International Refereed Journal of Engineering and Science (IRJES)
ISSN (Online) 2319-183X, (Print) 2319-1821
Volume 2, Issue 4(April 2013), PP.27-31
www.irjes.com

The Transition From Newtonian Mechanics to Galilean Transformation Via Calculus of Variation

Dr.Sami. H.Altoum
(Ph.D Mathematics)
Umm Al-qura University –KSA-University College of Al- Qunfudah

Abstract: *The aim of this research to show some standard of fact of special relativity theory can be derived from the calculus of variations. To give the essential of the method ,it suffices to suppose that M is one –dimensional ,so that the Newtonian picture is of a particle of mass m moving on a line with coordinate x and potential energy $V(x)$.We deduced Galilean transformation via calculus of variations*

Keywords*: Variational Principle**,** Galilean Transformations, Newtonian Mechanics*

I. Introduction

The Galilean transformation is used to transform between the coordinates of two reference frames which differ only by constant relative motion within the constructs of Newtonian physics. This is the passive transformation point of view. The equations below, although apparently obvious, break down at speeds that approach the speed of light owing to physics described by relativity theory.Galileo formulated these concepts in his description of uniform motion. The topic was motivated by Galileo's description of the motion of a ball rolling down a ramp, by which he measured the numerical value for the acceleration of gravity near the surface of the Earth. The Galilean symmetries can be uniquely written as the composition of a rotation, a translation and a uniform motion of space-time. Let **x** represent a point in three-dimensional space, and *t*a point in one-dimensional time. A general point in space-time is given by an ordered pair (x,t). In this research we deduced galilean transformation via so we define a variational principle is a scientific principle used within the calculus of variation, which develops general methods for finding functions which minimize or maximize the value of quantities that depends upon those functions. For example, to answer this question: "What is the shape of a chain suspended at both ends?" we can use the variational principle that the shape must minimize the gravitational potential energy.

II. Variational Principle

In classical mechanics the motion of a system with n degree of freedom $x^i\big|_{i=1}^{n}$, motion of point mass can be expressed by the variational principle

$$\delta S = 0, \tag{1}$$

where the action S is a function of the motion $x = x(t)$ of the system given by the Lagrangian $L = L\left(t, x, \dot{x}\right)$,

$$S = \int L\left(t, x \dot{x}\right), \tag{2}$$

from the principle (11) ,then the following equation of motion in the form of Euler-Lagrange equations

$$\frac{\partial L}{\partial x^i} - \frac{d}{dt}\left(\frac{\partial L}{\partial x^i}\right) = 0. \tag{3}$$

As the generalized momentum is normally defined by $P_i = \frac{\partial L}{\partial x^i}$,we can apply it to $\dot{x} = \dot{x}(t, x, p)$,where $p_i = p_i(t, x, p)$,can be solved with respect to x,if the

matrix $\frac{\partial p_i}{\partial x^j} = \frac{\partial^2 L}{\partial x^i \partial x^j}$,is regular, then the phase space $\{x, p\}, \dot{x} = \dot{x}(t, x, p)$.

Given Euler equations for the extremals in coordinates free from introducing the Cartan 1-form θL,a 1-differential forms on $T(M)\times\Re$. This can be defined by using a coordinate system. We find it is useful to see a more general definition in term of an arbitrary basis $(\omega_1,..., \omega_n)$ of 1-differential forms on an open set U of M,then denoted by y_i, the real –value functions defined by $\omega_i onT(M)$;hence, also on $(\omega_1,..., \omega_n)$, where $y_i(v) = \omega_i(v)$, for $v \in T(M)$.At any rate (ω_1, dy, dt), a local basis for differential forms on $T(M)\times\Re$,suppose that

$$dL = L_i\omega_i + L_{n+i}dy_i + L_i dt, \tag{3}$$

now we obtain

$$\theta(L) = L_{n+i}\omega - (L_{n+i}y_i - L)dt \text{ , where } \omega_i = dx_i . \tag{4}$$

