International Refereed Journal of Engineering and Science (IRJES)
ISSN (Online) 2319-183X, (Print) 2319-1821
Volume 2, Issue 3(March 2013), PP.43-53
www.irjes.com

Converging Algorithm for Calculation of Elements Current That Include Effects of Mutual Coupling

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Abstract:- *Phased array antennas are used with the intention to increase the efficiency and performance of communication system A high performance design requires the knowledge and control of the mutual coupling effects between the antenna elements, an inadequate handling of the mutual coupling may result in a reduced gain and a surprisingly high side-lobe level. There have been many methods suggested to identify' or compensate the mutual coupling effect in dipole antenna arrays.In this paper we make use of new rapidly converging algorithm for the calculation of the elements current that include the effects of mutual coupling.*

Keywords: *Phase Array Antenna , Converging Algorithm , Mutual Coupling*

I. INTRODUCTION

For modern satellite communication systems where several hundreds of Microstrip elements exist mutual coupling effects arise [8]. Radar Cross Section (RCS) of an aerospace vehicle significantly depends on the scattering of the signals by the antenna array mounted on it. And this scattering also depends on the mutual coupling effect [36]. When a radiating element is placed in an array environment, its electromagnetic properties are affected due to mutual coupling with other radiation elements of the array.
Mutual coupling is caused by the electromagnetic coupling between the array elements. It deteriorates the active input impedance of each array element and affects the radiation pattern of the whole array.
Besides, mutual coupling can also be the cause of blind scan angles, which should always be avoided within the scan range of the array.

Therefore, the development of antenna should start with the development of an accurate theoretical model that includes the effect of mutual coupling [4] The level of mutual coupling between Microstrip patch radiating elements is quite large up to -15 dB for adjacent elements [8]. The array should be optimized to give a minimum scan loss over the entire frequency band and scan range of interest [33] [34][35]

1.2 LINEAR AND PLANAR ANTENNA ARRAYS
1.2.1 Linear Array
1.2.1.1 The Mutual Impedance

The mutual impedance between two ports in a multi port network figure 5.1 is (defined as

$$Z_{ij} = \left.\frac{V_i}{I_j}\right|_{j^i=0} \quad (5\text{-}1)$$

Z_{ij}: is the mutual impedance between ports *i* and *j*.
V_i: is the voltage across port *i*.
I_j: is the culTent flowing into port/,
I_j^i: is the current into any port other than j.

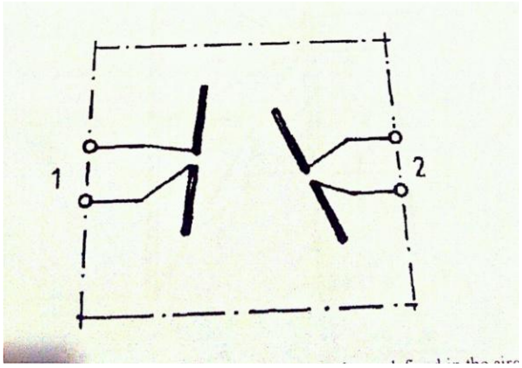

Figure 5.1. Two antennas as a network with mutual impedance defined in the circuit sense.

The mutual impedance between two dipoles with their terminals considered to be the two ports of a two-port network as shown in figure 5.1 is given under the usual assumption that it is unaffected by the presence of elements other than *i* and *j*, by [16]

$$Z_{ij} = \frac{j30}{\sin kl_1 . \sin kl_2} \int_{t2}^{t_2} [\frac{1}{r_1}\exp(-jkr_1) + \frac{1}{r_2}\exp(-jkr_2) - 2\cos kl_1 \frac{1}{r}\exp(-jkr)] \qquad .\sin k(l_2 - |\zeta_2|) d\zeta_2 \qquad (5\text{-}2)$$

Where

$$r = \left[y^2 + (z+\zeta_2)^2\right]^{1/2} \qquad (5\text{-}3)$$

$$r_1 = \left[y^2 + (z+\zeta_2 - l_i)^2\right]^{1/2} \qquad (5\text{-}4)$$

$$r_2 = \left[y^2 + (z+\zeta_2 - l_i)^2\right]^{1/2} \qquad (5\text{-}4)$$

The definitions of the variables are given in figure 5.2.

