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On Generalised β-Connectedness In Isotonic Spaces

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Abstract : *The purpose of this paper is to define and study β connectedness and β separated sets in generalized β closure spaces and discuss their properties. The notion of Z connectedness and strongly connectedness in isotonic spaces are also analysed.*
Keywords: *Generalized β closure spaces, β connectedness, β separated sets, Z connectedness, strongly connectedness.*

I. Introduction & Preliminaries

The structure of closure spaces is more general than that of topological spaces. Hammer studied closure spaces extensively and a recent study on these spaces can be found in Stadler [8,9], Harris[5],Habil and Elzenati [4].The following definition of a generalized closure space can be found in [4] and [9].Let X be a set, ℘(X) be its power set and cl: ℘(X)→ ℘(X) be any arbitrary set valued – function, called a closure function. We call cl (A), A⊂X, the closure of A and we call the pair (X,cl) a generalized closure space.

An operator cl : P(X) →P(X) is called **grounded** if cl(ϕ)= ϕ, **isotonic** if A⊆ B⊆ X implies cl(A) ⊆ cl(B), **expansive** if A⊆ cl(A)for every A⊆ X, **idempotent** if cl (cl(A)) = cl(A) for every A⊆X and **additive** if cl(A ∪ B) ⊆ cl(A) ∪ cl(B) for all subsets A and B of X.

Definition: 1.1.

(i) The space (X, cl) is said to be **isotonic** if cl is grounded and isotonic.
(ii) The space (X, cl) is said to be a **neighborhood space** if cl is grounded, expansive and isotonic.
(iii) The space (X, cl) is said to be a **closure space** if cl is grounded, expansive, isotonic and idempotent.
(iv) The space (X, cl) is said to be a **Cech closure space** if cl is grounded, expansive, isotonic and additive.
(v) A subset A of X is said to be **closed** if cl(A) = A. It is open if its complement is closed.

II. Generalized β-Closure Space

Definition 2.1:

(1) A generalized β-closure space is a pair (X,βcl) consisting of a set X and a β-closure function βcl, a function from the power set of X to itself.
(2) The β-closure of a subset A of X, denoted βcl, is the image of A under βcl.
(3) The β-exterior of A is βExt(A) = X\ βcl(A), and the β-Interior of A is βInt(A) = X \ βcl(X\A).
(4) A is β -closed if A = β cl(A), A is β -open if A = β Int(A)and N is a β -neighborhood of x if x ε βInt(N).

Definition 2.2: We say that a β -closure function β cl defined on X is:
(1) β -grounded if βcl(ϕ) = ϕ
(2) β -isotonic if βcl(A) ⊆ βcl(B) whenever A ⊆ B
(3) β -enlarging if A ⊆ βcl(A) for each subset A of X
(4) β -idempotent if βcl(A) = βcl(βcl(A)) for each subset A of X
(5) β -sub-linear if βcl(A ∪ B) ⊆ βcl(A) ∪ βcl(B) for all A,B ⊆ X
(6) β -additive if $\cup_{i\varepsilon I}$ β cl(A_i) = β cl($\cup_{i\varepsilon I}$ A_i) for A_i ⊆ X

III. β – Separated Sets

Definition 3.1: In an generalized β-closure space (X, βcl), two subsets A, B ⊆ X are called β - separated if β cl(A) ∩ B = A ∩ β cl(B) = ϕ.

Proposition 3.2: In an generalized β-closure space (X, βcl), the following two conditions are equivalent for all A, B ⊆ X:

1. A, B are β - separated.

2. There are $U \in N(A)$ and $V \in N(B)$ such that $A \cap V = U \cap B = \phi$.

Proof: By the definition of neighborhood if there exists U, V such that $A \subseteq \beta\, int(U)$, $B \subseteq \beta\, int(V)$, $A \cap V = \phi$, and $U \cap B = \phi$. We know that $A \cap V = \phi$ implies $V \subseteq X \setminus A$ and by isotonic property $B \subseteq \beta int(V) \subseteq \beta int(X \setminus A)$. Then $X \setminus \beta\, int(X \setminus A) \subseteq X \setminus \beta int(V)$ implies $\beta cl(A) \subseteq X \setminus \beta int(V) \subseteq X \setminus B$ and thus $\beta cl(A) \cap B = \phi$. Similarly $A \cap \beta cl(B) = \phi$.

