International Refereed Journal of Engineering and Science (IRJES)
ISSN (Online) 2319-183X, (Print) 2319-1821
Volume 2, Issue 4(April 2013), PP.22-28
www.irjes.com

Unique Common Fixed Point Theorem for Three Pairs of Weakly Compatible Mappings Satisfying Generalized Contractive Condition of Integral Type

[1]Kavita B. Bajpai, [2]Manjusha P. Gandhi
Karmavir Dadasaheb Kannamwar College of Engineering, Nagpur, India
Yeshwantrao Chavan College of Engineering, Wanadongri, Nagpur, India

Abstract: *We prove some unique common fixed point result for three pairs of weakly compatible mappings satisfying a generalized contractive condition of Integral type in complete G-metric space.The present theorem is the improvement and extension of Vishal Gupta and Naveen Mani [5] and many other results existing in literature.*
Keywords: *Fixed point , Complete G- metric space , G-Cauchy sequence , Weakly compatible mapping , Integral Type contractive condition.*

I. Introduction

Generalization of Banach contraction principle in various ways has been studied by many authors. One may refer Beg I. & Abbas M.[2] , Dutta P.N. & Choudhury B.S.[3] ,Khan M.S., Swaleh M. & Sessa, S.[9] , Rhoades B.E.[12] , Sastry K.P.R. & Babu G.V.R.[13] , Suzuki T.[15] . Alber Ya.I. & Guerre-Delabriere S. [1] had proved results for weakly contractive mapping in Hilbert space , the same was proved by Rhoades B.E.[12] in complete metric space.

Jungck G.[6] proved a common fixed point theorem for commuting mappings which is the extension of Banach contraction principle. Sessa S.[14] introduced the term "Weakly commuting mappings" which was generalized by Jungck G.[6] as "Compatible mappings". Pant R.P.[11] coined the notion of "*R*-weakly commuting mappings", whereas Jungck G.& Rhoades B.E. [8] defined a term called "weakly compatible mappings" in metric space.

Fisher B. [4] proved an important Common Fixed Point theorem for weakly compatible mapping in complete metric space.

Mustafa in collaboration with Sims [10] introduced a new notation of generalized metric space called G- metric space in 2006. He proved many fixed point results for a self mapping in G- metric space under certain conditions.

Now we give some preliminaries and basic definitions which are used through-out the paper.

Definition 1.1: Let X be a non empty set, and let $G : X \times X \times X \to R^+$ be a function satisfying the following properties:

(G_1) $G(x, y, z) = 0$ if $x = y = z$

(G_2) $0 < G(x, x, y)$ for all $x, y \in X$,with $x \neq y$

(G_3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$, with $y \neq z$

(G_4) $G(x, y, z) = G(x, z, y) = G(y, z, x)$ (Symmetry in all three variables)

(G_5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$, for all $x, y, z, a \in X$ (rectangle inequality)

Then the function G is called a generalized metric space, or more specially a G- metric on X, and the pair (X, G) is called a G–metric space.

Definition 1.2: Let (X, G) be a G - metric space and let $\{x_n\}$ be a sequence of points of X , a point $x \in X$ is said to be the limit of the sequence $\{x_n\}$, if $\lim_{n,m \to +\infty} G(x, x_n, x_m) = 0$, and we say that the sequence $\{x_n\}$ is G - convergent to x or $\{x_n\}$ G -converges to x .

Thus, $x_n \to x$ in a G - metric space (X,G) if for any $\in > 0$ there exists $k \in N$ such that $G(x, x_n, x_m) < \in$, for all $m,n \geq k$

Proposition 1.3: Let (X,G) be a G - metric space. Then the following are equivalent:

i) $\{x_n\}$ is G - convergent to x

ii) $G(x_n, x_n, x) \to 0$ as $n \to +\infty$

iii) $G(x_n, x, x) \to 0$ as $n \to +\infty$

iv) $G(x_n, x_m, x) \to 0$ as $n,m \to +\infty$

Proposition 1.4 : Let (X,G) be a G - metric space. Then for any x , y , z , a in X it follows that

i) If $G(x, y, z) = 0$ then $x = y = z$

ii) $G(x, y, z) \leq G(x, x, y) + G(x, x, z)$

iii) $G(x, y, y) \leq 2G(y, x, x)$

iv) $G(x, y, z) \leq G(x, a, z) + G(a, y, z)$

v) $G(x, y, z) \leq \frac{2}{3}\left(G(x, y, a) + G(x, a, z) + G(a, y, z)\right)$

vi) $G(x, y, z) \leq \left(G(x, a, a) + G(y, a, a) + G(z, a, a)\right)$

Definition 1.5: Let (X,G) be a G - metric space. A sequence $\{x_n\}$ is called a G - Cauchy sequence if for any $\in > 0$ there exists $k \in N$ such that $G(x_n, x_m, x_l) < \in$ for all $m,n,l \geq k$, that is $G(x_n, x_m, x_l) \to 0$ as $n,m,l \to +\infty$.