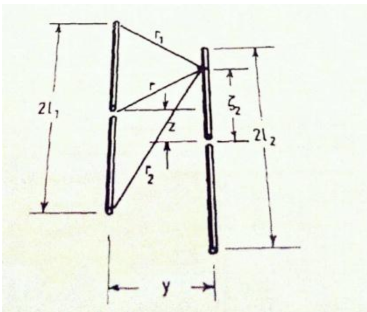

Figure 5.2. Geometry of two radiators defining the variables

The mutual impedance between any two dipoles of known dimensions and position is calculated directly by performing numerical integration.

1.2.1.2 The Active Impedance

While the mutual impedances between the elements in an array are determined by the physical properties of the array i.e., the dimensions of the elements and their positions, the mutual coupling between elements is also a function excitation of the excitation of the radiators. Thus, for an array of N elements,

$$V_n \, Z_{1n} \, I_1 + Z_{2n} I_2 + ... + Z_{nn} I_n + ... + Z_{nn} I_N$$

$$Z_{an} = \frac{V_n}{I_n}$$

$$Z_{an} = Z_{1n}\frac{I_1}{I_n} + Z_{1n} = Z_{1n}\frac{I_2}{I_n} \ldots + Z_{nn} + \ldots + Z_{Nn}\frac{I_N}{I_n}$$

$$Z_{an} = \sum_{i=1}^{N} Z_{1n}\frac{I_1}{I_n} + Z_{nn} \qquad (5\text{-}6)$$

In equation (5-6) Z_{an} is termed the active impedance of the nth radiator, and N' denotes the exclusion of $i=n$ from the summation. The first term on the right-hand side is what Elliott [16] calls the mutual coupling term and can only be calculated if the element currents are known, and vice versa.
Here I shall consistently refer only to the active impedance and reserve the expression "mutual coupling' for the phenomenon itself.

1.2.1.3 Element Currents: An Iterative Algorithm

In order to obtain the element currents, we make use of an iterative algorithm. In principle it involves taking a first guess (which does not have to be an educated one) of the values of the element currents. These currents are then used in equation (5-6) to calculate the active impedance of each element. Once the (very approximate) active impedances are known, a new current flowing in each element port can be calculated. These new currents include the effects of the mutual coupling, and are a better approximation to the actual currents that have to be determined. The crucial step is in choosing the procedure for selecting the new currents for the new iteration

Let the current at the beginning of each cycle he termed the starting current of that cycle, and the current at the end of a cycle of calculations be called the final current. The starting current of the new cycle is chosen to be one half of the sum of the starting current and the final current of the previous cycle. It turns out that if this is done, the estimate of the current improves rapidly.

The process now has to be repeated, in order to improve the estimate of the final current. until an acceptably accurate estimate has been arrived at. When exactly this has been achieved is determined by computing an error trem that will be described below.

Referring to figure 5.3(a) let, at the terminals of antenna i,
Z_{ij} = mutual impedances between elements i and j.
V_i = voltage measured at the antenna terminals.
I_i^{sk} = antenna current at the start of *kth* iteration.
I_i^{fk} = antenna current at the end of *kth*. iteration.
Z_{ai}^{k} = antenna active impedance, at the end of the *kth* iteration.
Also,
Z_0= The output impedance of the amplifiers in figure 5.3(a) and (b).
En= The element driving voltage distribution, equal to the Thevenin equivalent voltage with each amplifier terminated in Z_o as in figure 5.3(a).

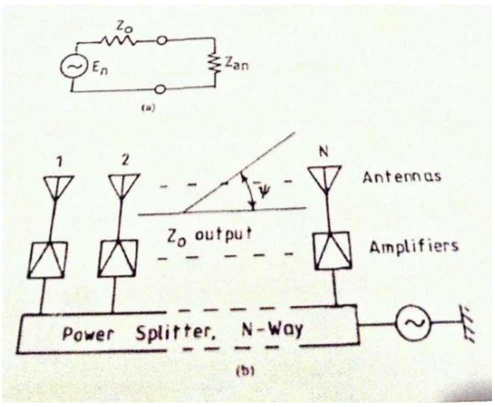

Figure 5.3. Array distribution along the power splitter,

Note that the driving voltage distribution can have a progressive phase shift of equal phase shifts across the width of the array.
In the first cycle, we assume that the mutual impedances are zero, and calculate the staring element current matrix for n = 1 to N, as simply the driving voltage divided by the sum of the element self-impedance and the generator impedance

$$\left[I_n^{s1}\right] = \left[\frac{E_n}{(Z_{nn} + Z_0)}\right] \quad (5\text{-}7)$$

