any time, based on the mean arrival rate and the lane type. They are the minimum number of lanes to open, the average waiting time of vehicles in queue and the optimum number of lanes to open.

4.3.1 Minimum number of lanes to open

Seongmoon (2009) argues that in designing a toll plaza the mean arrival rate λ_i needs to be less than the mean service rate μ_i for each lane type n_i in order for the queuing system not to have infinite queues, that is $\frac{\lambda_i}{n_i} < \mu_i \; \forall i$ (2.7). The minimum value of n_i that satisfies this inequality b_i, is the minimum number of lanes needed to process all vehicles. Opoku – Boahen et al. (2013) establishes the capacity of the manual and E -zone lanes on the motorway as μ_M= 5.9 veh/min and μ_E= 8.6 veh/min respectively, thus the minimum number of lanes to open for various traffic volumes can be calculated. Beginning from one arriving vehicle per minute, Table 3 and 4 present solutions of the minimum number of manual and E–zone lanes to open respectively, for various volumes of traffic arrivals in the peak hour.

Table 3 Minimum number of manual lanes required for various traffic volumes in peak hour

Mean arrival rate (λ_M)		Minimum number of manual lanes required (b_M)
Vehicles per hour	Vehicles per minute	
60 - 300	1 - 5	1
360 - 660	6 - 11	2
720 - 1020	12 - 17	3
1080 - 1380	18 - 23	4
1440 - 1740	24 - 29	5
1800 - 2100	30 - 35	6
2160 - 2460	36 - 41	7
2520 - 2820	42 - 47	8
2880 - 3180	48 - 53	9
3240 - 3540	54 - 59	10
3600 - 3840	60 - 64	11
3900 - 4200	65 - 70	12
4260 - 4560	71 - 76	13
4620 - 4920	77 - 82	14
4980 - 5280	83 - 88	15
5340 - 5640	89 - 94	16
5700 - 6000	95 - 100	17
6060 - 6360	101 - 106	18
6420 - 6720	107 - 112	19
6780 - 7080	113 - 118	20

Decimal values for each collection or lane type solution were rounded up to the nearest whole number in order to maintain feasibility and guarantee satisfying the required service standard, rounding upwards also ensures a decrease in user cost as opposed to operating costs. The minimum number of toll lanes required does not provide an optimum solution, but it can be used as an effective criterion to design a toll plaza and decrease user costs (waiting time) by designing to meet the minimum requirement of a future date rather than of the present. If the current traffic volume and yearly growth rate on the said road is known, then the traffic volume that will prevail in a given year from now can be ascertained and used for the design, this would help management to know beforehand when to expect minimum conditions at the plaza and to be proactive rather than reactive in intervening.

Table 4 Minimum number of E - zone lanes required for various traffic volumes in peak hour

Mean arrival rate (λ_E)		Minimum number of E – zone lanes required (b_E)
Vehicles per hour	**Vehicles per minute**	
60 - 480	1 - 8	1
540 - 1020	9 - 17	2
1080 - 1500	18 - 25	3
1560 - 2040	26 - 34	4
2100 - 2580	35 - 43	5
2640 - 3060	44 - 51	6
3120 - 3600	52 - 60	7
3660 - 4080	61 - 68	8
4140 - 4620	69 - 77	9
4680 - 5160	78 - 86	10
5220 - 5640	87- 94	11
5700 - 6180	95 - 103	12
6240 - 6660	104 - 111	13
6720 - 7200	112 - 120	14
7260 - 7740	121 - 129	15
7800 - 8220	130 - 137	16
8280 - 8760	138 - 146	17
8820 - 9240	147 - 154	18
9300 - 9780	155 - 163	19
9840 - 10320	164 - 172	20

Table 5 and 6 show the mean arrival rate per hour to the Accra and Tema toll plazas respectively as at April 2010 and the minimum required number of lanes to open to have a stable queuing system with finite queues.

Table 5 Mean arrival rate and minimum number of lanes required at the Accra toll plaza

Mean arrival rates (per minute)				Minimum required number of lanes to open		
Time	**Total (λ_i)**	**E zone (λ_E)**	**Manual (λ_M)**	**E zone (b_E)**	**Manual (b_M)**	**Total $b_E + b_M$**
6:00 - 7:00	13.5	0.4	13.1	1	3	4
7:00 - 8:00	17.6	0.5	17.1	1	3	4
8:00 - 9:00	20.4	0.6	19.8	1	4	5
9:00 - 10:00	23.0	0.7	22.3	1	4	5
10:00 - 11:00	17.7	0.5	17.2	1	3	4
11:00 - 12:00	23.2	0.7	22.5	1	4	5
12:00 - 13:00	15.0	0.5	14.6	1	3	4
13:00 - 14:00	21.0	0.6	20.4	1	4	5
14:00 - 15:00	20.4	0.6	19.8	1	4	5
15:00 - 16:00	19.7	0.6	19.1	1	4	5
16:00 - 17:00	18.5	0.6	17.9	1	4	5
17:00 - 18:00	23.0	0.7	22.3	1	4	5
18:00 - 19:00	16.3	0.5	15.8	1	3	4
19:00 - 20:00	16.2	0.5	15.7	1	3	4
20:00 - 21:00	14.6	0.4	14.1	1	3	4
21:00 - 22:00	6.2	0.2	6.0	1	2	3
22:00 - 23:00	3.7	0.1	3.6	1	1	2
23:00 - 00:00	3.7	0.1	3.6	1	1	2
00:00 -01:00	3.0	0.1	2.9	1	1	2
01:00 - 02:00	2.1	0.1	2.0	1	1	2
02:00 - 03:00	1.6	0.0	1.6	1	1	2
03:00 - 04:00	2.0	0.1	1.9	1	1	2
04:00 - 05:00	3.7	0.1	3.6	1	1	2
05:00 - 06:00	7.5	0.2	7.3	1	2	3