Let us verify that θL remains unchanged when a different basis $(\omega^\bullet)$,of
1-form for U is used. We state another property of $\theta(L)$ if $t \to \sigma(t), a \le t \le b$
is a curve in M and if

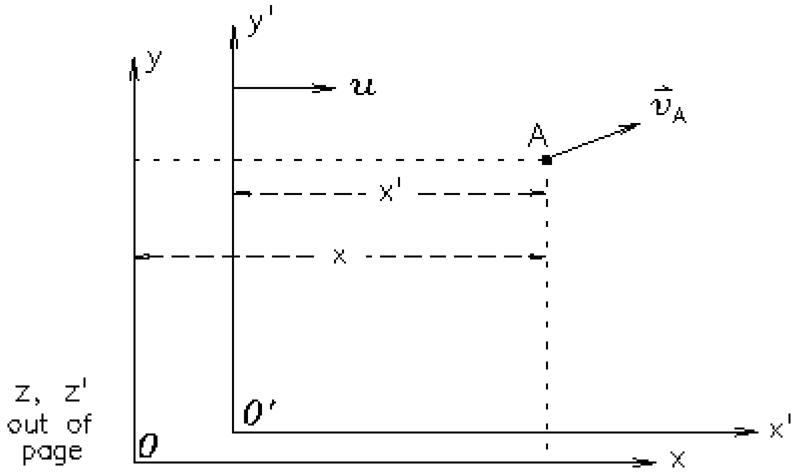
$$t \to (\sigma^\bullet(t), t) = \gamma(t) \tag{5}$$

is extended curve on $T(M)\times\Re$,then

$$\int_a^b L(\sigma^\bullet(t), t) = \gamma(t). \tag{6}$$

III. Galilean Transformations

Well, this innocuous looking claim has some very perplexing logical consequences with regard to relative velocities, where we have expectations that follow, seemingly, from self-evident common sense. For instance, suppose the propagation velocity of ripples (water waves) in a calm lake is 0.5 m/s. If I am walking along a dock at 1 m/s and I toss a pebble in the lake, the guy sitting at anchor in a boat will see the ripples move by at 0.5 m/s but I will see them dropping back relative to me! That is, I can "outrun" the waves. In mathematical terms, if all the velocities are in the same direction (say, along *x*), we just add relative velocities: if *v*ies the velocity of the wave relative to the water and *u* is my velocity relative to the water, then *v*', the velocity of the wave relative to me, is given by *v*' = *v* - *u*. This common sense equation is known as the Galilean velocity transformation- a big name for a little idea, it would seem. With a simple diagram, we can summarize the common-sense Galilean transformations.



First of all, it is self-evident that $t'=t$, otherwise nothing would make any sense at all. Nevertheless, we include this explicitly. Similarly, if the relative motion of O' with respect to O is only in the x direction, then $y'=y$ and $z'=z$, which were true at $t=t'=0$, must remain true at all later times. In fact, the only coordinates that differ between the two observers are x and x'. After a time t, the distance (x') from O' to some obect A is *less* than the distance (x) from O to A by an amount ut, because that is how much *closer* O' has *moved* to A in the interim. Mathematically, $x' = x - ut$.

The velocity $\vec{v}_A$ of A in the reference frame of O also looks different when viewed from O' - namely, we have to subtract the relative velocity of O' with respect to$O$, which we have labeled $\vec{u}$. In this case we picked $\vec{u}$ along , x' so that the vector $\vec{v}'_A = \vec{v}_A - \vec{u}$ subtraction becomes just $v'_{Ax} = v_{Ax} - u$ while $v'_{Ay} = v_{Ay}$ and$v'_{Az} = v_{Az}$. Let's summarize all these "coordinate transformations:"