Proposition 1.6: Let (X,G) be a G - metric space .Then the following are equivalent:

i) The sequence $\{x_n\}$ is G - Cauchy;

ii) For any $\in > 0$ there exists $k \in N$ such that $G(x_n, x_m, x_m) < \in$ for all $m,n \geq k$

Proposition 1.7: A G - metric space (X,G) is called G -complete if every G -Cauchy sequence is G - convergent in (X,G).

Proposition 1.8: Let (*X*, G) be a G- metric space. Then the function G(*x*, *y*, *z*) is jointly continuous in all three of its variables.

Definition 1.9 : Let f and g be two self – maps on a set X . Maps f and g are said to be commuting if $fgx = gfx$, for all $x \in X$

Definition 1.10 : Let f and g be two self – maps on a set X . If $fx = gx$, for some $x \in X$ then x is called coincidence point of f and g.

Definition 1.11: Let f and g be two self – maps defined on a set X , then f and g are said to be weakly compatible if they commute at coincidence points. That is if $fu = gu$ for some $u \in X$, then $fgu = gfu$.

The main aim of this paper is to prove a unique common fixed point theorem for three pairs of weakly compatible mappings satisfying Integral type contractive condition in a complete G – metric space.

The result is the extension of the following theorem of Vishal Gupta and Naveen Mani [5].

II. Theorem

Let S and T be self compatible maps of a complete metric space (X, d) satisfying the following conditions

i) $S(X) \subset T(X)$

ii) $\psi \int_0^{d(Sx,Sy)} \varphi(t)\,dt \leq \psi \int_0^{d(Tx,Ty)} \varphi(t)\,dt - \phi \int_0^{d(Tx,Ty)} \varphi(t)\,dt$

for each $x, y \in X$ where $\psi : [0,+\infty) \to [0,+\infty)$ is a continuous and non decreasing function and $\phi : [0,+\infty) \to [0,+\infty)$ is a lower semi continuous and non decreasing function such that $\psi(t) = \phi(t) = 0$ if and only if $t = 0$ also $\varphi : [0,+\infty) \to [0,+\infty)$ is a “Lebesgue-integrable function” which is summable on each compact subset of R^+ , nonnegative, and such that for each $\in > 0$, $\int_0^{\in} \varphi(t)\,dt > 0$. Then S and T have a unique common fixed point.

III. MAIN RESULT

Theorem 2.1 : Let (X,G) be a complete G-metric space and $L,M,N,P,Q,R : X \to X$ be mappings such that

i) $L(X) \subset P(X)$, $M(X) \subset Q(X)$, $N(X) \subset R(X)$

ii) $$\xi\left\{\int_0^{G(Lx,My,Nz)} f(t)\,dt\right\} \le \xi\left\{\int_0^{G(Px,Qy,Rz)} f(t)\,dt\right\} - \eta\left\{\int_0^{G(Px,Qy,Rz)} f(t)\,dt\right\} \quad \text{---------------------(2.1.1)}$$

for all $x,y,z \in X$ where $\xi : [0,\infty) \to [0,\infty)$ is a continuous and non-decreasing function , $\eta : [0,\infty) \to [0,\infty)$ is a lower semi continuous and non-decreasing function such that $\xi(t) = \eta(t) = 0$ if and only if $t = 0$, also $f : [0,\infty) \to [0,\infty)$ is a Lebesgue integrable function which is summable on each compact subset of R^+ , non negative and such that for each $\in > 0$,

$\int_0^{\in} f(t)\,dt > 0$

iii) The pairs (L,P) , (M,Q) , (N,R) are weakly compatible.

Then L,M,N,P,Q,R have a unique common fixed point in X.