The arrival rates were obtained by finding the mean of hourly volumes on working days (with the exception of Friday) from the 13th to the 20th of April 2010. 97% of the mean arrival rate to each plaza per hour was assigned to the manual lanes whilst 3% was assigned to the E – zone lanes, this was in accordance with the proportion of lane type usage prevailing (Opoku – Boahen et al., 2013). Table 5 and 6 show that the capacity of four manual lanes which was prevailing at both plazas was the minimum number of lanes required to ensure a stable queuing system during both morning and evening peak hours and this accounted for the long queues and delays.

Table 6 Mean arrival rate and minimum number of lanes required at the Tema toll plaza

Mean arrival rate (per minute)				**Minimum required number of lanes to open**		
TIME	**Total** $(\boldsymbol{\lambda_i})$	**E zone** $(\boldsymbol{\lambda_E})$	**Manual** $(\boldsymbol{\lambda_M})$	**E zone** $(\boldsymbol{b_E})$	**Manual** $(\boldsymbol{b_M})$	**Total** $\boldsymbol{b_E + b_M}$
6:00 - 7:00	17.9	0.5	17.4	1	3	4
7:00 - 8:00	21.6	0.6	20.9	1	4	5
8:00 - 9:00	21.7	0.6	21.0	1	4	5
9:00 - 10:00	16.4	0.5	15.9	1	3	4
10:00 - 11:00	16.9	0.5	16.4	1	3	4
11:00 - 12:00	14.0	0.4	13.6	1	3	4
12:00 - 13:00	14.6	0.4	14.2	1	3	4
13:00 - 14:00	15.2	0.5	14.7	1	3	4
14:00 - 15:00	18.1	0.5	17.6	1	3	4
15:00 - 16:00	18.3	0.5	17.7	1	4	5
16:00 - 17:00	18.0	0.5	17.4	1	3	4
17:00 - 18:00	15.6	0.5	15.1	1	3	4
18:00 - 19:00	22.6	0.7	21.9	1	4	5
19:00 - 20:00	14.1	0.4	13.7	1	3	4
20:00 - 21:00	10.9	0.3	10.6	1	2	3
21:00 - 22:00	8.1	0.2	7.9	1	2	3
22:00 - 23:00	5.7	0.2	5.5	1	1	2
23:00 - 00:00	3.7	0.1	3.6	1	1	2
00:00 -01:00	4.0	0.1	3.8	1	1	2
01:00 - 02:00	3.1	0.1	3.0	1	1	2
02:00 - 03:00	3.3	0.1	3.2	1	1	2
03:00 - 04:00	4.4	0.1	4.2	1	1	2
04:00 - 05:00	4.5	0.1	4.3	1	1	2
05:00 - 06:00	12.7	0.4	12.3	1	3	4

4.3.2 Average waiting time in queue

Table 7 shows in minutes the 85th percentile and average waiting time delays of vehicles in the manual lanes on the motorway as put forward by Opoku – Boahen et al. (2013) in our performance evaluation of the lanes on the motorway. It can be seen from Table 7 that the difference between the 85th percentile delays and average delays for each hour of measurement ranged between 0.2 to 0.4 minutes with an average value of 0.3 minutes. This could be used as a simple rule of thumb to predict what the average waiting time on an existing or new toll facility ought to be once we decide on a desired level of service and vice versa.

Table 7 Comparison between the 85th percentile and average waiting time delays on the Accra - Tema motorway, (Opoku – Boahen, et al., 2013)

	Accra plaza								**Tema plaza**
85th % - tile (min)	2.0	2.3	2.1	2.4	2.0	3.1	1.9	2.3	1.1
Avg. Delay (min)	1.8	1.9	1.8	2.0	1.7	2.7	1.7	2.0	0.9
Difference	0.3	0.4	0.3	0.4	0.3	0.4	0.2	0.3	0.2

Fig 1 shows a scatter plot of the 85th percentile delay and average delay data from Table 7, the plot indicates a positive linear association between the two sets of data and thus the 85th percentile delay tends to increase with an increase in average waiting time. Regression analysis indicates that the relationship between the 85th percentile delay and Average delay can be described by the equation

$$85^{th} Percentile\ delay\ (y) = 0.066 + 1.128\ Average\ delay\ (x) \quad (4.1)$$

The correlation coefficient (R) between the two sets of data was 0.993 which is a near perfect correlation with no substantial variation from the line; we can therefore also use the regression equation to predict the average waiting time that ought to be achieved on a facility once we select a desired level of service and vice versa.