Coordinates:

$$x' = x - ut$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

Velocities:

$$v'_{A_x} = v_{A_x} - u$$
$$v'_{A_y} = v_{A_y}$$
$$v'_{A_z} = v_{A_z}$$

This is all so simple and obvious that it is hard to focus one's attention on it. We take all these properties for granted - and therein lies the danger.A uniform motion, with velocity **v**, is given by $(x,t) \mapsto (x+tv,t)$where **v** isin $\Re^3$.A translation is given by:

$$(x,t) \mapsto (x+a, t+b) \tag{7}$$

where **a** in $\Re^3$ and b in $\Re$. A rotation is given by $(x,t) \to (Gx,t)$where is an ort$G:\Re^3 \to \Re^3$hogonal transformation. As a Lie group, the Galilean transformations have dimensions 1

IV. Newton Lagrange Mechanics and Galilean Transformations

The Newtonian Lagrangian

$$L = \frac{1}{2}\dot{x}^2 - \frac{V(x)}{m}. \tag{8}$$

This Lagrangian is of the non homogeneous, regular type, so that its extremals come with their own parametrization.This parameter, of course, is identified with the physical time.

Now L defines a whole class of Lagrangians as $V(x)$ runs over the class of suitable functions. Let ϕ be a transformation of $(x,t)-space$ into itself which Lagrangian L' of the same class, that is,

$$L' = \frac{1}{2}\dot{x}^2 - \frac{V'(x)}{m}, \tag{9}$$

So that if $t \to (x(t),t)$ is an extremal of L , $t \to (\phi(t),t)$ is an extremal of L' . Now it is easily seen that this requires that $\phi^*(dt) = dt$ .That is $\phi^*(t) = t + cons\tan t : (\phi_*)^* = L$ .To determine the explicit possibilities for ϕ ,suppose $\phi^*(x) = f(x,t)$. Then

$$(\phi_*)^*(L') = \frac{1}{2}\left(\frac{\partial f}{\partial x}\right)^2 \dot{x}^2 - V'(f(x,t)) = 2L, \tag{10}$$

For some constant α or

$$\frac{\partial f}{\partial x} = \pm 1 \text{ or } \qquad f = \pm x + g(t). \tag{11}$$

Then the condition

$$\frac{V'(f(x,t))}{m} = \frac{V(x)}{m} \tag{12}$$

Forces $g(t) = \beta = cons\tan t.$

Finally, then ,we see that ϕ must be of the form

$$\phi(x,t) = (\pm x + \alpha, t + \beta). \tag{13}$$

Hence the symmetry group is the group of rigid motions of the real line. We also see that the symmetry group permuting this class of Lagrangians is the symmetry group of the Lagrangian for the extremals of the free Lagrangian.

Now let us are looking for maps ϕ of $(x,t)-space$ into itself which permute the extremals of

$$L = \frac{1}{2}\dot{x}^2 \tag{14}$$

ϕ is of the form

$$\phi(x,t) = (f(x,t), t + \beta). \tag{15}$$

The extended transformation of ϕ is

$$\phi_* : (x,\dot{x},t) \to \left(f(x,t), \frac{\partial f}{\partial x}\dot{x} + \frac{\partial f}{\partial t}, t + \beta \right). \tag{16}$$

Now $\theta(L) = \dot{x}dx - \frac{1}{2}\dot{x}^2 dt$;hence

$$(\phi_*)^*(\theta(L)) = \left(\frac{\partial f}{\partial x}\dot{x} + \frac{\partial f}{\partial t}\right)\left(\frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial t}dt\right) - \frac{1}{2}\left(\frac{\partial f}{\partial x}\dot{x} + \frac{\partial f}{\partial t}\right)^2 dt. \tag{17}$$

Setting the coefficient of $d\dot{x} \wedge dt$ in $\phi^*(d\theta(L)) - d\theta(L)$ equal to zero gives

$$\frac{\partial f}{\partial x}\frac{\partial f}{\partial t} - \left(\frac{\partial f}{\partial x}\dot{x} + \frac{\partial f}{\partial t}\right)\frac{\partial f}{\partial x} = -\dot{x}. \tag{18}$$