Now suppose A and B are β- separated. Then $\beta cl(A) \cap B = \phi$ if and only if $B \subseteq X \setminus \beta cl(A) = \beta int(X \setminus A)$, i.e.,$X \setminus A \in N(B)$. Thus there is $V \in N(B)$ such that $A \cap V = \phi$.

Similarly we have $A \cap \beta cl(B) = \phi$ if and only if $X \setminus B \in N(A)$, i.e., there is $U \in N(A)$ with $U \cap B = \phi$.

Proposition 3.3: Let $(X, \beta cl)$, be an generalized β-closure space and let $A, B \subseteq Y \subseteq X$. Then A and B are β -separated in $(X, \beta\, cl)$ if and only if A and B are β-separated in $(Y, \beta cl_Y)$.

Proof. Suppose $A, B \subseteq Y$ and they are β- separated in X. Then we have $A \cap \beta cl(B) = \phi$ implies $A \cap \beta cl(B) \cap Y = A \cap \beta cl_Y(B) = \phi$. and $\beta cl(A) \cap B = \phi$ implies $\beta cl_Y(A) \cap B = \phi$.

Conversely, If A and B are β- separated in Y. Assume $A \cap \beta cl_Y(B) = \phi$.

We have $A \cap \beta cl_Y(B) = A \cap Y \cap \beta cl(B) = A \cap \beta cl(B) = \phi$ since $A \subseteq Y$ .Hence $A \cap \beta cl(B) = \beta cl(A) \cap B = \phi$.

Proposition 3.4: In neighborhood spaces let $f : (X, \beta cl) \rightarrow (Y, \beta cl)$ be β- continuous and suppose $A, B \subseteq Y$ are β -separated. Then $f^{-1}(A)$ and $f^{-1}(B)$ are β -separated.

Proof. Suppose A and B are β-separated in Y then we have $\beta\, cl(A) \cap B = \phi$ and $A \cap \beta\, cl(B) = \phi$ we now prove that $f^{-1}(A)$ and $f^{-1}(B)$ are β separated in X. Let us take $\beta\, cl(A) \cap B = \phi$. Then $f^{-1}(\beta\, cl(A)) \cap f^{-1}(B) = \phi$. Since f is β continuous, $\beta cl(f^{-1}(A)) \subseteq f^{-1}(\beta cl(A))$, hence $\beta cl(f^{-1}(A)) \cap f^{-1}(B) \subseteq f^{-1}(\beta cl(A)) \cap f^{-1}(B)$ implies $\beta cl(f^{-1}(A)) \cap f^{-1}(B) = \phi$.

Similarly $\beta\, cl(B) \cap A = \phi$ implies $\beta\, cl(f^{-1}B) \cap f^{-1}(A) = \phi$. Hence $f^{-1}(A)$ and $f^{-1}(B)$ are β-separated.

IV. β connectedness

Definition 4.1: A set $Y \in P(X)$ is β-connected in a generalized β closure space $(X, \beta cl)$ if it is not a disjoint union of a nontrivial β-separated pair of sets $A, Y \setminus A, A \neq \phi, Y$.

Definition 4.2: Let $(X, \beta\, cl)$ be a space and $x \in X$. The component $C(x)$ of x in X is the union of all β-connected subsets of X containing x.

Theorem 4.3: A set $Z \in P(X)$ is β- connected in an generalized β closure space $(X, \beta\, cl)$ if and only if for each proper subset $A \subseteq Z$ holds $[\beta\, cl(A) \cap (Z \setminus A)] \cup [\beta\, cl(Z \setminus A) \cap A] \neq \phi$. This is known as Hausdorff-Lennes condition.