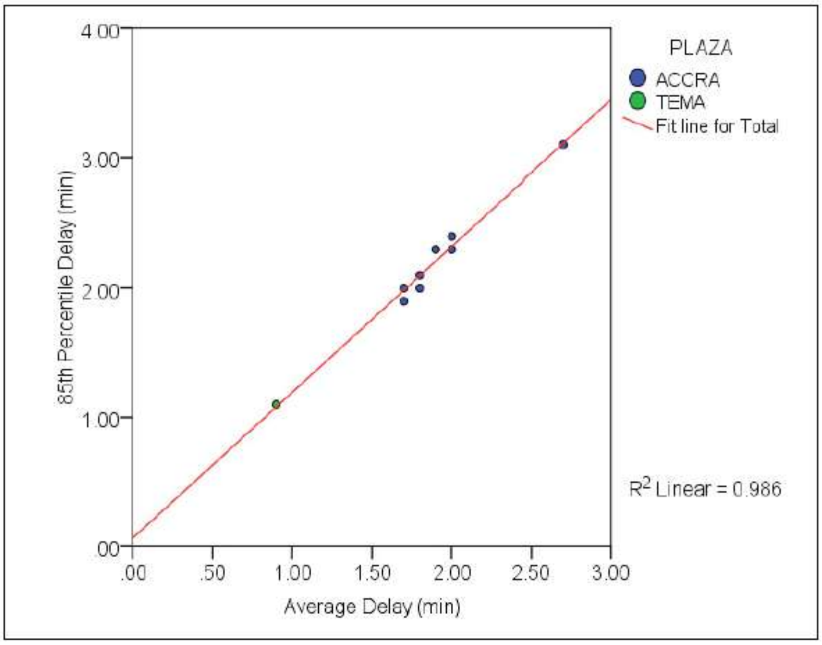

Figure 1 Scatter plot of the 85th percentile delays and average waiting delays measured on the Accra – Tema Motorway

Having established the capacity of the manual and E -zone lanes on the motorway as μ_M= 5.9 veh/min and μ_E= 8.6 veh/min respectively, with a service time standard deviation of 0.05 min (3 sec) for the manual lane and zero for the E –zone lane (Opoku – Boahen et al., 2013), we can use the average waiting time parameter as an effective design criterion by selecting a desired LOS and hence an 85th percentile delay value desired for a facility (whether existing or proposed) and apply either the regression equation (4.1) or rule of thumb to estimate what the average waiting time W_i should be. Equation (2.11) could then be used to estimate the number of either lanes n_i needed to achieve such an average waiting time W_i based on the mean arrival rate λ_i in the peak hour (whether current or future) for the toll facility in question.

$$W_i = \frac{\lambda_i(\mu_i^2\sigma_i^2 + 1)}{2n_i\mu_i\left(\mu_i - \frac{\lambda_i}{n_i}\right)} \quad \forall i, \qquad (2.11)$$

Table 8 shows the toll plaza level of service criteria put forward by klodzinski and Al – deek (2002), a lower bound 85th percentile delay of around 0.3 and 0.6 min/veh is required to ensure levels of service of B and C respectively, using the regression equation (4.1) put forward in this paper; these 85th percentile delays would be achieved with average waiting times of 0.21 min/veh and 0.47 min/veh respectively. Similarly an upper bound 85th percentile delay of 0.2 min/veh is needed to ensure a level of service of A and this works to keeping an average waiting time of 0.12 min/veh or less.

Table 8 Toll plaza level of service criteria, (Klodzinski and Al – Deek, 2002)

Level of service	85th percentile delay (minutes/vehicle)
A	≤ 0.2
B	> 0.2 – 0.5
C	> 0.5 – 0.8
D	> 0.8 –1.3
E	> 1.3 – 1.9
F	> 1.9

The mean arrival rate to the manual lanes at the peak hour during data collection as shown in Table 5 and 6 were 22.3 veh/min and 21.9 veh/min at the Accrra and Tema plazas respectively, using equation (2.11) we can conclude that we needed to open 5 manual lanes to ensure an average waiting time of 0.47 min/veh at both plazas and hence LOS of C and 6 manual lanes to ensure an average waiting time of 0.21 min/veh and hence LOS of B at peak hours. Furthermore, we needed to open 7 manual lanes to keep average waiting times at

0.12 min/veh and ensure a LOS of A. Table 9 and 10 provide solutions for the number of manual and E -zone lanes to open respectively for various mean arrival rates to ensure an average waiting time of 0.47 min/veh and hence to obtain a LOS of C.