Now the active impedance matrix is calculated from

$$Z_{ai}^{k} = \sum_{i=1}^{N} Z_{1n} \frac{I_i^{sk}}{I_n^{sk}} + Z_{nn} \quad (5\text{-}8)$$

The final (i.e., at the end of the cycle) current matrix is calculated from

$$\left[I_n^{fk}\right] = \left[\frac{E_n}{(Z_{an} + Z_0)}\right] \quad (5\text{-}9)$$

The new- starting current matrix is obtained as

$$\left[I_n^{s(k+1)}\right] = \frac{1}{2}\left[I_n^{sk} + I_n^{fk}\right] \quad (5\text{-}10)$$

After each iteration. an error size is calculated as

$$\Xi = \sum_{i=1}^{N} \left|\mathrm{Re}\left[I_i^{fk}\right] - \mathrm{Re}\left[I_i^{sk}\right] + \left|\mathrm{Im}\left[I_i^{fk}\right] - \left|\mathrm{Im}\left[I_i^{sk}\right]\right.\right.\right|^2 \quad (5\text{-}11)$$

The error size is used as an indication of the convergence of the procedure. In practice, the error decreases almost exactly logarithmically while the currents are converging to the correct values. As soon as the decrease in error deviates from the logarithmic law, the final values of currents are being approached and the procedure can be slopped.

1.2.1.4 Directivity and Radiation Pattern

While the purpose of the exercise is to obtain the element currents, the effects of mutual coupling are also observable in the change in directivity and side-lobe level between cases where the mutual coupling is taken into consideration and cases where it is not. The radiation pattern is calculated very simply as

$$A(\psi) = \left|\sum_{i=1}^{N} I_i^{f} \exp(-jkd\ \cos\psi)\ \right|^2 \quad (5\text{-}12)$$

Where d and ψ are defined in figure 53(b).

The directivity is calculated from equations presented by Cheng [29].

$$D = \frac{[I]^r [A][I]}{[I]^r [B][I]} \quad (5\text{-}13)$$

Where
[*I*] is the matrix describing the currents in each of the elements.
[*I*] is the complex conjugate transpose of [*I*][A] is a complex N x N matrix with elements $A_{mn,}$.
[B] is an N x N matrix with elements B_{mn}, and form. n 1,2, 3 N,

$$A_{mn} = \exp(\, jk[m - n] \sin \psi_0 \,) \qquad (5\text{-}14)$$

For radiators with an isotropic radiation pattern [30].

$$B_{mn} = \frac{\sin(k[m-n]d}{(k[m-n]d)} \qquad (5\text{-}15)$$

While for dipole-type radiators,

B*mn* =sin(k[m-n]d]/(k[m-n]d) + cos(k[m-n]) d]/(k[m-n]d)^2+sin (k[m-n]d)/(k[m-n]d)^3

$$B_{mn}=2/3. \qquad (5\text{-}16)$$

1.2.2 PLANAR ARRAY

1.2.2.1 The Mutual Impedance

The general formula for the mutual impedance of the planar array can be written as:

$$Z_{ij} = \frac{j30}{\sin kl_i . \sin kl_j} \int_{ij}^{ij} [\frac{1}{r_1} \exp(-jkr_1) + \frac{1}{r_2} \exp(-jkr_2) - 2\cos kl_i \frac{1}{r} \exp(-jkr)]$$

$$.\sin k(l_j - |\zeta| d\zeta \qquad (5\text{-}17)$$

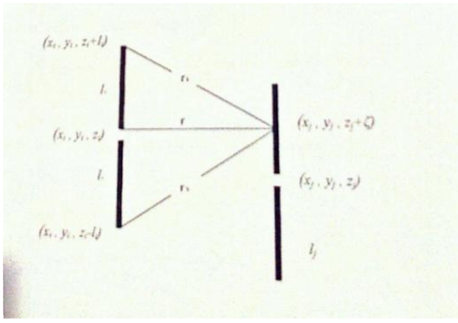

Figure 5.6, Geometry of two radiators for three dimension configuration.

