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Observations On Icosagonal Pyramidal Number

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ABSTRACT:- *We obtain different relations among Icosagonal Pyramidal number and other two, three and four dimensional figurate numbers.*

Keyword:- *Polygonal number, Pyramidal number, Centered polygonal number, Centered pyramidal number, Special number*
MSC classification code: 11D99

I. INTRODUCTION

Fascinated by beautiful and intriguing number patterns, famous mathematicians, share their insights and discoveries with each other and with readers. Throughout history, number and numbers [2,3,7-15] have had a tremendous influence on our culture and on our language. Polygonal numbers can be illustrated by polygonal designs on a plane. The polygonal number series can be summed to form "solid" three dimensional figurate numbers called Pyramidal numbers [1,4,5 and 6] that can be illustrated by pyramids. In this communication we deal with Icosagonal Pyramidal numbers given by $p_n^{20} = \frac{6n^3 + n^2 - 5n}{2}$ and various interesting relations among these numbers are exhibited by means of theorems involving the relations.

Notation

p_n^m = Pyramidal number of rank n with sides m

$t_{m,n}$ = Polygonal number of rank n with sides m

jal_n = Jacobsthal Lucas number

$ct_{m,n}$ = Centered Polygonal number of rank n with sides m

cp_n^m = Centered Pyramidal number of rank n with sides m

g_n = Gnomonic number of rank n with sides m

p_n = Pronic number

$carl_n$ = Carol number

mer_n = Mersenne number, where n is prime

cul_n = Cullen number

Tha_n = Thabit ibn kurrah number

II. INTERESTING RELATIONS

1) $p_n^{20} = 3p_n^6 + p_n^5 - 2t_{3,n}$

Proof

$$2p_n^{20} = \left(4n^3 + 3n^2 - n\right) + 2\left(n^3 + n^2\right) - 4\left(n^2 + n\right)$$

$$= 6p_n^{20} + 2p_n^5 - 4t_{3,n}$$

$$p_n^{20} = 3p_n^6 + p_n^5 - 2t_{3,n}$$

2) $p_n^{20} + n = cp_n^5 + p_n^5$

Proof

$$2p_n^{20} = \left(5n^3 - 3n\right) + \left(n^3 + n^2\right) - 2n$$

$$= 2cp_n^5 + 2p_n^5 - 2n$$

$$p_n^{20} + n = cp_n^5 + p_n^5$$

3) $p_{n-1}^{20} - p_{n+1}^{20} = 2g_n + 1$

Proof

$$p_{n+1}^{20} = 6n^3 + 19n^2 - 15n + 2$$

$$p_{n-1}^{20} = 6n^3 + 19n^2 - 7n + 5$$

$$2p_{n-1}^{20} - 2p_{n+1}^{20} = 8n - 2$$

$$p_{n-1}^{20} - p_{n+1}^{20} = 2(2n-1) + 1$$

$$p_{n-1}^{20} - p_{n+1}^{20} = 2g_n + 1$$

4) The following triples are in arithmetic progression

i) $\left(p_n^8, p_n^{14}, p_n^{20}\right)$

Proof

$$p_n^{20} + p_n^8 = \frac{6n^3 + n^2 - 5n}{2} + \frac{2n^3 + n^2 - n}{2}$$

$$= 2\left(\frac{4n^3 + 2n^2 - 6n}{2}\right)$$

$$= 2p_n^{14}$$

ii) $\left(p_n^4, p_n^{12}, p_n^{20}\right)$

Proof

$$p_n^4 + p_n^{20} = \frac{2n^3 + 3n^2 + n}{6} + \frac{6n^3 + n^2 - 5n}{2}$$

$$= 2\left(\frac{10n^3 + 3n^2 - 14n}{6}\right)$$

$$= 2p_n^{12}$$

iii) $\left(p_n^{12}, p_n^{16}, p_n^{20}\right)$

Proof

$$p_n^{12} + p_n^{20} = \frac{10n^3 + 3n^2 - 7n}{6} + \frac{6n^3 + n^2 - 5n}{2}$$

$$= 2\left(\frac{14n^3 + 3n^2 - 11n}{6}\right)$$

$$= 2p_n^{16}$$

5) $p_n^{20} - 5\left(p_n^5 - t_{3,n}\right) = 0$

Proof

$$2p_n^{20} - 2p_n^5 = 4\left(n^3 + n^2\right) - 5\left(n^2 + n\right)$$

$$= 4\left(2p_n^5\right) - 5\left(2t_{3,n}\right)$$

6) $2\sum_{n=1}^{N} \frac{p_n^{20}}{n} + 5 = 6p_N^4 + t_{3,N}$

Proof

$$2\sum_{n=1}^{N} \frac{p_n^{20}}{n} = \sum\left(6n^2 + n - 5\right)$$

$$= 6\sum n^2 + \sum n - 5$$

$$2\sum_{n=1}^{N} \frac{p_n^{20}}{n} + 5 = 6p_N^4 + t_{3,N}$$

7) $2p_{2^n}^{20} = 2^{n+1}\left(Tha_{2n} - 1\right) + 2^n mer_n$

Proof

$$2p_{2^n}^{20} = 6\left(2^{3n}\right) + \left(2^{2n}\right) - 5\left(2^n\right)$$

$$\frac{2p_{2^n}^{20}}{2^n} = 2\left(3\left(2^{2n}\right) - 1\right) + \left(2^n - 1\right) - 2$$