Table 9 Number of manual lanes required for various traffic volumes in peak hour to ensure an 85th percentile delay of 0.6 min/veh or average waiting time delay of 0.47 min/veh and hence a LOS of C

Mean arrival rate (λ_M)		Number of manual lanes required (n_M)
Vehicles per hour	**Vehicles per minute**	
60 - 240	1 - 4	1
300 - 540	5 - 9	2
600 - 840	10 - 14	3
900 - 1140	15 - 19	4
1200 - 1440	20 - 24	5
1500 - 1740	25 - 29	6
1800 - 2040	30 - 34	7
2100 -2340	35 - 39	8
2400 - 2640	40 - 44	9
2700 - 2940	45 - 49	10
3000 - 3240	50 - 54	11
3300 -3540	55 - 59	12
3600 - 3840	60 - 64	13
3900 - 4140	65 - 69	14
4200 - 4380	70 - 73	15
4440 - 4680	74 - 78	16
4740 - 4980	79 - 83	17
5040 - 5280	84 - 88	18
5340 - 5580	89 - 93	19
5640 -5880	94 - 98	20

Table 10 Number of E-zone lanes required for various traffic volumes in peak hour to ensure an 85th percentile delay of 0.6 min/veh or average waiting time delay of 0.47 min/veh and hence LOS of C

Mean arrival rate (λ_E)		Number of E-zone lanes required (n_E)
Vehicles per hour	**Vehicles per minute**	
60 - 420	1 -7	1
480 - 900	8 -15	2
960 - 1320	16 - 22	3
1380 - 1800	23 - 30	4
1860 - 2280	31 - 38	5
2340 - 2700	39 - 45	6
2760 - 3180	46 - 53	7
3240 - 3660	54 - 61	8
3720 - 4080	62 - 68	9
4140 - 4560	69 - 76	10
4620 - 5040	77 - 84	11
5100 - 5460	85 - 91	12
5520 - 5940	92 - 99	13
6000 - 6420	100 -107	14
6480 - 6840	108 - 114	15
6900 - 7320	115 - 122	16
7380 - 7800	123 - 130	17
7860 - 8220	131 - 137	18
8280 - 8700	138 - 145	19
8760 - 9180	146 - 153	20

Provided in the appendix also is the solutions for the number of manual and E - zone lanes to open respectively for various mean arrival rates to ensure an average waiting time of 0.21 min/veh and hence obtain a LOS of B.

4.3.3 Optimum number of lanes

An Abridged version of Seongmoon's (2009) non-linear integer programming model was used to obtain the unconstrained optimal number of lanes to open for each lane type, in order to minimize Z, which is the operating and user-waiting cost for various traffic volumes.

Minimise

$$Z = \sum_{i=1}^{K} (C_i n_i + d\lambda_i W_i), \qquad (2.6)$$

Subject to

$$\frac{\lambda_i}{n_i} < \mu_i \quad \forall i, \qquad (2.7)$$

$$W_i = \frac{\lambda_i(\mu_i^2 \sigma_i^2 + 1)}{2n_i\mu_i\left(\mu_i - \frac{\lambda_i}{n_i}\right)} \quad \forall i, \qquad (2.11)$$

$$n_i = integer \quad \forall i, \qquad (2.13)$$

Where C_i is the operating cost of a type i lane, d is the value of time of a motorist in queue and W_i is the average waiting time of a vehicle in a type i lane. The operating cost for the manual lane per hour C_M was ascertained as US$ 0.54, while the operating cost of the E – zone lane was ascertained as US$ 2.38 per hour. The value of time d of an arriving vehicle was estimated as US$ 0.38 per hour for off peak hours and US$ 0.56 per hour for peak hours.

For any given mean arrival rate, If more toll lanes are opened above the optimal solution then Z increases with the cost of operating the lanes outweighing the waiting time cost of motorists, however if fewer toll lanes are opened then Z increases with the cost of the waiting time of arrivals outweighing the operating cost of the lanes, the model was thus used to obtain the number of lanes to open in each hour to minimise the total cost Z for both the manual and E – zone lanes based on the mean arrival rates to either lane types at both the Accra and Tema toll plazas respectively. Table 11 shows the optimal number of manual and E - zone lanes to open (n_M^* ,n_E^*) at the Accra toll plaza for each hour, based on the mean arrival rate to the toll plaza for each hour already presented in Table 5.

Table 11 Optimal solution and corresponding mean wait times and cost for the Accra toll plaza

Time	n_E^*	n_M^*	W_E (min)	W_M (min)	W (min)	C_O $	C_W $	Z $
6:00 - 7:00	1	4	0.0029	0.11	90	4.54	0.57	5.11
7:00 - 8:00	1	5	0.0038	0.13	130	5.08	1.21	6.29
8:00 - 9:00	1	6	0.0045	0.12	140	5.62	1.30	6.92
9:00 - 10:00	1	7	0.0051	0.11	145	6.16	1.35	7.51
10:00 - 11:00	1	5	0.0038	0.13	133	5.08	1.24	6.32
11:00 - 12:00	1	7	0.0051	0.11	149	6.16	1.39	7.55
12:00 - 13:00	1	4	0.0032	0.15	130	4.54	0.83	5.37
13:00 - 14:00	1	6	0.0046	0.12	152	5.62	1.42	7.04
14:00 - 15:00	1	6	0.0044	0.12	138	5.62	1.29	6.91
15:00 - 16:00	1	6	0.0043	0.11	123	5.62	1.15	6.77
16:00 - 17:00	1	5	0.0040	0.14	153	5.08	1.43	6.51
17:00 - 18:00	1	7	0.0051	0.11	144	6.16	1.34	7.50
18:00 - 19:00	1	5	0.0035	0.11	101	5.08	0.94	6.02
19:00 - 20:00	1	5	0.0035	0.10	99	5.08	0.92	6.00
20:00 - 21:00	1	4	0.0031	0.14	116	4.54	0.74	5.28
21:00 - 22:00	1	2	0.0013	0.10	35	3.46	0.22	3.68
22:00 - 23:00	1	1	0.0008	0.14	30	2.92	0.19	3.11
23:00 - 00:00	1	1	0.0008	0.14	30	2.92	0.19	3.11
00:00 -01:00	1	1	0.0006	0.09	15	2.92	0.10	3.02
01:00 - 02:00	1	1	0.0004	0.05	6	2.92	0.04	2.96
02:00 - 03:00	1	1	0.0003	0.03	3	2.92	0.02	2.94
03:00 - 04:00	1	1	0.0004	0.04	5	2.92	0.03	2.95
04:00 - 05:00	1	1	0.0008	0.14	30	2.92	0.19	3.11
05:00 - 06:00	1	2	0.0016	0.15	64	3.46	0.40	3.86