This implies, since $(x, \dot{x}, t)$ are independent variables,

$$\frac{\partial f}{\partial x} = \pm 1 \quad \text{or} \quad \frac{\partial f}{\partial x} = \pm x + g(t). \tag{19}$$

Hence,

$$(\phi_*)^*(\theta(L)) = \left(\pm \dot{x} + \frac{dg}{dt}\right)(\pm dx + dg) - \frac{1}{2}\left(\pm \dot{x} + \frac{dg}{dt}\right)^2 dt. \tag{20}$$

Setting the coefficient of $dx \wedge dt$ in $d(\phi_*)^*(\theta(L)) - d\theta(L)$ equal to zero gives

$$\frac{d^2 g}{dt^2} = 0 \quad \text{or} \quad g(t) = \gamma t + \alpha . \tag{21}$$

Finally, then, ϕ is of the form:

$$(x,t) \to (\pm x, \gamma t + \alpha, t + \beta). \tag{22}$$

Equation (22) is a Galilean transformation and equivalence equation (7).

Conclusion:

In this research we deduced Galilean transformation via calculus of variations**,** also

Its defining property can be but more physically in the following way:

If $s \to (x(s), t(s))$ is a curve in space-time, let $\left(\frac{dx}{ds} / \frac{dt}{ds}\right)$ be the velocity of the curve. The Galilean transformations permute the curves of constant velocity. The coefficient γ is the increment given to the velocity. The new coordinates for space-time introduced by a Galilean transformation then represent physically a coordinate system moving at constant velocity with respect to the old.

References:

[1] Abraham Albert Ungar –Analytic Hyperbolic Geometry and Albert Einstein's Special Theory Relativity-World Scientific publishing Co-Pte.Ltd, (2008).

[2] Al Fred Grany, Modern Differential Geometry of Curves and Surfaces with Mathematica, CRC Press (1998).

[3] Aubin Thierry, Differential Geometry-American Mathematical Society(2001).

[4] Aurel Bejancu &Hani Reda Faran-Foliations and Geometric Structures, Springer Adordrecht, the Netherlands(2006).

[5] Bluman G.W & Kumei. S , Symmetry and Differential Equations New York :Springer-Verlag (1998).

[6] David Bleaker-Gauge Theory and Variational Principle, Addison- Wesley Publishing Company, (1981).

[7] Differential Geometry and the calculus of Variations, Report Hermann-New York and London, (1968).

[8] Edmund Bertschinger-Introduction to Tenser Calculus for General Relativity, (2002).

[9] M. Lee. John-Introduction to Smooth Manifolds-Springer Verlag, (2002).

[10] Elsgolts,L., Differential Equations and Calculus of variations, Mir Publishers,Moscow,1973.

[11] Lyusternik,L,A., The shortest Lines:Varitional Problems, Mir Publishers,Moscow,1976.

[12] Courant ,R. and Hilbert,D. ,Methods of Mathematical Physics,Vols.1 and 2,Wiley – Interscince, New York,1953.

[13] Hardy,G.,Littlewood,J.E.and Polya,G.,Inqualities,(Paperback edition),Cambrige University Press,London,1988.

[14] Tonti,E.,Int. J. Engineering Sci.,22,P.1343,1984.

[15] Vladimirov,V.S.,A Collection of problems of the Equations of Mathematical Physics, Mir Publishers,Moscow,1986.

[16] Komkov,V.,Variational Principles of Continuum Mechanics with Engineering Applications,Vol.1,D.Reidel Publishing Co.,Dordecht,Holland,1985.

[17] Nirenberg,L.,Topic in Calculus of Variations (edited by M.Giaquinta),P.100,Springer – Verlag,Berlin 1989.

Dr. Sami Hajazi Mustafa

received his PhD degree in Differential Geometry from Alneelain University, Sudan in 2007. He was a head of Department of Mathematics in Academy of Engineering Sciences in Sudan. Now he is Assistant Professor of Mathematics, University college of Alqunfudha, Umm Al Qura University , Saudi Arabia.