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# To Find the Equality of the Rainbow Connection Number towards the Diameter using Interval Graphs

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***Abstract :*** *Coloring takes its name from the map coloring application, we assign labels to vertices. When the numerical value of the labels is unimportant, we call them colors to indicate that they may be elements of any set. In graph theory, a connected component of an undirected graph is a subgraph in which any two vertices are connected to each other by paths. The rainbow connection number of a connected graph is the minimum number of colors needed to color its edges, so that every pair of its vertices is connected by at least one path in which no two edges are colored the same. In this paper we show that the rainbow connection number of an interval graph, which are of the form the rainbow connection number is equal to the diameter of the graph G plus one*
***Keywords:*** *Diameter, eccentricity, interval graph, rainbow connection number, rainbow path.*

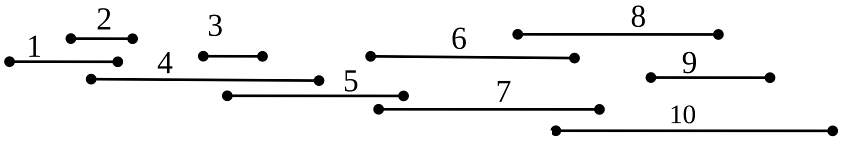
## I. Introduction

A Graph $G=(V,E)$ is an interval graph, if the vertex set $V$ can be put into one-to-one correspondence with a set of intervals $I$ on the real line $R$, such that two vertices are adjacent in $G$ if and only if their corresponding intervals have non-empty intersection. The set $I$ is called an interval representation of $G$ and $G$ is referred to as the intersection graph $I$. The intervals $I=\{i_1,i_2,i_3,\ldots\ldots,i_n\}$ be an interval family, where each $i_i$ is an interval on the real line and $i_i=[a_i,b_i]$ for i= 1,2,.....,$n$ here $a_i$ is called left end point labeling and $b_i$ is called the right end point labeling of $i_i$ . Without loss of generality we assume that all end points of the interval in $I$ are distinct numbers between 1 and 2$n$. Two intervals $i$ and $j$ are said to be intersect each other if they have non-empty intersection. An edge color of a graph is a function from its edge set to the set of natural numbers. A path in an edge colored graph with no two edges sharing the same color is called a Rainbow Path. An edge colored graph is said to be rainbow connected if every pair of vertices is connected by at least one rainbow path, such a color is called a rainbow color of the graph. The minimum number of colors required to a connected graph is called its rainbow connection number [1, 2, 5], denoted by $rc(G)$. In graph theory, the rainbow connection number of a complete graph is one that of a path is its length that of an even cycle is its diameter [3]. The diameter of a graph $G$ is the maximum of eccentricity of all its vertices and is denoted by $diam(G)$, that is $\text{diam}(G) = \max\{ecc(v):v\in V(G)\}$, where the maximum distance from a vertex $u$ to any vertex of $G$ is called eccentricity of the vertex $v$ and is denoted by $ecc(v)$, that is $ecc(v)=\max\{d(u,v):u\in V(G)\}$, where as the distance between two vertices $u$ and $v$ of a graph G is the length of the shortest path between them and is denoted by $d_G(u,v)$ or $d(u,v)$. To the rainbow colored graph it is enough to ensure that every edge of some spanning tree in the graph gets a distinct color. Hence it is an interval graph with minimum degree $\delta(G)$ [4] is two and the diameter "diam(G)".

## II. Main Theorems

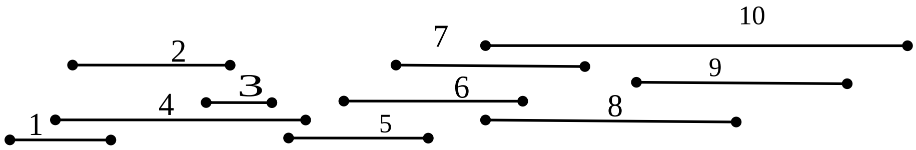
**Theorem (1):** Let G be an interval graph corresponding to an interval family $I= \{i_1,i_2,i_3,i_4,\ldots,i_n\}$. Let i and j be any two intervals in I such that $i\in D$, $j \neq 1$ and j is contained in i and if there is at least one interval to the left of j that intersects j and there is no interval $k \neq i$ to the right of j that intersects j then rainbow connection number rc (G) = diam (G) +1

**Proof :** Let G be an interval graph corresponding to an interval family $I=\{i_1,i_2,i_3,i_4,\ldots\ldots,i_n\}$ . Let i and j be any two intervals in I such that $i\in D$, $j \neq 1$ and j is contained in i and if there is at least one interval to the left of j that intersects j and there is no interval $k \neq i$ to the right of j that intersects j then rainbow connection number rc(G) < diam(G)+1.This is a contradiction to our statement. Hence our assumption that there is no interval to the left of j that intersects j and there is an interval $k \neq i$ to the right of j that intersects j is wrong, and this statement is shown in the following edge colored graph.