From figure 5.6 we can find that

$$r = ((x_j - x_i)^2 (y_j - y_i)^2 + (z_j + \zeta - z_i)^2)^{1/2} \qquad (5\text{-}18)$$

$$r = ((x_j - x_i)^2 (y_j - y_i)^2 + (z_j + \zeta - z_i - l_i)^2)^{1/2} \qquad (5\text{-}19)$$

$$r_2 = ((x_j - x_i)^2 (y_j - y_i)^2 + (z_j + \zeta - z_i + l_i)^2)^{1/2} \qquad (5\text{-}20)$$

1.2.2.2 The Radiation Pattern

The radiation pattern can be calculated simply by [3]:
E(total) = [E(single element at reference point)] x array factor] (5-21)

The array factor can be calculated by:

$$AF = \sum_{m=1}^{M} \exp(\, j(m-1)(kd_x \sin\theta \cos\phi)) \sum_{m=1}^{M} \exp(\, j(n-1)(kd_y \sin\theta \cos\phi) \qquad (5\text{-}22)$$

Where 1r is the uniform amplitude excitation of the entire array.

And ,

$$E_\theta = j\eta \frac{I_0 \exp(jkr)}{2\pi r}\left[\frac{\cos(\frac{kl}{2}\cos\theta) - \cos(\frac{kl}{2})}{\sin\theta}\right] \quad (5\text{-}23)$$

Where E8 is the electric field component in the far field

1.2.2.3 The Directivity

The Directivity for planar array configuration is given as [3]:

$$D_0 = 4\pi \frac{F(\theta,\phi)\big|_{\max}}{\left[\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta . d\theta d\phi\right]} \quad (5\text{-}24)$$

Where F(θ,ϕ) is the array factor.

II. CALCULATIONS

2.1 Calculation for Linear Array

The following parameters were taken as an input:
f=3 GH_Z, is the operating frequency.
N=9. is the number of radiating elements.
d=0.5 λ . is the spacing between elements multiplied by wavelength.
L=0.5 λ . is the length of the dipole multiplied by wavelength.

2.1 Calculation Steps

2.1.1 Considering Mutual Coupling Effects

When taking mutual coupling into consideration the following steps should be applied:
Step 1. Calculate mutual impedance Z_{ij} using equation (5-2), this yields N x N matrix. The mutual impedance data are used as input to the subroutine for calculating the impedance and element currents.
Step 2. Calculate starting current for the first cycle using the self impedances only using equation (5-7), this yields Nxl matrix
Step 3. Calculate active impedance using equation (5-8), this yields Nxl matrix Step 4. Calculate final current using equation (5-9),this yields Nxl matrix Step 5. Calculate new starting current using equation (5-10)
Step 6. Calculate error size using equation (5-11)
Step 7. Check if error is less than minimum acceptable error then proceed to next step otherwise go to step 3 and loop again.
Figure 5.4 shows the magnitude of the error during the iterative process. It can be clearly seen that for approximately 15 iterations the error decreases logarithmically, after which the rate of decrease tapers off. At this stage, the process is terminated, because the error has become extremely small and the approximation is acceptably close to the true answer.
Step 8. Calculate radiation pattern using equation (5-12)
Step 9. Calculate directivity using equation (5-13)
Step 10. Plot radiation Pattern
These steps are summarized in a flowchart in appendix A

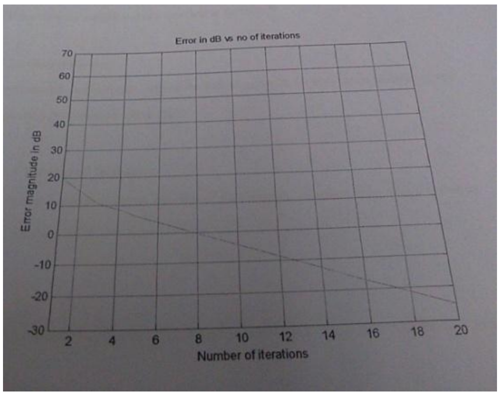

Figure 5.4, Magnitude of the error as a function of the number of iteration for linear array.