Definition 4.4 : Connectedness is closely related to separation .Two sets $A, B \in P(X)$ are β - separated if there are β-neighborhoods $N' \in N(A)$ and $N'' \in N(B)$ such that $A \cap N' = N'' \cap B = \phi$; they are β separated if there are β- neighborhoods $N' \in N(A)$ and $N'' \in N(B)$ such that $N' \cap N'' = \phi$.

Theorem 4.5: A neighborhood space $(X, \beta cl)$ is β connected if and only if there are no nonempty disjoint β-open (closed) sets H and K in X with $X = H \cup K$.

Proof : Suppose that X is β-disconnected. Then $X = H \cup K$, where H, K are β- separated, disjoint sets. Since $H \cap \beta\, cl(K) = \phi$, we have $\beta\, cl(K) \subseteq X \setminus H \subseteq K$ and then, by definition of expanding $\beta\, cl(K) = K$. Since $\beta\, cl(H) \cap K = \phi$, a similarly H is a β-closed set.

Conversely, suppose that H and K are disjoint β -open sets such that $X = H \cup K$.

Now $K = X \setminus H$, and H is an β-open set, hence K is a β-closed set. Thus $H \cap \beta cl(K) = H \cap K = \phi$. Similarly $\beta\, cl(H) \cap K = \phi$. Thus H and K are β -separated and therefore X is β disconnected.

Proposition 4.6: If X and Y are β-connected in an generalized β closure space $(X, \beta cl)$ and $X \cap Y = \phi$,then $X \cup Y$ is β-connected.

Proof: We use the Hausdorff-Lennes condition

$$[\beta cl(A) \cap (Y \cup Z) \setminus A] \cup [A \cap \beta cl((Y \cup Z) \setminus A)] =$$

$$[\beta cl(A) \cap (Y \setminus A)] \cup [\beta cl(A) \cap (Z \setminus A)] \cup [A \cap \beta cl((Y \setminus A) \cup (Z \setminus A))] \supseteq$$

$$\{[\beta cl(A) \cap (Y \setminus A)] \cup [A \cap \beta cl(Y \setminus A)]\} \cup \{[A \cap \beta cl(Z \setminus A)] \cup \beta cl(A) \cap (Z \setminus A)]\}$$

If $A \cap Y$ or $A \cap Z$ is a proper subset of Y or Z, then one of the expression is non empty .Both the expressions are empty if and only if either A=Z and $A \cap Y = \phi$ or $A \cap Z = \phi$ and A=Y. This is impossible if $Y \cap Z \neq \phi$

Theorem 4.7: If (X, βcl) is a neighborhood space then βcl(Z) is β-connected whenever Z is β-connected.

Proof: Set A' = Z ∩ A and A″ = A\Z. We then use the Hausdorff-Lennes condition

$[\beta cl(A) \cap (\beta cl(Z) \setminus A)] \cup [\beta cl(\beta cl(Z) \setminus A) \cap A] \supseteq$

$[(\beta cl(A') \cup \beta cl(A'')) \cup (Z \setminus A')] \cup [\beta cl(Z - A') \cap (A' \cup A'')] \supseteq$

$\{[\beta cl(A') \cap (Z \setminus A')] \cup [\beta cl(Z \setminus A') \cap A']\}\ [\beta cl(Z - A') \cap A'']$

Here we have used $Z - A \subseteq (\beta cl(Z) \setminus A') \setminus A'$ which is true only if $A \subseteq \beta cl(A)$ holds. If $A' \neq \phi$ then the term in the braces is nonempty because Z is βconnected. By assumption if $A' = \phi$ then $A'' \subseteq \beta cl(Z) \setminus Z$ is nonempty and hence $\beta cl(Z \setminus A') \cap A'' = \beta cl(Z) \cap A'' = \phi$. Thus βcl(Z) is β-connected.