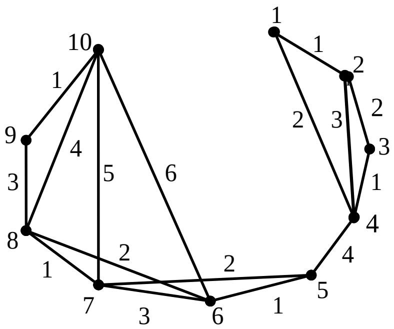



Interval family I

Hence if G is an interval graph corresponding to an interval family I={$i_1$,$i_2$,$i_3$,$i_4$,…..,$i_n$}. Let i and j be any two intervals in I such that $i \in D$, $j \neq 1$ and j is contained in i and if there is at least one interval to the left of j that intersects j and there is no interval $k \neq i$ to the right of j that intersects j then rainbow connection number rc(G) = diam(G)+1.




Interval family I




Interval graph

An interval graph G as follows from an interval family I as in figure, since an interval graph indicated by the neiborhood sets.

| | | |
|---|---|---|
| nbd[1]={1,2,4}, | nbd[2]={1,2,3,4}, | nbd[3]={2,3,4}, |
| nbd[4]={1,2,3,4,5}, | nbd[5]={4,5,6,7}, | nbd[6]={5,6,7,8,10}, |
| nbd[7]={5,6,7,8,10}, | nbd[8]={6,7,8,9,10}, | nbd[9]={8,9,10}, |
| nbd [10]={6,7,8,9,10}, | | |

Interval graph G corresponding to an interval family I.

Let us assume that I= {1, 2… 10} = {$v_1$, $v_{2…}$ $v_{10}$} since,

| | | | |
|---|---|---|---|
| $C(v_1,v_2)=1$ | $C(v_3,v_4)=1$ | $C(v_6,v_7)=3$ | $C(v_7,v_{10})=5$ |
| $C(v_1,v_4)=2$ | $C(v_4,v_5)=4$ | $C(v_6,v_8)=2$ | $C(v_8,v_9)=3$ |
| $C(v_2,v_3)=2$ | $C(v_5,v_6)=1$ | $C(v_6,v_{10})=6$ | $C(v_8,v_{10})=4$ |
| $C(v_2,v_4)=3$ | $C(v_5,v_7)=2$ | $C(v_7,v_8)=1$ | $C(v_9,v_{10})=1$ |

The set of colors are C = {1,2,2,3,1,4,1,2,3,2,6,1,5,3,4,1}

C= {1,2,3,4,5,6}

Therefore rainbow connection number of a graph G is given by rc (G) = 6

The distances of the paths from interval graph G as follows,

| | | | | |
|---|---|---|---|---|
| $d(v_1,v_1)=0$ | $d(v_2,v_1)= 1$ | $d(v_3,v_1)=2$ | $d(v_4,v_1)=1$ | $d(v_5,v_1)=2$ |
| $d(v_1,v_2)=1$ | $d(v_2,v_2)=0$ | $d(v_3,v_2)=1$ | $d(v_4,v_2)=1$ | $d(v_5,v_2)=2$ |
| $d(v_1,v_3)=2$ | $d(v_2,v_3)=1$ | $d(v_3,v_3)=0$ | $d(v_4,v_3)=1$ | $d(v_5,v_3)=2$ |
| $d(v_1,v_4)=1$ | $d(v_2,v_4)=1$ | $d(v_3,v_4)=1$ | $d(v_4,v_4)=0$ | $d(v_5,v_4)=1$ |
| $d(v_1,v_5)=2$ | $d(v_2,v_5)=2$ | $d(v_3,v_5)=2$ | $d(v_4,v_5)=1$ | $d(v_5,v_5)=0$ |
| $d(v_1,v_6)=3$ | $d(v_2,v_6)=3$ | $d(v_3,v_6)=3$ | $d(v_4,v_6)=2$ | $d(v_5,v_6)=1$ |
| $d(v_1,v_7)=3$ | $d(v_2,v_7)=3$ | $d(v_3,v_7)=3$ | $d(v_4,v_7)=2$ | $d(v_5,v_7)=1$ |
| $d(v_1,v_8)=4$ | $d(v_2,v_8)=4$ | $d(v_3,v_8)=4$ | $d(v_4,v_8)=3$ | $d(v_5,v_8)=2$ |
| $d(v_1,v_9)=5$ | $d(v_2,v_9)=5$ | $d(v_3,v_9)=5$ | $d(v_4,v_9)=4$ | $d(v_5,v_9)=3$ |
| $d(v_1,v_{10})=4$ | $d(v_2,v_{10})=4$ | $d(v_3,v_{10})=4$ | $d(v_4,v_{10})=3$ | $d(v_5,v_{10})=2$ |