Proof : Let x_0 be an arbitrary point of X and define the sequence $\{x_n\}$ in X such that

$$y_n = Lx_n = Px_{n+1}\ ,\ y_{n+1} = Mx_{n+1} = Qx_{n+2}\ ,\ y_{n+2} = Nx_{n+2} = Rx_{n+3}$$

Consider , $$\xi\left\{\int_0^{G(y_n,y_{n+1},y_{n+2})} f(t)\,dt\right\} = \xi\left\{\int_0^{G(Lx_n,Mx_{n+1},Nx_{n+2})} f(t)\,dt\right\}$$

$$\le \xi\left\{\int_0^{G(Px_n,Qx_{n+1},Rx_{n+2})} f(t)\,dt\right\} - \eta\left\{\int_0^{G(Px_n,Qx_{n+1},Rx_{n+2})} f(t)\,dt\right\}$$

$$= \xi\left\{\int_0^{G(y_{n-1},y_n,y_{n+1})} f(t)\,dt\right\} - \eta\left\{\int_0^{G(y_{n-1},y_n,y_{n+1})} f(t)\,dt\right\} \quad \text{--------------(2.1.2)}$$

$$\le \xi\left\{\int_0^{G(y_{n-1},y_n,y_{n+1})} f(t)\,dt\right\}$$

Since ξ is continuous and has a monotone property ,

$$\therefore \int_0^{G(y_n,y_{n+1},y_{n+2})} f(t)\,dt \le \int_0^{G(y_{n-1},y_n,y_{n+1})} f(t)\,dt \quad \text{------------------(2.1.3)}$$

Let us take $\delta_n = \int_0^{G(y_n,y_{n+1},y_{n+2})} f(t)\,dt$, then it follows that δ_n is monotone decreasing and lower bounded sequence of numbers.

Therefore there exists $k \ge 0$ such that $\delta_n \to k$ as $n \to \infty$. Suppose that $k > 0$

Taking limit as $n \to \infty$ on both sides of (2.1.2) and using that η is lower semi continuous , we get , $\xi(k) \le \xi(k) - \eta(k) < \xi(k)$, which is a contradiction. Hence $k = 0$.

This implies that $\delta_n \to 0$ as $n \to \infty$ i.e. $\int_0^{G(y_n,y_{n+1},y_{n+2})} f(t)\,dt \to 0$ as $n \to \infty$.----------------(2.1.4)

Now , we prove that $\{y_n\}$ is a G- Cauchy sequence. On the contrary , suppose it is not a G- Cauchy sequence.

$\therefore$ There exists $\in > 0$ and subsequences $\{y_{m(i)}\}$ and $\{y_{n(i)}\}$ such that for each positive integer i , $n(i)$ is minimal in the sense that , $G(y_{n(i)}, y_{m(i)}, y_{m(i)}) \geq \in$ and $G(y_{n(i-1)}, y_{m(i)}, y_{m(i)}) < \in$

Now , $\in \leq G(y_{n(i)}, y_{m(i)}, y_{m(i)}) \leq G(y_{n(i)}, y_{m(i-1)}, y_{m(i-1)}) + G(y_{m(i-1)}, y_{m(i)}, y_{m(i)})$

$$< \in + G(y_{m(i-1)}, y_{m(i)}, y_{m(i)}) \qquad \text{----------------(2.1.5)}$$

Let $0 < \alpha = \int_0^{\in} f(t)\,dt \leq \int_0^{G(y_{n(i)}, y_{m(i)}, y_{m(i)})} f(t)\,dt \leq \int_0^{\in + G(y_{m(i-1)}, y_{m(i)}, y_{m(i)})} f(t)\,dt$

Taking $i \to \infty$, and using (2.1.4) , we get , $\lim_{i\to\infty} \int_0^{G(y_{n(i)}, y_{m(i)}, y_{m(i)})} f(t)\,dt = \alpha$ -------------(2.1.6)