Table 11 also gives what the mean waiting time of arrivals to either lane types would be for the optimal solution (W_E, W_M) as well as the mean total waiting time of all arrivals to the plaza for each hour W. The time between 7:00 – 12:00 hrs and 13:00 – 20:00 hrs were considered as peak hours and therefore attracted the higher cost in terms of value of time. C_O represents the total operating cost for operating the optimal number of manual and E – zone lanes for each hour, while C_W gives the total waiting time cost of all arrivals to both lane types for the optimal solution. Z represents the sum of C_O and C_w. Table 11 indicates that to minimise the operation and user cost at the Accra plaza; three additional manual lanes should have been added to the existing four for operation during peak hours, and thus we needed to open seven manual lanes during the peak hour. The solution also shows that the number of lanes kept open should vary from hour to hour, the practice of keeping all four manual lanes open between 21:00 - 6:00 hrs was not cost effective; as it allured no significant benefit to the road user and imposed unnecessary cost to the toll operator. It would be cost effective to operate two lanes between 21:00 – 22:00 hr and 5:00 – 6:00 hr and only one lane between 22:00 – 6:00 hrs. For example, if one manual lane is operated in the hour 2:00 – 3:00 am, mean waiting time of a manual lane arrival would be 0.03 min (1.8 sec) with the total waiting time of all arrivals in the hour being 3 min (180 sec).

Table 12 compares the operating and user waiting time cost for the then existing situation of one E – zone lane and four manual lanes, with the optimal solution put forward in Table 11. The comparison in Table 12 shows that the optimal solution for the Accra plaza would have resulted in a savings in terms of the toll user's time cost of up to US$ 79.02 each day compared to the then existing situation, with a loss of US$ 1.10, and hence a net gain of US$ 77.92 each day.. The toll operator also stood to gain rather than lose US$ 1.62 daily with the optimal solution. Though the gain to the toll operator is small (US$ 1.62 daily), it's a gain and not a loss, it indicates that the operating cost that would have been incurred with the implementation of the optimal solution is about the same as not implementing it, but implementation would have reduced waiting times at the plaza. In total, the implementation of the optimal solution would have resulted in a total daily savings of US$ 79.54 in terms of operation and user waiting cost at the Accra plaza.

Table 12 Operation and user waiting cost for the then existing situation and the optimal solution at the Accra toll plaza

Time	C_O($)			C_W($)			Z ($)		
	Existing	**Optimal**	**diff**	**Existing**	**Optimal**	**diff**	**Existing**	**Optimal**	**diff**
6:00 - 7:00	4.54	4.54	0.00	0.57	0.57	0.00	5.11	5.11	0.00
7:00 - 8:00	4.54	5.08	-0.54	2.31	1.21	1.10	6.85	6.29	0.56
8:00 - 9:00	4.54	5.62	-1.08	5.39	1.30	4.08	9.93	6.92	3.00
9:00 - 10:00	4.54	6.16	-1.62	19.50	1.35	18.15	24.04	7.51	16.53
10:00 -11:00	4.54	5.08	-0.54	2.39	1.24	1.14	6.93	6.32	0.60
11:00 -12:00	4.54	6.16	-1.62	23.18	1.39	21.79	27.72	7.55	20.17
12:00 - 13:00	4.54	4.54	0.00	0.83	0.83	0.00	5.37	5.37	0.00
13:00 - 14:00	4.54	5.62	-1.08	6.60	1.42	5.17	11.14	7.04	4.09
14:00 - 15:00	4.54	5.62	-1.08	5.25	1.29	3.96	9.79	6.91	2.88
15:00 - 16:00	4.54	5.62	-1.08	4.13	1.15	2.98	8.67	6.77	1.90
16:00 - 17:00	4.54	5.08	-0.54	2.92	1.43	1.49	7.46	6.51	0.95
17:00 - 18:00	4.54	6.16	-1.62	19.10	1.34	17.76	23.64	7.50	16.14
18:00 - 19:00	4.54	5.08	-0.54	1.65	0.94	0.71	6.19	6.02	0.17
19:00 - 20:00	4.54	5.08	-0.54	1.61	0.92	0.69	6.15	6.00	0.15
20:00 - 21:00	4.54	4.54	0.00	0.74	0.74	0.00	5.28	5.28	0.00
21:00 - 22:00	4.54	3.46	1.08	0.07	0.22	-0.15	4.61	3.68	0.93
22:00 - 23:00	4.54	2.92	1.62	0.02	0.19	-0.17	4.56	3.11	1.45
23:00 - 00:00	4.54	2.92	1.62	0.02	0.19	-0.17	4.56	3.11	1.45
00:00 -01:00	4.54	2.92	1.62	0.01	0.10	-0.08	4.55	3.02	1.54
01:00 - 02:00	4.54	2.92	1.62	0.01	0.04	-0.03	4.55	2.96	1.59
02:00 - 03:00	4.54	2.92	1.62	0.00	0.02	-0.02	4.54	2.94	1.60
03:00 - 04:00	4.54	2.92	1.62	0.01	0.03	-0.03	4.55	2.95	1.59
04:00 - 05:00	4.54	2.92	1.62	0.02	0.19	-0.17	4.56	3.11	1.45
05:00 - 06:00	4.54	3.46	1.08	0.11	0.40	-0.29	4.65	3.86	0.79
Loss			-11.9			-1.10			
Gain			13.5			79.02			
Total			1.62			77.92			79.54