| | | | | |
|---|---|---|---|---|
| $(v_6,v_1)=3$ | $d(v_7,v_1)=3$ | $d(v_8,v_1)=4$ | $d(v_9,v_1)=5$ | $d(v_{10},v_1)=4$ |
| $d(v_6,v_2)=3$ | $d(v_7,v_2)=3$ | $d(v_8,v_2)=4$ | $d(v_9,v_2)=5$ | $d(v_{10},v_2)=4$ |
| $d(v_6,v_3)=3$ | $d(v_7,v_3)=3$ | $d(v_8,v_3)=4$ | $d(v_9,v_3)=5$ | $d(v_{10},v_3)=4$ |
| $d(v_6,v_4)=2$ | $d(v_7,v_4)=2$ | $d(v_8,v_4)=3$ | $d(v_9,v_4)=4$ | $d(v_{10},v_4)=3$ |
| $d(v_6,v_5)=1$ | $d(v_7,v_5)=1$ | $d(v_8,v_5)=2$ | $d(v_9,v_5)=3$ | $d(v_{10},v_5)=2$ |
| $d(v_6,v_6)=0$ | $d(v_7,v_6)=1$ | $d(v_8,v_6)=1$ | $d(v_9,v_6)=2$ | $d(v_{10},v_6)=1$ |
| $d(v_6,v_7)=1$ | $d(v_7,v_7)=0$ | $d(v_8,v_7)=1$ | $d(v_9,v_7)=2$ | $d(v_{10},v_7)=1$ |
| $d(v_6,v_8)=1$ | $d(v_7,v_8)=1$ | $d(v_8,v_8)=0$ | $d(v_9,v_8)=1$ | $d(v_{10},v_8)=1$ |
| $d(v_6,v_9)=2$ | $d(v_7,v_9)=2$ | $d(v_8,v_9)=1$ | $d(v_9,v_9)=0$ | $d(v_{10},v_9)=1$ |
| $d(v_6,v_{10})=1$ | $d(v_7,v_{10})=1$ | $d(v_8,v_{10})=1$ | $d(v_9,v_{10})=1$ | $d(v_{10},v_{10})=0$ |

Eccentricities of each vertex are as follows,

$ecc(v_1) = \max\{d(v_1,v_1), d(v_1,v_2), d(v_1,v_3), d(v_1,v_4), d(v_1,v_5), d(v_1,v_6), d(v_1,v_7), d(v_1,v_8), d(v_1,v_9),d(v_1,v_{10}) \}$
$= \max\{0,1,2,1,2,3,3,4,5,4\}= 5$

$ecc(v_2) = \max\{d(v_2,v_1), d(v_2,v_2), d(v_2,v_3), d(v_2,v_4), d(v_2,v_5), d(v_2,v_6), d(v_2,v_7), d(v_2,v_8), d(v_2,v_9), d(v_2,v_{10}) \}$
$= \max\{1,0,1,2,2,3,3,4,5,4\} = 5$

$ecc(v_3) = \max\{d(v_3,v_1), d(v_3,v_2), d(v_3,v_3), d(v_3,v_4), d(v_3,v_5), d(v_3,v_6), d(v_3,v_7), d(v_3,v_8), d(v_3,v_9), d(v_3,v_{10}) \}$
$= \max\{2,1,0,1,2,3,3,4,5,4\}=5$

$ecc(v_4) = \max\{d(v_4,v_1), d(v_4,v_2), d(v_4,v_3), d(v_4,v_4), d(v_4,v_5), d(v_4,v_6), d(v_4,v_7), d(v_4,v_8), d(v_4,v_9),d(v_4,v_{10})\}$
$= \max\{1,1,1,0,1,2,2,3,4,3\}=4$

$ecc(v_5) = \max\{d(v_5,v_1),d(v_5,v_2),d(v_5,v_3),d(v_5,v_4),d(v_5,v_5),d(v_5,v_6),d(v_5,v_7),d(v_5,v_8),d(v_5,v_9),d(v_5,v_{10})\}$
$= \max\{2,2,2,1,0,1,1,2,3,2\}=3$

$ecc(v_6) = \max\{d(v_6,v_1), d(v_6,v_2) ,d(v_6,v_3), d(v_6,v_4), d(v_6,v_5), d(v_6,v_6), d(v_6,v_7) ,d(v_6,v_8), d(v_6,v_9) ,d(v_6,v_{10})\}$
$= \max\{3,3,3,2,1,0,1,1,2,1\}=3$