Now , using rectangular inequality , we have

$$G(y_{n(i)}, y_{m(i)}, y_{m(i)}) \leq G(y_{n(i)}, y_{n(i-1)}, y_{n(i-1)}) + G(y_{n(i-1)}, y_{m(i-1)}, y_{m(i-1)}) + G(y_{m(i-1)}, y_{m(i)}, y_{m(i)}) \qquad \text{----(2.1.7)}$$

$$G(y_{n(i-1)}, y_{m(i-1)}, y_{m(i-1)}) \leq G(y_{n(i-1)}, y_{n(i)}, y_{n(i)}) + G(y_{n(i)}, y_{m(i)}, y_{m(i)}) + G(y_{m(i)}, y_{m(i-1)}, y_{m(i-1)}) \qquad \text{----(2.1.8)}$$

$$\therefore \int_0^{G(y_{n(i)}, y_{m(i)} y_{m(i)})} f(t)\;dt \leq \int_0^{G(y_{n(i)}, y_{n(i-1)}, y_{n(i-1)}) + G(y_{n(i)}, y_{m(i-1)}, y_{m(i-1)}) + G(y_{m(i-1)}, y_{m(i)}, y_{m(i)})} f(t)\,dt$$

and

$$\int_0^{G(y_{n(i-1)}, y_{m(i-1)}, y_{m(i-1)})} f(t)\,dt \leq \int_0^{G(y_{n(i-1)}, y_{n(i)}, y_{n(i)}) + G(y_{n(i)}, y_{m(i)}, y_{m(i)}) + G(y_{m(i)}, y_{m(i-1)}, y_{m(i-1)})} f(t)\,dt$$

Taking limit as $i \to \infty$ and using (2.1.4) , (2.1.6) we get

$$\int_0^{G(y_{n(i-1)}, y_{m(i-1)}, y_{m(i-1)})} f(t)\,dt \leq \alpha \leq \int_0^{G(y_{n(i-1)}, y_{m(i-1)}, y_{m(i-1)})} f(t)\,dt$$

This implies that , $\lim_{i\to\infty} \int_0^{G(y_{n(i-1)}, y_{m(i-1)}, y_{m(i-1)})} f(t)\,dt = \alpha$ ---------------------(2.1.9)

Now , from (2.1.1) , we have ,

$$\xi\left\{\int_0^{G(y_{n(i)}, y_{m(i)}, y_{m(i)})} f(t)\,dt\right\} \leq \xi\left\{\int_0^{G(y_{n(i-1)}, y_{m(i-1)}, y_{m(i-1)})} f(t)\,dt\right\} - \eta\left\{\int_0^{G(y_{n(i-1)}, y_{m(i-1)}, y_{m(i-1)})} f(t)\,dt\right\}$$

$\therefore$ Taking limit as $i \to \infty$ and using (2.1.6) , (2.1.8) we will have , $\xi(\alpha) \leq \xi(\alpha) - \eta(\alpha) < \xi(\alpha)$ which is a contradiction. Hence we have $\alpha = 0$.

Hence $\{y_n\}$ is a G- Cauchy sequence. Since (X, G) is a complete G-metric space , there exists a point $u \in X$ such that $\lim_{n\to\infty} y_n = u$

i.e. $\lim_{n\to\infty} Lx_n = \lim_{n\to\infty} Px_{n+1} = u$, $\lim_{n\to\infty} Mx_{n+1} = \lim_{n\to\infty} Qx_{n+2} = u$, $\lim_{n\to\infty} Nx_{n+2} = \lim_{n\to\infty} Rx_{n+3} = u$

As $Lx_n \to u$ and $Px_{n+1} \to u$, therefore we can find some $h \in X$ such that $Qh = u$.

$$\therefore \xi\left\{\int_0^{G(Lx_n, Mh, Mh)} f(t)\,dt\right\} \leq \xi\left\{\int_0^{G(Lx_n, Mh, Nx_{n+1})} f(t)\,dt\right\} \leq \xi\left\{\int_0^{G(Px_n, Qh, Rx_{n+1})} f(t)\,dt\right\} - \eta\left\{\int_0^{G(Px_n, Qh, Rx_{n+1})} f(t)\,dt\right\}$$

On taking limit as $n \to \infty$, we get , $\xi\left\{\int_0^{G(u, Mh, Mh)} f(t)\,dt\right\} \leq \xi(0) - \eta(0)$

$\therefore \quad \xi\left\{\int_0^{G(u,Mh,Mh)} f(t)\,dt\right\}=0$, which implies that $Mh=u$.

Hence $Mh=Qh=u$ i.e. h is the point of coincidence of M and *Q*.

Since the pair of maps M and Q are weakly compatible , we write $MQh=QMh$ i.e. $Mu=Qu$.