Theorem 4.8: If $f : (X, \beta cl) \to (Y, \beta cl)$ is a β-continuous function between neighborhood spaces, and A is β connected in X, then f(A) is β-connected in Y.

Proof : Suppose f(A) is not β- connected. Then f(A) = U ∪ V , where U and V are β-separated and non-empty. Thus $f^{-1}(U)$ and $f^{-1}(V)$ are β-separated. Clearly $A' = A \cap f^{-1}(U)$ and $A'' = A \cap f^{-1}(V)$ are both non-empty and also β-separated. Further A' ∪ A″ = A and hence A is not β-connected.

Theorem 4.9:Let (X, βcl) be a generalized β-closure space with β-grounded β-isotonic β-enlarging βCl. Then, the following are equivalent:

(1) (X, βcl) is β-connected,

(2) X cannot be a union of nonempty disjoint βopen sets.

Proof :. (1) ⇒(2): Let X be a union of nonempty disjoint β-open sets A and B. Then, X = A ∪ B and this implies that B = X\A and A is a β-open set. Thus, B is β-closed and hence A ∩ β cl(B) = A ∩ B = ϕ. Likewise, we obtain βcl(A) ∩ B = ϕ. Hence, A and B are β-closure-separated and hence X is not β-connected. This is a contradiction.

(2) ⇒ (1): Suppose that *X* is not β-connected. Then X = A ∪B, where A, B are disjoint β- separated sets, i.e A∩βcl(B) = βcl(A) ∩B = ϕ. We have βcl(B) ⊂ X\A⊂ B. Since βcl is β-enlarging, we get βcl(B) = B and hence, B is β-closed. By using βcl(A) ∩B = ϕ. and similarly, it is obvious that A is β-closed. This is a contradiction.

Definition 4.10: Let (X, βcl) be a generalized β-closure space with β–grounded β–isotonic βcl. Then, (X, βcl) is called a T_1-β-grounded β-isotonic space if βcl({x}) ⊂ {x} for all x ∈ X.

Theorem 4.11: Let (X, βcl) be a generalized β-closure space with β-grounded β-isotonic βcl. Then, the following are equivalent:

(1) (X, βcl) is β-connected,

(2) Any β-continuous function f: X → Y is constant for all T_1- β-grounded β-isotonic spaces Y = {0, 1}.

Proof :. (1)⇒(2): Let X be β- connected. Suppose that f: X → Y is β- continuous and it is not constant. Then there exists a set U ⊂ X such that $U = f^{-1}(\{0\})$ and $X \setminus U = f^{-1}(\{1\})$. Since f is βcontinuous and Y is T_1- β- grounded β-isotonic space, then we have $\beta cl(U) = \beta cl(f^{-1}(\{0\})) \subset f^{-1}(\beta cl\{0\}) \subset f^{-1}(\{0\}) = U$ and hence $\beta cl(U) \cap (X \setminus U) = \phi$.Similarly we have $U \cap \beta cl(X \setminus U) = \phi$. This is a contradiction. Thus, f is constant.

(2)⇒ (1): Suppose that X is not βconnected. Then there exist β-closure- separated sets U and V such that U ∪ V = X. We have βcl(U)⊂U and βcl(V) ⊂V and X\U ⊂V. Since βcl is β-isotonic and U and V are β- closure-separated, then βcl(X\U) ⊂βcl(V) ⊂ X\U. If we consider the space (Y, β cl) by Y = {0, 1}, βcl(ϕ) = ϕ ,βcl({0}) = {0}, βcl ({1}) = {1} and βcl (Y) = Y, then the space (Y, βcl) is a T_1- β-grounded β-isotonic space. We define the function f: X → Y as f(U) = {0} and f(X\U) = {1}. Let A≠ϕ and A ⊂Y. If A = Y, then $f^{-1}(A) = X$ and hence $\beta cl(X) = \beta cl(f^{-1}(A)) \subset X = f^{-1}(A) = f^{-1}(\beta cl(A))$.