2.1.2 Neglecting Mutual Coupling Effects

When neglecting mutual coupling effects the steps will be reduced as follows:
Step 1. Calculate mutual impedance Z1, using equation (5-2), where i and] are equal and whenever i does not equal to j, the mutual impedance will be zero.
This yields N x N matrix.
Step 2. Calculate elements current equation (5-7), this yields Nxl matrix. And this will be the final current.
Step 3. Calculate radiation pattern using equation (5-12)
Step 4. Calculate directivity using equation (5-13)
Step 5. Plot radiation pattern

III. RESULTS

3.1 RESULTS FOR LINEAR ARRAY

Applying the above process, we find the following results for both cases, considering and neglecting the mutual coupling effects:

3.1.1 The Mutual impedance

The mutual impedance matrix consists of 81 elements, of which only nine are unique.
These unique elements are shown in table 5.1.

Table 5.1 Mutual Impedances for a Nine Element Linear Array with O.52 Interelement Spacing

Element	With Mutual Coupling	Without Mutual Coupling
Z(l,l)	72.9831 +42.2710i	72.9831 +42.27l0i
Z(l,2)	-12.5070 -29.8687i	0
Z(1,3)	4.0036 +17.7065i	0
Z(1,4)	-1.8835 -12.27971	0
Z(1,5)	1.0820+9.3457i	0
Z(1,6)	-0.6994 7.52861	0
Z(l,7)	0.4884 +6.2978i	0
Z(l.8)	-0.3600—5.4107i	0
Z(l,9)	0.2762+4.7415i	0

3.1.2 The Active Impedance

The active impedance matrix consists of 9 elements, and it is calculated only when mutual coupling is considered. The active impedance matrix is shown in table 5.2

Table 5.2 Active Impedances for a Nine Element Linear Array with 0.5k Interelement Spacing

Element With Mutual Coupling	Element With Mutual Coupling
Za(1,l)	65.7954 +17.3293i
Za(2,l)	48.7605 + 1.4786i
Za(3.1)	56.8363 + 4.9915i
Za(4,l)	51.0484 + 3.2037j
Za(5,1)	55.9752 ± 4,4049i
Za(6,l)	5 1.0484 + 3.2037i
Za(7,1)	56.8363 +4.9915i
Za(8,1)	48.7605 ± 1.4786i
Za(9,1)	65.7954 +17.3293i

3.1.3 The Element Currents

The elements current is shown in Table 5.3, for both cases, considering and neglecting mutual coupling. Table 5.3 Elements Current for a Nine Element Linear Array with 0.52. Interelement Spacing

Element	With Mutual Coupling	Without Mutual Coupling
I_f (l, l)	8.4467 - 1.2641i	7.2721 - 2.4995i
I_f (2,1)	10.1232-0.1516i	7.2721 -2.4995i
I_f (3,1)	9.3397 -0.4364i	7.2721 - 2.4995i
I_f (4,1)	9.8863 - O.3134i	7.2721 - 2.4995i
I_f (5,1)	9.4199 O.3915i	7.2721 - 2.4995i
I_f (6,l)	9.8863 - 0.3134i	7.2721 -2.4995i
I_f (7,1)	9.3397 - 0.4364i	7.2721 - 2.4995i
I_f (8,l)	10.1232-0.1516i	7.2721 -2.4995i
I_f (K9,1)	8.4467-1.2641i	7.2721 -2.4995i

3.1.4 The Directivity

The directivity is shown in table 5.4, for both cases, considering and neglecting mutual coupling.

Table 5.4 Directivity in dB for a Nine Element Linear Array with 0.52. Interelement Spacing

	With Mutual Coupling	Without Mutual Coupling
Directivity (dB)	17.6431	17.7529

3.1.5 Radiation Pattern Comparison in Two Dimension

Figure 5.5 shows the radiation pattern of the array, both with and without taking the mutual coupling into consideration for the array with uniform aperture illumination.

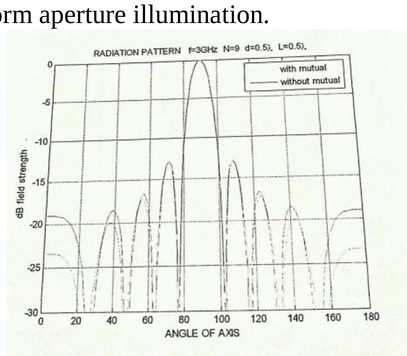

Figure 5.5, Radiation pattern for both the case where mutual coupling is taken into consideration (blue line), and where it is not (red line), for a uniformly illuminated aperture.