$ecc(v_7) = \max\{ d(v_7,v_1), d(v_7,v_2), d(v_7,v_3), d(v_7,v_4), d(v_7,v_5), d(v_7,v_6), d(v_7,v_7), d(v_7,v_8), d(v_7,v_9), d(v_7,v_{10})\}$
$= \max\{3,3,3,2,1,1,0,1,2,1\}=3$

$ecc(v_8) = \max\{ d(v_8,v_1), d(v_8,v_2), d(v_8,v_3), d(v_8,v_4), d(v_8,v_5), d(v_8,v_6), d(v_8,v_7), d(v_8,v_8), d(v_8,v_9), d(v_8,v_{10}) \}$
$= \max\{4,4,4,3,2,1,1,0,1,1\}=4$

$ecc(v_9)=\max\{ d(v_9,v_1), d(v_9,v_2), d(v_9,v_3), d(v_9,v_4), d(v_9,v_5), d(v_9,v_6), d(v_9,v_7), d(v_9,v_8), d(v_9,v_9), d(v_9,v_{10}) \}$
$= \max\{5,5,5,4,3,2,2,1,0,1\}=5$

$ecc(v_{10})=\max\{d(v_{10},v_1),d(v_{10},v_2),d(v_{10},v_3),d(v_{10},v_4),d(v_{10},v_5),d(v_{10},v_6),d(v_{10},v_7),d(v_{10},v_8),d(v_{10},v_9),d(v_{10},v_{10}\}$
$= \max\{4,4,4,3,2,1,1,1,1,0\}=4$

The diameter of the graph 'G' is the maximum of all its vertices and is denoted by diam(G) .

That is $diam(G) = \max\{ e(v):v\in V(G) \}$
$= \max\{ecc(v_1), ecc(v_2), ecc(v_3), ecc(v_4), ecc(v_5), ecc(v_6), ecc(v_7), ecc(v_8), ecc(v_9), ecc(v_{10})\}$

$diam(G) = 5$

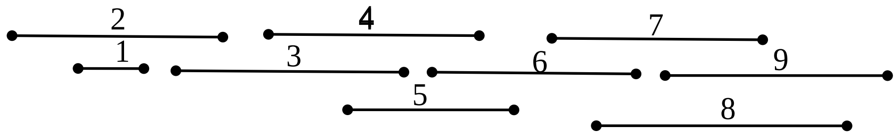
Therefore $rc(G) = 6$, $diam(G)+1= 6$

i.e. $rc(G) = diam(G)+1$ .

**Theorem (2):** If i and j are any two intervals of 'I' such that j is contained in i and there is an interval $k \neq i$, that intersects j then $rc(G) = diam(G)+1$ .

**proof:** Let G be an interval graph corresponding to an interval family $I=\{i_1,i_2,i_3,\ldots\ldots,i_n\}$ . If i and j are any two intervals of I such that j is contained in i and there is an interval $k \neq i$ that intersects j . If possible suppose that there is no other interval $k\neq i$ that intersects j to the right of j , this is shown in the following edge colored graph clearly .

Consider the following interval family I is as follows

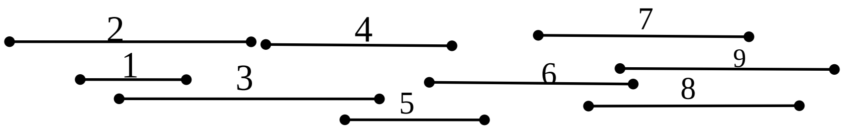



Interval family I

In the above edge cololored graph, we can easily seen that $rc(G) + 3 = diam (G) +1$, which is contradiction to our statement. Hence our assumption that there is no other interval $k \neq i$ that intersect j to the right of j is wrong. There must be an interval $k \neq i$ that intersects j.

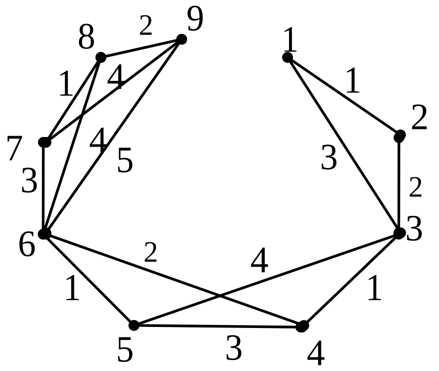
Now to prove the theorem, consider following interval family I