If A = {0}, then $f^{-1}(A) = U$ and hence $\beta cl(U) = \beta cl(f^{-1}(A)) \subset U = f^{-1}(A) = f^{-1}(\beta cl(A))$. If A = {1}, then $f^{-1}(A) = X \setminus U$ and hence $\beta cl(X \setminus U) = \beta cl(f^{-1}(A)) \subset X \setminus U = f^{-1}(A) = f^{-1}(\beta cl(A))$. Hence, f is βcontinuous. Since f is not constant, this is a contradiction.

Theorem 4.12: Let *f* : (X, βcl) → (Y, βcl) and g : (Y, βcl) → (Z, βcl) be β continuous functions. Then, gof : X → Z is β continuous.

Proof :. Suppose that f and g are β continuous. For all A ⊂ Z we have

$\beta cl(gof)^{-1}(A) = \beta cl(f^{-1}(g^{-1}(A))) \subset f^{-1}(\beta cl(g^{-1}(A))) \subset f^{-1}(g^{-1}(\beta cl(A)))$

$= (gof)^{-1}(\beta cl(A))$. Hence, gof : $X \to Z$ is β continuous.

Theorem 4.13: Let (X, βcl) and (Y, βcl) be generalized β-closure spaces with β-grounded β-isotonic βcl and **f** : (X, βcl) → (Y, βcl) be a β continuous function onto Y . If X is β connected, then Y is β connected.

Proof : Let us suppose that $\{0, 1\}$ is a generalized β-closure spaces with β-grounded β-isotonic βCl and g : Y → $\{0, 1\}$ is a β continuous function. Since **f** is β continuous, gof : X → $\{0, 1\}$ is β continuous. Since X is β-connected, gof is constant and hence *g* is constant. Therefore Y is β connected.

Definition 4.14: Let (Y, βcl) be a generalized β-closure space with β-grounded β-isotonic βcl and more than one element. A generalized β-closure space (X, βcl) with β-grounded β-isotonic βcl is called Y – β connected if any β- continuous function **f** : X → Y is constant.

Theorem 4.15: Let (Y, βcl) be a generalized β -closure space with β -grounded β -isotonic β – enlarging βcl and more than one element. Then every Y - β connected generalized β -closure space with β - grounded β -isotonic is β connected.

Proof : Let (X, βcl) be a Y - β connected generalized β -closure space with β - grounded β -isotonic βcl. Suppose that **f** : X → $\{0, 1\}$ is a β continuous function, where $\{0, 1\}$ is a T_1- β -grounded β -isotonic space. Since Y is a generalized β -closure space with β -grounded β -isotonic β -enlarging βcl and more than one element, then there exists a β continuous injection
g : $\{0, 1\}$ → Y .gof : X → Y is β continuous. Since X is Y - β connected, then gof is constant. Thus, **f** is constant and hence, by theorem 5.11 X is β connected.

Theorem 4.16: Let (X, βcl) and (Y, βcl) be generalized β -closure spaces with β -grounded β -isotonic βcl and **f** : (X, βcl) → (Y, βcl) be a β continuous function onto Y . If X is Z - β connected, then Y is Z - β connected.

Proof : Suppose that g : Y → Z is a β continuous function. Then gof : X → Z is β continuous. Since X is Z- β connected, then gof is constant. This implies that g is constant. Thus, Y is Z- β connected.

5.Strongly Connected Spaces

Definition 5.1: A generalized β -closure space (X, βcl) is strongly β connected if there is no countable collection of pairwise β -closure-separated sets $\{A_n\}$ such that $X = \cup A_n$.

Theorem 5.2: Every strongly β connected generalized β closure space with β -grounded β -isotonic βcl is β connected.

Theorem 5.3: Let (X, βcl) and (Y, βcl) be generalized β -closure spaces with β -grounded β -isotonic βcl and **f** : (X, βcl) → (Y, βcl) be a β- continuous function onto Y . If X is strongly β connected, then Y is strongly β- connected.