3.2 Output Results

Applying the above process, we find the following results for both cases, considering and neglecting the mutual effects:

3.2.1 The Mutual Impedance

The mutual impedance matrix consists of 656i elements; only nine elements are shown in table 5.5.

Table 5.5 Mutual Impedances For Planar Array 9x9 Elements With 0.5 λ Interelement Spacing on x-axis and v-axis

Element	With Mutual Coupling	Without Mutual Coupling
Z(I,l,1,1)	72.9831 +42.2710i	72.9831 +42.2710i
Z(2,l,l,l)	-12.5070 -29.8687i	0
Z(3,1,1,1)	4.0036+17.7065i	0
Z(4,1,l,1)	-1.8835 -12.2797i	0
Z(5,1,1,1)	1.0820 + 9.3457i	0
Z(6,l,l,l)	-0.6994—7.5286i	0
Z(7,1,1,1)	0.4884+6.2978i	0
Z(8,1,1,1)	-0.3600—5.4107i	0
Z(9,1,1,1)	0.2762+4.7415i	0

3.2.2 The Active Impedance

The active impedance matrix consists of 81 elements, and it is calculated only when mutual coupling is considered. Only nine elements of the active impedance matrix is shown in table 5,6

Table 5.6 Active Impedances For Planar Array 9x9 Element Array With 0.5k Interelement Spacing on x-axis and y-axis

Element	With Mutual Coupling
(Z_a(l, l)	90.842 + 16.763i
(Z_a(2,l)	30.371 +41.966i
Za(4,1)	33.575 ±0.43429j
Za(5,1)	76.488 +2.0661i
Za(6,1)	6.1559e+062 -4.5711e+060i
Za(7,1)	84.242 +25.654i
Za(8,1)	47.362 +37.338i
Za(9,1)	99.287-5.9481i
Za(3,1)	81.967 7.3224i

3.2.3 The Element Currents

The elements current is shown in Table 5,7, for both cases, considering and neglecting mutual coupling.

Table 5.7 Elements Current For Planar Array 9x9 Element Array With 0.52. Inter element Spacing on x-axis and y-axis

Element	With Mutual Coupling	Without Mutual Coupling
I_f(1,1)	7.0010-0.8332i	20.0000
I_f (2,1)	9.7767-5.1050i	20.0000
I_f (3,i)	7.5544+0.4192i	20.0000
I_f (4.l)	11.9650 - 0.0622i	20.0000
I_f (5,l)	7.9038-0i291i	20.0000
I_f (6,1)	0.0000 + 0.0000i	20.0000
I_f (7.l)	7.1868 - 1.3734i	20.0000
I_f (8,l)	8.9541-3.4339i	20.0000
I_f (9,1)	6.6879+0.2665i	20.0000

3.2.2.4 The Directivity

The directivity is shown in table 5.8, for both cases, considering and neglecting mutual coupling.

Table 5.8 Directivity in dB For Planar Array 9x9 Element Array With 0.5. Interelement Spacing on x-axis and v-axis

parameter	**With Mutual Coupling**	**Without Mutual Coupling**
Directivity (dbB)	8.9131	14.0788

3.2.2.5 Array Factor Comparison in Three Dimensions

Figure 5.8 through 5.10 shows the array factor pattern of the array with and without mutual coupling and a comparison between both of them.

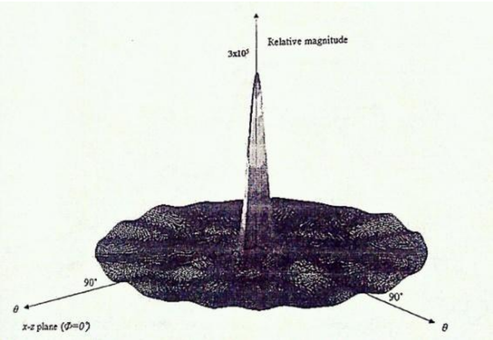

Figure 5.8, Array factor with mutual coupling for 9x9 planar array planar array with

$$d_x d_x = 0.5\lambda, and\ L = 0.5\lambda$$

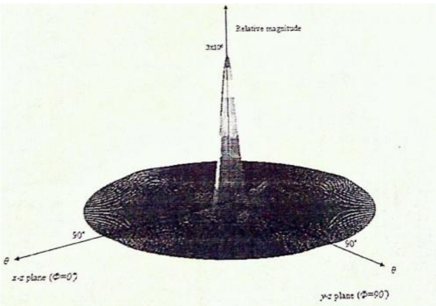

Figure 5.9. Array factor without mutual coupling for 9x9 planar array with $d_x d_y = 0.5\lambda, and\ L = 0.5\lambda$