Table 17 in the appendix also presents the optimal solution as to the number of either lane types to open per hour at the Tema toll plaza based on the mean hourly arrival rates to the Tema plaza presented in Table 6. The time between 6:00 – 11:00 hrs and 14:00 – 19:00 hrs were considered as peak hours in the case of the

Tema plaza. Additionally, Table 18 in the appendix compares the operating and user waiting time cost for the then existing situation of one E – zone lane and four manual lanes at the Tema plaza with the optimal solution put forward in Table 17. The optimal solution for the Tema plaza gave similar findings as the solution for the Accra plaza.

Engineers wanting to decide on the optimal number of lanes to open during design would foremost be concerned with what happens in the peak or busiest hour. Table 13 and 14 present solutions as to the optimum number of manual and E - zone lanes to open respectively for various traffic volumes in the peak hour. The solutions were carried out considering the toll lane operating costs as discussed as well as the estimated peak hour waiting time value of US$ 0.56 per hour.

Table 13 Optimum number of manual lanes to open for various traffic volumes in peak hour

Mean arrival rate (λ_M)		Optimum number of Manual lanes to open (n_M^*)
Vehicles per hour	Vehicles per minute	
60 - 240	1- 4	1
300 - 480	5 - 8	2
540 - 660	9 - 11	3
720 - 840	12 - 14	4
900 - 1080	15 - 18	5
1140 - 1260	19 - 21	6
1320 - 1500	22 - 25	7
1560 - 1680	26 - 28	8
1740 -1860	29 - 31	9
1920 - 2100	32 - 35	10
2160 - 2280	36 - 38	11
2340 - 2520	39 - 42	12
2580 - 2700	43 - 45	13
2760 - 2880	46 - 48	14
2940 - 3120	49 - 52	15
3180 - 3300	53 - 55	16
3360 - 3480	56 - 58	17
3540 - 3720	59 - 62	18
3780 - 3900	63 - 65	19
3960 - 4140	66 - 69	20

Table 14 Optimum number of E – zone lanes to open for various traffic volumes in peak hour

Mean arrival rate (λ_E)		Optimum number of E – zone lanes to open (n_E^*)
Vehicles per hour	Vehicles per minute	
60 - 420	1 - 7	1
480 - 840	8 - 14	2
900 - 1260	15 - 21	3
1320 - 1680	22 - 28	4
1740 - 2040	29 - 34	5
2100 - 2460	35 - 41	6
2520 - 2820	42 - 47	7
2880 - 3240	48 - 54	8
3300 - 3600	55 - 60	9
3660 - 3960	61 - 66	10
4020 - 4380	67 - 73	11
4440 - 4740	74 - 79	12
4800 - 5160	80 - 86	13
5220 - 5520	87 - 92	14
5580 - 5940	93 - 99	15
6000 - 6300	100 - 105	16
6360 - 6660	106 - 111	17
6720 - 7080	112 - 118	18
7140 - 7440	119 - 124	19
7500 - 7860	125 - 131	20

It is expected that the value of time and the operating cost estimates used in the solution for the optimum number of lanes put forward may change over time and hence the optimum solution may also change with time. The Highway Authority may have to establish and update these cost parameters with time to reflect changes that may occur. Alternatively, in the same way as the Road Agencies fix prices of tolls paid by motorists and update these prices from year to year, the Agencies could from time to time fix these cost parameters (value of time and operating cost of lane types) at specific values so as to reflect any goals or targets they want to achieve at the plazas in line with any policy or aspirations of the agency. However any changes which may occur are more likely to be an increase in these cost parameters rather than a decrease and hence the optimum number of lanes that would have to be opened would then be more than that put forward in this paper for each specific rate of arrival, thus the optimum solution put forward could always serve as a base solution in respect of the number of lanes to open.