Also , $Mx_{n+1}\to u$ and $Qx_{n+2}\to u$, $\therefore$ we can find some $v\in X$ such that $Pv=u$.

$$\therefore \quad \xi\left\{\int_0^{G(Lv,Mx_{n+1},Mx_{n+1})} f(t)\,dt\right\}\le\xi\left\{\int_0^{G(Lv,Mx_{n+1},Nx_{n+2})} f(t)\,dt\right\}\le\xi\left\{\int_0^{G(Pv,Qx_{n+1},Rx_{n+2})} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Pv,Qx_{n+1},Rx_{n+2})} f(t)\,dt\right\}$$

On taking limit as $n\to\infty$, we get , $\xi\left\{\int_0^{G(Lv,u,u)} f(t)\,dt\right\}\le\xi(0)-\eta(0)$

$\therefore \quad \xi\left\{\int_0^{G(Lv,u,u)} f(t)\,dt\right\}=0$, which implies that $Lv=u$. Hence we have $Lv=Pv=u$ i.e. *v* is the point of coincidence of L and *P*. Since the pair of maps *L* and *P* are weakly compatible , we can write $LPv=PLv$ i.e. $Lu=Pu$.

Again , $Nx_{n+2}\to u$ and $Rx_{n+3}\to u$, therefore we can find some $w\in X$ such that $Rw=u$.

$$\therefore \quad \xi\left\{\int_0^{G(Lx_n,Mx_{n+1},Nw)} f(t)\,dt\right\}\le\xi\left\{\int_0^{G(Px_n,Qx_{n+1},Rw)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Px_n,Qx_{n+1},Rw)} f(t)\,dt\right\}$$

On taking limit as $n\to\infty$, we get , $\xi\left\{\int_0^{G(u,u,Nw)} f(t)\,dt\right\}\le\xi(0)-\eta(0)$

i.e. $\xi\left\{\int_0^{G(u,u,Nw)} f(t)\,dt\right\}=0$, which implies that $Nw=u$.

Thus we get $Nw=Rw=u$ i.e. *w* is the coincidence point of N and *R*.

Since the pair of maps *N* and *R* are weakly compatible , we have $NRw=RNw$ i.e. $Nu=Ru$

Now , we show that *u* is the fixed point of *L*.

$$\text{Consider , } \xi\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}=\xi\left\{\int_0^{G(Lu,Mh,Nw)} f(t)\,dt\right\}\le\xi\left\{\int_0^{G(Pu,Qh,Rw)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Pu,Qh,Rw)} f(t)\,dt\right\}$$

$$\therefore \quad \xi\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}\le\xi\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}$$

$$\text{i.e. } \xi\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}\le\xi\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}$$

i.e. $\xi\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}<\xi\left\{\int_0^{G(Lu,u,u)} f(t)\,dt\right\}$, which is a contradiction. $\therefore$ we get $Lu=u$

$\therefore$ $Lu=Pu=u$ i.e. u is fixed point of L and *P*.

Now , we prove that *u* is fixed point of M.

$$\text{Consider , } \xi\left\{\int_0^{G(u,u,Mu)} f(t)\,dt\right\}=\xi\left\{\int_0^{G(Lu,Mu,Nw)} f(t)\,dt\right\}\le\xi\left\{\int_0^{G(Pu,Qu,Rw)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Pu,Qu,Rw)} f(t)\,dt\right\}$$

$$\therefore \quad \xi\left\{\int_0^{G(u,u,Mu)} f(t)\,dt\right\}\le\xi\left\{\int_0^{G(u,Mu,u)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(u,Mu,u)} f(t)\,dt\right\}$$

i.e. $\xi\left\{\int_0^{G(u,u,Mu)} f(t)\,dt\right\}<\xi\left\{\int_0^{G(u,u,Mu)} f(t)\,dt\right\}$, which is a contradiction. $\therefore$ we get $Mu=u$

Hence $Mu=Qu=u$ i.e. u is fixed point of M and Q.

At last we prove that u is fixed point of N.