**Illustration:**




Interval family I

The corresponding interval graph G with the edge colors are as follows,




Interval graph G with the edge colors

Interval graph G corresponding to an interval family I .
Let us assume that I={1,2,3,----9} = {$v_1,v_2,\ldots..v_9$ }
Since $c(v_1,v_2)=1$, $c(v_1,v_3)=3$, $c(v_2,v_3)=2$, $c(v_3,v_4)=1$, $c(v_6,v_7)=3$, $c(v_6,v_8)=4$
$c(v_6,v_9)=5$, $c(v_7,v_8)=1$, $c(v_7,v_9)=4$, $c(v_8,v_9)=2$
the set of colors are {1,3,2,1,4,3,2,1,3,4,5,1,4,2}
= {1,2,3,4,5}
Therefore rainbow connection number r.c(G) = 5
The distances of the path lengths from interval graph 'G' as follows,

| | | | | |
|---|---|---|---|---|
| $d(v_1,v_1)=0$ | $d(v_2,v_1)=1$ | $d(v_3,v_1)=1$ | $d(v_4,v_1)=2$ | $d(v_5,v_1)=2$ |
| $d(v_1,v_2)=1$ | $d(v_2,v_2)=0$ | $d(v_3,v_2)=1$ | $d(v_4,v_2)=2$ | $d(v_5,v_2)=2$ |
| $d(v_1,v_3)=1$ | $d(v_2,v_3)=1$ | $d(v_3,v_3)=0$ | $d(v_4,v_3)=1$ | $d(v_5,v_3)=1$ |
| $d(v_1,v_4)=2$ | $d(v_2,v_4)=2$ | $d(v_3,v_4)=1$ | $d(v_4,v_4)=0$ | $d(v_5,v_4)=1$ |
| $d(v_1,v_5)=2$ | $d(v_2,v_5)=2$ | $d(v_3,v_5)=1$ | $d(v_4,v_5)=1$ | $d(v_5,v_5)=0$ |
| $d(v_1,v_6)=3$ | $d(v_2,v_6)=3$ | $d(v_3,v_6)=2$ | $d(v_4,v_6)=1$ | $d(v_5,v_6)=1$ |
| $d(v_1,v_7)=4$ | $d(v_2,v_7)=3$ | $d(v_3,v_7)=3$ | $d(v_4,v_7)=2$ | $d(v_5,v_7)=2$ |
| $d(v_1,v_8)=4$ | $d(v_2,v_8)=4$ | $d(v_3,v_8)=3$ | $d(v_4,v_8)=2$ | $d(v_5,v_8)=2$ |
| $d(v_1,v_9)=4$ | $d(v_2,v_9)=4$ | $d(v_3,v_9)=3$ | $d(v_4,v_9)=2$ | $d(v_5,v_9)=2$ |
| $d(v_6,v_1)=3$ | $d(v_7,v_1)=4$ | $d(v_8,v_1)=4$ | $d(v_9,v_1)=4$ | |
| $d(v_6,v_2)=3$ | $d(v_7,v_2)=4$ | $d(v_8,v_2)=4$ | $d(v_9,v_2)=4$ | |
| $d(v_6,v_3)=2$ | $d(v_7,v_3)=3$ | $d(v_8,v_3)=3$ | $d(v_9,v_3)=3$ | |
| $d(v_6,v_4)=1$ | $d(v_7,v_4)=2$ | $d(v_8,v_4)=2$ | $d(v_9,v_4)=2$ | |
| $d(v_6,v_5)=1$ | $d(v_7,v_5)=2$ | $d(v_8,v_5)=2$ | $d(v_9,v_5)=2$ | |
| $d(v_6,v_6)=0$ | $d(v_7,v_6)=1$ | $d(v_8,v_6)=1$ | $d(v_9,v_6)=1$ | |
| $d(v_6,v_7)=1$ | $d(v_7,v_7)=0$ | $d(v_8,v_7)=1$ | $d(v_9,v_7)=1$ | |
| $d(v_6,v_8)=1$ | $d(v_7,v_8)=1$ | $d(v_8,v_8)=0$ | $d(v_9,v_8)=1$ | |
| $d(v_6,v_9)=1$ | $d(v_7,v_9)=1$ | $d(v_8,v_9)=1$ | $d(v_9,v_9)=0$ | |

The eccentricity of the vertices is as follows ,

$$ecc(v)=\max_{x\in V(G)}(d(v,x))$$

$ecc(v_1)=\max\{d(v_1,v_1), d(v_1,v_2), d(v_1,v_3), d(v_1,v_4), d(v_1,v_5), d(v_1,v_6), d(v_1,v_7), d(v_1,v_8), d(v_1,v_9), \}$
$=\max\{0,1,1,2,2,3,4,4,4\} = 4$
$ecc(v_2)=\max\{d(v_2,v_1), d(v_2,v_2), d(v_2,v_3), d(v_2,v_4), d(v_2,v_5), d(v_2,v_6), d(v_2,v_7), d(v_2,v_8), d(v_2,v_9), \}$
$=\max\{1,0,1,2,2,3,4,4,4\} = 4$