V. CONCLUSION AND RECOMMENDATION

5.1 Conclusions

- As at April 2010 the number of four manual lanes open in either direction on the Accra - Tema motorway was the minimum number of lanes required for a stable queuing system during peak hours thus drivers enjoyed no time savings benefit during the peak hours and hence the long queues.
- With manual lane mean arrival rates of 22.3 veh/min and 21.9 veh/min in the peak hour at the Accra and Tema plazas respectively, we needed to open five manual lanes in either direction to ensure a level of service of C, six manual lanes in either direction to ensure a level of service of B and seven manual lanes in either direction to ensure a level of service of A.
- The optimum number of lanes to open as at April 2010 to minimise operation and user waiting time cost during the peak hour was seven manual lanes and one E –zone lane
- The optimal solution would have actually reduced the operating cost of the toll operator on the motorway in addition to reducing the waiting time of arrivals and hence aside from the initial cost of construction of the additional lanes has no downside to it.
- From the three criteria examined as decision making tools is clear that the solutions vary from hour to hour in response to the arrival rate thus the practice of keeping all existing lanes open for 24 hours is in appropriate. Though the number of four manual lanes which was existing was inadequate for the peak hours, the optimum solution for the night off peak hours between 1:00 - 5: 00 hrs was only 1 manual lane, thus it makes no sense to operate all the lanes around these times.
- With the regression equation put forward in this paper we are now able to predict the level of service based on the average waiting time and vice versa.

5.2 Recommendations

- Decisions as to the number of lanes to open in either plazas on the Accra - Tema motorway and indeed in other plazas with the same lane types, should be done by considering the solutions put forward in this paper in respect of the three criteria: Minimum number of lanes, Average waiting time and Optimum number of lanes.
- Solutions put forward in respect of the minimum number of lanes to open on its own could serve as an effective toll plaza management tool by ensuring that at every point in time the number of lanes opened is more than the minimum requirement.
- Since the Average waiting time has been found to have a near perfect positive linear correlation with the LOS, the Highway Authority could set a sector policy or goal in respect of an average waiting time value (say 0.47 min/veh) as a target standard not to be exceeded either in design or management of any toll plaza.
- The operating cost of the lane types and the waiting time costs of users may vary from year to year, effort should be made to update these parameters from year to year and thus update the optimal solutions.
- Alternatively the operating cost of the operator for any lane and waiting time costs can be fixed by the Ghana Highway Authority in the same way as toll prices are fixed from year to year (above the actual measured values of these parameters) to reflect the strategic goals of the agency, The agency could come out with projections of these parameters from year to year to aid in design and to act as a check.
- The practice of keeping all lanes open for the off peak hours should be stopped, the number of lanes opened could be varied in line with the arriving traffic to reduce cost. Furthermore, workers could still be paid for the closed lanes but be given the time off as an incentive.

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APPENDIX

Table 15 Number of manual lanes required for various traffic volumes in peak hour to ensure an 85th percentile delay of 0.3 min/veh or average waiting time delay of 0.21 min/veh and hence a LOS of B

Mean arrival rate (λ_M)		Number of manual lanes required (n_M)
Vehicles per hour	Vehicles per minute	
60 - 240	1 - 4	1
300 - 480	5 - 8	2
540 - 720	9 - 12	3
780 - 960	13 - 16	4
1020 - 1200	17 - 20	5
1260 - 1440	21 - 24	6
1500 - 1680	25 - 28	7
1740 - 1920	29 - 32	8
1980 - 2160	33 - 36	9
2220 - 2460	37 - 41	10
2520 - 2700	42 - 45	11
2760 - 2940	46 - 49	12
3000 - 3180	50 - 53	13
3240 - 3420	54 - 57	14
3480 - 3660	58 - 61	15
3720 - 3900	62 - 65	16
3960 - 4140	66 - 69	17
4200 - 4380	70 - 73	18
4440 - 4620	74 - 77	19
4680 - 4920	78 - 82	20

Table 16 Number of E-zone lanes required for various traffic volumes in peak hour to ensure an 85th percentile delay of 0.3 min/veh or average waiting time delay of 0.21 min/veh and hence LOS of B

Mean arrival rate (λ_E)		Number of E -zone lanes required (n_E)
Vehicles per hour	Vehicles per minute	
60 - 360	1 - 6	1
420 - 780	7 - 13	2
840 - 1200	14 - 20	3
1260 - 1560	21 - 26	4
1620 - 1980	27 - 33	5
2040 - 2400	34 - 40	6
2460 - 2820	41 - 47	7
2880 - 3180	48 - 53	8
3240 - 3600	54 - 60	9
3660 - 4020	61 - 67	10
4080 - 4440	68 -74	11
4500 - 4800	75 - 80	12
4860 - 5220	81 - 87	13
5280 - 5640	88 - 94	14
5700 - 6060	95 - 101	15
6120 - 6420	102 - 107	16
6480 - 6840	108 - 114	17
6900 - 7260	115 - 121	18
7320 -7620	122 - 127	19
7680 - 8040	128 - 134	20

Table 17 Optimal solution and corresponding mean wait times and cost for the Tema toll plaza