Consider , $\xi\left\{\int_0^{G(u,u,Nu)} f(t)\,dt\right\}=\xi\left\{\int_0^{G(Lu,Mu,Nu)} f(t)\,dt\right\}\le\xi\left\{\int_0^{G(Pu,Qu,Ru)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Pu,Qu,Ru)} f(t)\,dt\right\}$

i.e. $\xi\left\{\int_0^{G(u,u,Nu)} f(t)\,dt\right\}\le\xi\left\{\int_0^{G(u,u,Ru)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(u,u,Ru)} f(t)\,dt\right\}$

i.e. $\xi\left\{\int_0^{G(u,u,Nu)} f(t)\,dt\right\}<\xi\left\{\int_0^{G(u,u,Ru)} f(t)\,dt\right\}$, which means $\xi\left\{\int_0^{G(u,u,Nu)} f(t)\,dt\right\}<\xi\left\{\int_0^{G(u,u,Nu)} f(t)\,dt\right\}$ as $Nu=Ru$.

Which implies that $Nu=u$. Hence we get $Nu=Ru=u$.

i.e. u is fixed point of N and R.

Thus u is the common fixed point of L,M,N,P,Q and R.

Now , we prove that u is the unique common fixed point of L,M,N,P,Q and R.

If possible , let us assume that μ is another fixed point of L,M,N,P,Q and R.

$$\therefore\ \xi\left\{\int_0^{G(u,u,\mu)} f(t)\,dt\right\}=\xi\left\{\int_0^{G(Lu,Mu,N\mu)} f(t)\,dt\right\}\le\xi\left\{\int_0^{G(Pu,Qu,R\mu)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(Pu,Qu,R\mu)} f(t)\,dt\right\}$$

$$=\xi\left\{\int_0^{G(u,u,\mu)} f(t)\,dt\right\}-\eta\left\{\int_0^{G(u,u,\mu)} f(t)\,dt\right\}$$

i.e. $\xi\left\{\int_0^{G(u,u,\mu)} f(t)\,dt\right\}<\xi\left\{\int_0^{G(u,u,\mu)} f(t)\,dt\right\}$, which is again a contradiction.

Hence finally we will have $u=\mu$.

Thus u is the unique common fixed point of L,M,N,P,Q and R.

Corollary 2.2: Let (X,G) be a complete G-metric space and $L,M,N,P:X\to X$ be mappings such that

i) $L(X)\subset P(X)$, $M(X)\subset P(X)$, $N(X)\subset P(X)$

ii) $\xi\left\{\int_0^{G(Lx,My,Nz)} f(t)\ dt\right\}\le\xi\left\{\int_0^{G(Px,Py,Pz)} f(t)\ dt\right\}-\eta\left\{\int_0^{G(Px,Py,Pz)} f(t)\ dt\right\}$

for all $x,y,z\in X$ where $\xi:[0,\infty)\to[0,\infty)$ is a continuous and non-decreasing function , $\eta:[0,\infty)\to[0,\infty)$ is a lower semi continuous and non-decreasing function such that $\xi(t)=\eta(t)=0$ if and only if $t=0$, also $f:[0,\infty)\to[0,\infty)$ is a Lebesgue integrable function which is summable on each compact subset of R^+, non negative and such that for each $\in>0$,

$\int_0^{\in} f(t)\,dt>0$

iii) The pairs (L,P) , (M,P) , (N,P) are weakly compatible.

Then L,M,N,P have a unique common fixed point in X.

Proof : By taking $P=Q=R$ in Theorem 2.1 we get the proof.

Corollary 2.3: Let (X,G) be a complete G-metric space and $L,P:X\to X$ be mappings such that

i) $L(X) \subset P(X)$

ii) $\xi\left\{\int_0^{G(Lx,Ly,Lz)} f(t)\, dt\right\} \le \xi\left\{\int_0^{G(Px,Py,Pz)} f(t)\, dt\right\} - \eta\left\{\int_0^{G(Px,Py,Pz)} f(t)\, dt\right\}$

for all $x, y, z \in X$ where $\xi : [0,\infty) \to [0,\infty)$ is a continuous and non-decreasing function , $\eta : [0,\infty) \to [0,\infty)$ is a lower semi continuous and non-decreasing function such that $\xi(t) = \eta(t) = 0$ if and only if $t = 0$, also $f : [0,\infty) \to [0,\infty)$ is a Lebesgue integrable function which is summable on each compact subset of R^+ , non negative and such that for each $\in > 0$,

$\int_0^{\in} f(t)\, dt > 0$

iii) The pair (L, P) is weakly compatible.

Then L, P have a unique common fixed point in X.

Proof: By substituting $L = M = N$ and $P = Q = R$ in Theorem 2.1 we get the proof.

Remark: The Corollary 2.3 is the result proved by Vishal Gupta and Naveen Mani [5] in complete metric space.

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