$ecc(v_3) = \max\{d(v_3,v_1), d(v_3,v_2), d(v_3,v_3), d(v_3,v_4), d(v_3,v_5), d(v_3,v_6), d(v_3,v_7), d(v_3,v_8), d(v_3,v_9)\}$
$= \max\{1,1,0,1,1,2,3,3,3\} = 3$
$ecc(v_4) = \max\{d(v_4,v_1), d(v_4,v_2), d(v_4,v_3), d(v_4,v_4), d(v_4,v_5), d(v_4,v_6), d(v_4,v_7), d(v_4,v_8), d(v_4,v_9)\}$
$=\max\{2,2,1,0,1,1,2,2,2\} = 2$
$ecc(v_5) = \max\{d(v_5,v_1), d(v_5,v_2), d(v_5,v_3), d(v_5,v_4), d(v_5,v_5), d(v_5,v_6), d(v_5,v_7), d(v_5,v_8), d(v_5,v_9)\}$
$= \max\{2,2,1,1,0,1,2,2,2\} = 2$
$ecc(v_6) = \max\{d(v_6,v_1), d(v_6,v_2), d(v_6,v_3), d(v_6,v_4), d(v_6,v_5), d(v_6,v_6), d(v_6,v_7), d(v_6,v_8), d(v_6,v_9)\}$
$= \max\{3,3,2,1,1,0,1,1,1\} = 3$
$ecc(v_7) = \max\{d(v_7,v_1), d(v_7,v_2), d(v_7,v_3), d(v_7,v_4), d(v_7,v_5), d(v_7,v_6), d(v_7,v_7), d(v_7,v_8), d(v_7,v_9)\}$
$= \max\{4,4,3,2,2,1,0,1,1\} = 4$
$ecc(v_8) = \max\{d(v_8,v_1), d(v_8,v_2), d(v_8,v_3), d(v_8,v_4), d(v_8,v_5), d(v_8,v_6), d(v_8,v_7), d(v_8,v_8), d(v_8,v_9)\}$
$= \max\{4,4,3,2,2,1,1,0,1\} = 4$
$ecc(v_9) = \max\{d(v_9,v_1), d(v_9,v_2), d(v_9,v_3), d(v_9,v_4), d(v_9,v_5), d(v_9,v_6), d(v_9,v_7), d(v_9,v_8), d(v_9,v_9)\}$
$= \max\{4,4,3,2,2,1,1,1,0\}= 4$

The diameter of a graph G is the maximum of eccentricity of all its vertices and is denoted by diam(G) .
That is $diam(G) = \max\{e(v):v\in V(G)\}$
$= \max\{ecc(v_1), ecc(v_2), ecc(v_3), ecc(v_4), ecc(v_5), ecc(v_6), ecc(v_7), ecc(v_8), ecc(v_9)\}$
$= \max\{4,4,3,2,2,3,4,4,4\} = 4$

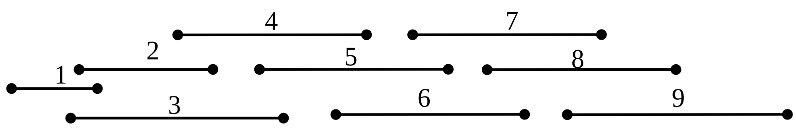
Therefore $rc(G)=5$ $\quad diam(G) + 1 = 5$
Therefore $rc(G) = diam(G) + 1$

**Theorem(3)**: If i, j, k are three consecutive intervals such that $i < j < k$ and i intersects j , j intersects k and i intersects k if $i\in D$ then i must intersects k+1 , then $rc(G) = diam(G)+1$ .

**proof:** Let G be an interval graph corresponding to an interval family $I=\{i_1,i_2,\ldots,i_n\}$. If i, j, k are three consecutive intervals such that $i < j < k$ and i intersects j , j intersects k , and i intersects k if $i\in D$ i.e. i is a dominated interval then i must intersects k+1 then $rc(G) = diam(G) + 1$ .

If possible suppose that for the intervals $i < j < k$ , i intersects j, j intersects k, i intersects k and the dominated interval i does not intersects k+1 then we get the contradiction
i.e. $rc(G) < diam(G) +1$ .This can be shown in the following illustration clearly.