Time	n_E^*	n_M^*	W_E	W_M	W	C_O	C_W	Z
			(min)	(min)	(min)	$	$	$
6:00 - 7:00	1	5	0.0039	0.13	137	5.08	1.28	6.36
7:00 - 8:00	1	6	0.0047	0.13	167	5.62	1.56	7.18
8:00 - 9:00	1	6	0.0048	0.13	170	5.62	1.58	7.20
9:00 - 10:00	1	5	0.0035	0.11	104	5.08	0.97	6.05
10:00 - 11:00	1	5	0.0036	0.12	114	5.08	1.06	6.14
11:00 - 12:00	1	4	0.0030	0.13	102	4.54	0.65	5.19
12:00 - 13:00	1	4	0.0031	0.14	119	4.54	0.75	5.29
13:00 - 14:00	1	4	0.0032	0.15	134	4.54	0.85	5.39
14:00 - 15:00	1	5	0.0039	0.14	143	5.08	1.33	6.41
15:00 - 16:00	1	5	0.0040	0.14	147	5.08	1.38	6.46
16:00 - 17:00	1	5	0.0039	0.13	139	5.08	1.30	6.38
17:00 - 18:00	1	5	0.0033	0.10	88	5.08	0.82	5.90
18:00 - 19:00	1	7	0.0050	0.10	137	6.16	1.28	7.44
19:00 - 20:00	1	4	0.0030	0.13	105	4.54	0.67	5.21
20:00 - 21:00	1	3	0.0023	0.14	88	4.00	0.56	4.56
21:00 - 22:00	1	2	0.0017	0.18	87	3.46	0.55	4.01
22:00 - 23:00	1	2	0.0012	0.08	26	3.46	0.17	3.63
23:00 - 00:00	1	1	0.0008	0.14	30	2.92	0.19	3.11
00:00 -01:00	1	1	0.0008	0.17	39	2.92	0.25	3.17
01:00 - 02:00	1	1	0.0006	0.10	17	2.92	0.11	3.03
02:00 - 03:00	1	1	0.0007	0.11	22	2.92	0.14	3.06
03:00 - 04:00	1	1	0.0009	0.24	60	2.92	0.38	3.30
04:00 - 05:00	1	1	0.0009	0.26	67	2.92	0.43	3.35
05:00 - 06:00	1	3	0.0027	0.21	155	4.00	0.98	4.98

Table 18 Operation and user waiting cost for the then existing situation and the optimal solution at the Tema toll plaza

Time	C_O($)			C_W($)			Z ($)		
	Existing	**0ptimal**	**diff**	**Existing**	**0ptimal**	**diff**	**Existing**	**0ptimal**	**diff**
6:00 -7:00	4.54	5.08	-0.54	2.49	1.28	1.21	7.03	6.36	0.67
7:00 -8:00	4.54	5.62	-1.08	8.44	1.56	6.88	12.98	7.18	5.80
8:00 -9:00	4.54	5.62	-1.08	8.79	1.58	7.20	13.33	7.20	6.12
9:00 -10:00	4.54	5.08	-0.54	1.72	0.97	0.75	6.26	6.05	0.21
10:00 - 11:00	4.54	5.08	-0.54	1.93	1.06	0.87	6.47	6.14	0.33
11:00 - 12:00	4.54	4.54	0.00	0.65	0.65	0.00	5.19	5.19	0.00
12:00 - 13:00	4.54	4.54	0.00	0.75	0.75	0.00	5.29	5.29	0.00
13:00 - 14:00	4.54	4.54	0.00	0.85	0.85	0.00	5.39	5.39	0.00
14:00 - 15:00	4.54	5.08	-0.54	2.63	1.33	1.30	7.17	6.41	0.76
15:00 - 16:00	4.54	5.08	-0.54	2.76	1.38	1.38	7.30	6.46	0.84
16:00 - 17:00	4.54	5.08	-0.54	2.53	1.30	1.24	7.07	6.38	0.70
17:00 - 18:00	4.54	5.08	-0.54	1.39	0.82	0.57	5.93	5.90	0.03
18:00 - 19:00	4.54	6.16	-1.62	14.78	1.28	13.50	19.32	7.44	11.88
19:00 - 20:00	4.54	4.54	0.00	0.67	0.67	0.00	5.21	5.21	0.00
20:00 - 21:00	4.54	4.00	0.54	0.30	0.56	-0.25	4.84	4.56	0.29
21:00 - 22:00	4.54	3.46	1.08	0.14	0.55	-0.41	4.68	4.01	0.67
22:00 - 23:00	4.54	3.46	1.08	0.06	0.17	-0.11	4.60	3.63	0.97
23:00 - 00:00	4.54	2.92	1.62	0.02	0.19	-0.17	4.56	3.11	1.45
00:00 -01:00	4.54	2.92	1.62	0.03	0.25	-0.22	4.57	3.17	1.40
01:00 - 02:00	4.54	2.92	1.62	0.02	0.11	-0.09	4.56	3.03	1.53
02:00 - 03:00	4.54	2.92	1.62	0.02	0.14	-0.12	4.56	3.06	1.50
03:00 - 04:00	4.54	2.92	1.62	0.03	0.38	-0.35	4.57	3.30	1.27
04:00 - 05:00	4.54	2.92	1.62	0.03	0.43	-0.39	4.57	3.35	1.23
05:00 - 06:00	4.54	4.00	0.54	0.47	0.98	-0.51	5.01	4.98	0.03
Loss			-7.56			-2.63			
Gain			12.96			34.88			
Total			5.40			32.25			37.65