$$= 2\left(Tha_{2n} - 1\right) + mer_n$$

$$2p_{2^n}^{20} = 2^{n+1}\left(Tha_{2n} - 1\right) + 2^n mer_n$$

8) The following equation represents Nasty number

i) $12\left(p_n^{20} - cp_n^{15} - 2p_n^6 + n\right)$

Proof

$$p_n^{20} = \frac{6n^3 + n^2 - 5n}{2}$$

$$= \frac{5n^3 - 3n}{2} + \frac{n^3 + n^2}{2} - n$$

$$= cp_n^{15} + 2p_n^6 + \frac{n^2}{2} - n$$

$$p_n^{20} - cp_n^{15} - 2p_n^6 + n = \frac{n^2}{2}$$

Multiply by 12, then the equation represents Nasty number

ii) $\frac{2p_n^{20}}{n} - n + 5$

Proof

$$2p_n^{20} = 6n^3 + n^2 - 5n$$

$$\frac{2p_n^{20}}{n} - n + 5 = 6n^2$$

9) $\frac{p_n^{20}}{n} - n = s_n + 2g_n - 4$

Proof

$$\frac{p_n^{20}}{n} = 6n^2 + n - 5$$

$$= \left(6n^2 - 6n + 1\right) + 2\left(2n - 1\right) + n - 4$$

$$\frac{p_n^{20}}{n} - n = s_n + 2g_n - 4$$

10) $2p_n^{20} + 9n = 2\left(nct_{12,n} - t_{7,n}\right)$

Proof

$$p_n^{20} = n\left(\frac{6n^2 + 6n + 1}{2} - \frac{5n - 6}{2}\right)$$

$$= nct_{12,n} - \left(\frac{5n^2 - 3n}{2}\right) - \frac{9n}{2}$$

$$= nct_{12,n} - t_{7,n} - \frac{9n}{2}$$

$$2p_n^{20} + 9n = 2\left(nct_{12,n} - t_{7,n}\right)$$

11) $2p_{n+2}^{20} - 4p_n^{11} - t_{66,n} \equiv 42\left(\mathrm{mod}\,107\right)$

Proof

$$2p_{n+2}^{20} = 6n^3 + 37n^2 + 69n + 42$$

$$= 2\left(3n^3 + n^2 - 2n\right) + \left(35n^2 - 34n\right) + 107n + 42$$

$$= 4p_n^{11} + t_{66,n} + 107n + 42$$

$$2p_{n+2}^{20} - 4p_n^{11} - t_{66,n} \equiv 42\left(\mathrm{mod}\,107\right)$$

12) $p_{n+3}^{20} - p_{n-3}^{20} = 2t_{56,n} + 29g_n + 176$

Proof

$$2p_{n+3}^{20} = 6n^3 + 55n^2 + 163n + 156$$

$$2p_{n-3}^{20} = 6n^3 - 53n^2 + 151n - 138$$

$$2\left(p_{n+3}^{20} - p_{n-3}^{20}\right) = 2\left(54n^2 + 6n + 147\right)$$

$$p_{n+3}^{20} - p_{n-3}^{20} = \left(54n^2 - 52n\right) + 29\left(2n - 1\right) + 176$$

$$p_{n+3}^{20} - p_{n-3}^{20} = 2t_{56,n} + 29g_n + 176$$

13) $$2\sum_{n=1}^{N} p_n^{20} = t_{3,N}\left(6t_{3,N} - 5\right) + sq_N$$

Proof

$$2\sum_{n=1}^{N} p_n^{20} = 6\sum n^3 + \sum n^2 - 5\sum n$$

$$= 6t_{3,N} + sq_N - 5t_{3,N}$$

$$2\sum_{n=1}^{N} p_n^{20} = t_{3,N}\left(6t_{3,N} - 5\right) + sq_N$$

14) $$4p_n^{20} + 7n = 6cp_n^{8} + cp_n^{12} + cp_n^{18} + 2cp_n^{9} + 4t_{3,n}$$

Proof

$$2p_n^{20} = 6cp_n^{8} + cp_n^{12} + 2t_{3,n} - 4n \quad (1)$$

$$2p_n^{20} = cp_n^{18} + 2cp_n^{9} + 2t_{3,n} - 3n \quad (2)$$

Add equation (1) and (2), we get

$$4p_n^{20} + 7n = 6cp_n^{8} + cp_n^{12} + cp_n^{18} + 2cp_n^{9} + 4t_{3,n}$$

15) $$\frac{2p_{2^n}^{20}}{2^n} = jal_{2n+2} + Tha_{2n} - car\,l_n - mer_n - 8$$

Proof

$$\frac{2p_{2^n}^{20}}{2^n} = 6\left(2^{2n}\right) + 2^n - 5$$

$$= \left(4\left(2^{2n}\right) + 1\right) + \left(3\left(2^{2n}\right) - 1\right) - \left(2^{2n} - 2^{n+1} - 1\right) - \left(2^n - 1\right) - 8$$

$$\frac{2p_{2^n}^{20}}{2^n} = jal_{2n+2} + Tha_{2n} - car\,l_n - mer_n - 8$$

16) $p_n^{20} - p_n^{14} + n$ is a cubic integer

Proof

$$p_n^{20} - p_n^{14} = \frac{6n^3 + n^2 - 5n}{2} - \frac{4n^3 + n^2 - 3n}{2}$$

$$= n^3 - n$$

$$p_n^{20} - p_n^{14} + n = n^3$$

17) $24\left(PEN_n - p_n^{20}\right) + 66p_n^{6} - 7n + p_n$ is biquadratic integer

Proof

$$24\left(PEN_n - p_n^{20}\right) = n^4 - 66n^3 - n^2 + 6n$$

$$= n^4 - 66n^3 - (n^2 + n) + 7n$$

$$24\left(PEN_n - p_n^{20}\right) + 66p_n^{6} - 7n + p_n = n^4$$

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