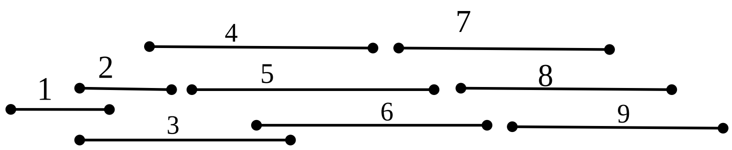
Consider the following interval family,



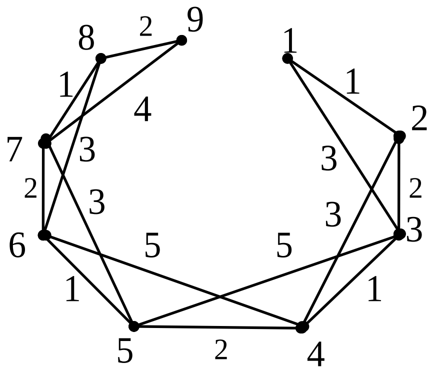

Interval family I

In the above interval graph with the edge colors we can easily seen that $rc (G) < diam (G) +1$.
Hence our assumption that the dominated interval i does not intersects k+1 is wrong . If i, j, k are three consecutive intervals such that $i < j < k$ and i intersects j, j intersects k and i intersects k and if $i\in D$ then i must intersects k+1 , then the rainbow connection number $rc(G) = diam(G) + 1$ .
This can be shown in the following illustration clearly.

**Illustration (3):**




Interval family I



Interval graph

An interval graph as follows from an interval family I as in figure (1) . Since an interval graph indicated by the neiborhood sets .
nbd[1]={1,2,3}, nbd[2]={1,2,3,4}, nbd[3]={1,2,3,4,5}, nbd[4]={2,3,4,5,6}
nbd[5]={3,4,5,6,7}, nbd[6]={4,5,6,7,8}, nbd[7]={5,6,7,8,9}, nbd[8]={6,7,8,9}
nbd[9]={7,8,9}.
Interval graph G corresponding to an interval family I .Let us assume that
$I=\{1,2,3,----------,9\}=\{v_1,v_2,v_{3,}\ldots\ldots..,v_9\}$ since
$C(v_1,v_2)=1$, $C(v_1,v_3)=1$, $C(v_2,v_3)=2$, $C(v_2,v_4)=3$, $C(v_3,v_4)=1$, $C(v_3,v_5)=5$, $C(v_4,v_5)=2$, $C(v_4,v_6)=5$, $C(v_5,v_6)=1$,
$C(v_5,v_7)=3$, $C(v_6,v_7)=2$, $C(v_6,v_8)=3$, $C(v_7,v_8)=1$, $C(v_7,v_9)=4$, $C(v_8,v_9)=2$.
Therefore the set of colors are
C ={1,1,2,3,1,5,2,5,1,3,2,3,1,4,2}
={1,2,3,4,5}
= 5.
Therefore rc(G)=5.
The distances of the path lengths from interval graph 'G' as follows,

| | | | | |
|---|---|---|---|---|
| $d(v_1,v_1)=0$ | $d(v_2,v_1)= 1$ | $d(v_3,v_1)=1$ | $d(v_4,v_1)=2$ | $d(v_5,v_1)=2$ |
| $d(v_1,v_2)=1$ | $d(v_2,v_2)=0$ | $d(v_3,v_2)=1$ | $d(v_4,v_2)=1$ | $d(v_5,v_2)=2$ |
| $d(v_1,v_3)=1$ | $d(v_2,v_3)= 1$ | $d(v_3,v_3)=0$ | $d(v_4,v_3)=1$ | $d(v_5,v_3)=1$ |
| $d(v_1,v_4)=2$ | $d(v_2,v_4)=1$ | $d(v_3,v_4)=1$ | $d(v_4,v_4)=0$ | $d(v_5,v_4)=1$ |
| $d(v_1,v_5)=2$ | $d(v_2,v_5)=2$ | $d(v_3,v_5)=1$ | $d(v_4,v_5)=1$ | $d(v_5,v_5)=0$ |
| $d(v_1,v_6)=3$ | $d(v_2,v_6)=2$ | $d(v_3,v_6)=2$ | $d(v_4,v_6)=1$ | $d(v_5,v_6)=1$ |
| $d(v_1,v_7)=3$ | $d(v_2,v_7)=3$ | $d(v_3,v_7)=2$ | $d(v_4,v_7)=2$ | $d(v_5,v_7)=1$ |
| $d(v_1,v_8)=4$ | $d(v_2,v_8)=3$ | $d(v_3,v_8)=3$ | $d(v_4,v_8)=2$ | $d(v_5,v_8)=2$ |
| $d(v_1,v_9)=4$ | $d(v_2,v_9)=4$ | $d(v_3,v_9)=4$ | $d(v_4,v_9)=3$ | $d(v_5,v_9)=3$ |