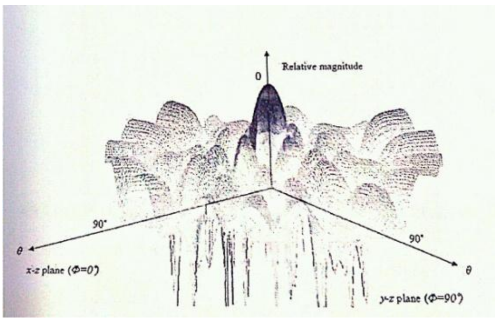

Figure 5.10. Array factor with mutual coupling in dB for 9x9 planar array with $d_x d_y = 0.5\lambda, and\ L = 0.5\lambda$

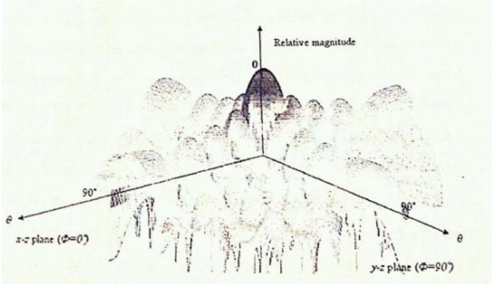

Figure 5.11, Array factor without mutual coupling in dB for 9x9 planar array planar array with
$d_x d_y = 0.5\lambda, and\ L = 0.5\lambda$

IV. CONCLUSION

Elements Current for a Nine Element Linear Array with 0.52. inter element Spacing is calculated for both cases ,considering and neglecting mutual coupling. Elements Current For Planar Array 9x9 Element Array With 0.52. inter element Spacing on x-axis and y-axis is calculated for both cases, considering and neglecting mutual coupling.

ACKNOWLEDGEMENT

We acknowledge the MSc student Farid Fouad Abdulrahman Bouges for his hard work and contribution.This article was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah. The authors, therefore, acknowledge with thanks DSR technical and financial support.

REFERENCES

[1] Shuguang CHEN and Ryuichi IWATA, "Mutual Coupling Effects in Microstrip Patch Phased Array Antenna" 0-7803-4478-2/98/S10.00 Q 1998 IEEE

[2] H. L. Sneha, Hema Singh, and R. M. Jha: "Mutual Coupling Effects for Radar Cross Section (RCS) of a Series-Fed Dipole Antenna Array", International Journal of Antennas and PropagationVolume 2012 (August 2012), Article ID 601532, 20 pagesdoi:10.1155/2012/601532

[3] JOHANNES A. G. MALHERBE, senior member IEEE. "Analysis of a Linear Antenna Array Including the Effects of Mutual Coupling", IEEE TRANSACTION ON EDUCATIONAL, VOL. 32, NO. 1, February 1989.

[4] Shuguang CHEN and Ryuichi IWATA, "Mutual Coupling Effects in Microstrip Patch Phased Array Antenna" 0-7803-4478-2/98/S10.00 Q 1998 IEEE

[5] NEWMAN, E.H., RICHMOND, J.H., and KWAN, B.W.: "Mutual Impedance Computation Between Microstrip Antennas", IEEE Trans., 1983, MTT-31, pp. 941-945
[6] POZAR, D.M.: "Input Impedance and Mutual Coupling of Rectangular Microstrip Antennas", IEEE Trans., 1982, AP-30, pp. 1191-1196

[7] VAN-LIL, RH., and VAN DE CAPELLE, A.R: "Transmission Line Model for Mutual Coupling Between Microstrip Antennas", IEEE Trans., 1984, AP-32, pp. 816-821.

[8] R. S . Elliott. "Antenna Theory and Design." Englewood Cliffs, NJ: Prentice-Hall, 1981.

[9] Constantine A. Balanis, "Antenna theory analysis and Design", Harper & Row, publishers, Newyork. 1982, and the new edition 1997.