| | | | |
|---|---|---|---|
| $d(v_6,v_1)=3$ | $d(v_7,v_1)=3$ | $d(v_8,v_1)=4$ | $d(v_9,v_1)=4$ |
| $d(v_6,v_2)=2$ | $d(v_7,v_2)=3$ | $d(v_8,v_2)=3$ | $d(v_9,v_2)=4$ |
| $d(v_6,v_3)=2$ | $d(v_7,v_3)=3$ | $d(v_8,v_3)=3$ | $d(v_9,v_3)=3$ |
| $d(v_6,v_4)=1$ | $d(v_7,v_4)=2$ | $d(v_8,v_4)=2$ | $d(v_9,v_4)=3$ |
| $d(v_6,v_5)=1$ | $d(v_7,v_5)=1$ | $d(v_8,v_5)=2$ | $d(v_9,v_5)=2$ |
| $d(v_6,v_6)=0$ | $d(v_7,v_6)=1$ | $d(v_8,v_6)=1$ | $d(v_9,v_6)=2$ |
| $d(v_6,v_7)=1$ | $d(v_7,v_7)=0$ | $d(v_8,v_7)=1$ | $d(v_9,v_7)=1$ |
| $d(v_6,v_8)=1$ | $d(v_7,v_8)=1$ | $d(v_8,v_8)=0$ | $d(v_9,v_8)=1$ |
| $d(v_6,v_9)=2$ | $d(v_7,v_9)=1$ | $d(v_8,v_9)=1$ | $d(v_9,v_9)=0$ |

The eccentricity of the vertices is as follows ,
$ecc(v_1) = \max\{d(v_1,v_1), d(v_1,v_2), d(v_1,v_3), d(v_1,v_4), d(v_1,v_5), d(v_1,v_6), d(v_1,v_7), d(v_1,v_8), d(v_1,v_9) \}$
$= \max\{0,1,1,2,2,3,3,4,4\}= 4$
$ecc(v_2) = \max\{d(v_2,v_1), d(v_2,v_2), d(v_2,v_3), d(v_2,v_4), d(v_2,v_5), d(v_2,v_6), d(v_2,v_7), d(v_2,v_8), d(v_2,v_9)\}$
$= \max\{1,0,1,1,2,2,3,3,4\}= 4$
$ecc(v_3) = \max\{d(v_3,v_1), d(v_3,v_2), d(v_3,v_3), d(v_3,v_4), d(v_3,v_5), d(v_3,v_6), d(v_3,v_7), d(v_3,v_8), d(v_3,v_9) \}$
$= \max\{1,1,0,1,1,2,2,3,4\}= 3$
$ecc(v_4) = \max\{d(v_4,v_1), d(v_4,v_2), d(v_4,v_3), d(v_4,v_4), d(v_4,v_5), d(v_4,v_6), d(v_4,v_7), d(v_4,v_8), d(v_4,v_9)\}$

$= \max\{2,1,1,0,1,1,2,2,3\} = 3$

$ecc(v_5) = \max\{d(v_5,v_1), d(v_5,v_2), d(v_5,v_3), d(v_5,v_4), d(v_5,v_5), d(v_5,v_6), d(v_5,v_7), d(v_5,v_8), d(v_5,v_9)\}$
$= \max\{2,2,1,1,0,1,1,2,3\} = 3$

$ecc(v_6) = \max\{d(v_6,v_1), d(v_6,v_2), d(v_6,v_3), d(v_6,v_4), d(v_6,v_5), d(v_6,v_6), d(v_6,v_7), d(v_6,v_8), d(v_6,v_9)\}$
$= \max\{3,2,2,1,1,0,1,1,2\} = 3$

$ecc(v_7) = \max\{d(v_7,v_1), d(v_7,v_2), d(v_7,v_3), d(v_7,v_4), d(v_7,v_5), d(v_7,v_6), d(v_7,v_7), d(v_7,v_8), d(v_7,v_9)\}$
$= \max\{3,3,3,2,1,1,0,1,1\} = 3$

$ecc(v_8) = \max\{d(v_8,v_1), d(v_8,v_2), d(v_8,v_3), d(v_8,v_4), d(v_8,v_5), d(v_8,v_6), d(v_8,v_7), d(v_8,v_8), d(v_8,v_9)\}$
$= \max\{4,3,3,2,2,1,1,0,1\} = 4$

$ecc(v_9) = \max\{d(v_9,v_1), d(v_9,v_2), d(v_9,v_3), d(v_9,v_4), d(v_9,v_5), d(v_9,v_6), d(v_9,v_7), d(v_9,v_8), d(v_9,v_9)\}$
$= \max\{4,4,3,3,2,2,1,1,0\} = 4$

The diameter of the graph 'G' is the minimum of the eccentricity of all its vertices and is denoted by diam(G)
That is diameter(G) = max{e(v:)v∈V(G)}

$diam(G) = \max\{ecc(v_1), ecc(v_2), ecc(v_3), ecc(v_4), ecc(v_5), ecc(v_6), ecc(v_7), ecc(v_8), ecc(v_9)\}$
$= \max\{4,4,3,3,3,3,3,4,4\} = 4$, $rc(G) = 5$, Therefore $rc(G)=diam(G)+1$.

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