Proof : Suppose that Y is not strongly β- connected. Then, there exists a countable collection of pairwise β- closure-separated sets $\{A_n\}$ such that $Y = \cup A_n$. Since $f^{-1}(A_n) \cap \beta cl(f^{-1}(A_m))$ $\subset f^{-1}(A_n) \cap f^{-1}(\beta\, cl(A_m)) = \phi$ for all n ≠ m, then the collection $\{f^{-1}(A_n)\}$ is pairwise β –closure-separated. This is a contradiction. Hence, Y is strongly β-connected.

Theorem 5.4: Let $(X, \beta cl_X)$ and $(Y, \beta cl_y)$ be generalized β -closure spaces. Then, the following are equivalent for a function **f** : X → Y

(1) **f** is β -continuous,

(2) $f^{-1}(\beta Int(B)) \subseteq \beta Int(f^{-1}(B))$ for each $B \subseteq Y$.

Theorem 5.5: Let (X,βcl) be a generalized β -closure space with β -grounded β –isotonic β -additive βcl. Then (X,βcl) is strongly β- connected if and only if (X, βcl) Y – β- connected for any countable T_1- β - grounded β -isotonic space (Y, βcl).

Proof : (⇒): Let (X,βcl) be strongly β- connected. Suppose that (X, βcl) is not Y – β- connected for some countable T_1- β -grounded β -isotonic space (Y, βcl). There exists a β continuous function f: X → Y which is not constant and hence K= f(X) is a countable set with more than one element. For each $y_n \in K$, there exists $U_n \subset X$ such that $U_n = f^{-1}(\{y_n\})$ and hence $Y = \cup U_n$.

Since f is β- continuous and Y is β -grounded, then for each n ≠ m, $U_n \cap \beta cl(U_m) =$

$f^{-1}(\{y_n\}) \cap \beta cl(f^{-1}(\{y_m\})) \subset f^{-1}(\{y_n\}) \cap f^{-1}(\beta Cl(\{y_m\})) \subset f^{-1}(\{y_n\})$ $\cap f^{-1}(\{y_m\}) = \phi$. This contradicts the strong β- connectedness of X. Thus, X is Y - β connected.

(⇐) :Let X be Y - β connected for any countable T_1- β -grounded β -isotonic space (Y, βcl). Suppose that

X is not strongly β- connected. There exists a countable collection of pairwise β -closure-separated sets $\{U_n\}$ such that $X = \cup U_n$ .Consider the space (Z, βcl), where Z is the set of integers and βcl : P(Z) → P(Z) is defined by βcl(K) =K for each K ⊂ Z .Clearly (Z, βcl) is a countable T_1- β -grounded β - isotonic space.Put $U_k \in \{U_n\}$.We define a function

$f: X \to Z$ by $f(U_k) = \{x\}$ and $f(X \setminus U_k) = \{y\}$ where x, y ∈ Z and x ≠ y. Since $\beta cl(U_k) \cap U_n = \phi$ for all n ≠ k, then $\beta cl(U_k) \cap U_{n \neq k} U_n = \phi$ and hence $\beta cl(U_k) \subset U_k$. Let

$\phi \neq K \subset Z$. If x, y ∈ K then $f^{-1}(K) = X$ and $\beta cl(f^{-1}(K)) = \beta cl(X) \subset X = f^{-1}(K)$

$= f^{-1}(\beta cl(K))$. If x∈ K and y ∉ K ,then $f^{-1}(K) = U_k$ and $\beta cl(f^{-1}(K)) = \beta cl(U_k) \subset U_k$

$= f^{-1}(K) = f^{-1}(\beta cl(K))$.If y∈ K and x∉ K then $f^{-1}(K) = X \setminus U_k$.On the other hand, for all n ≠k, $U_k \cap \beta cl(U_n) = \phi$ and hence $U_k \cap U_{n \neq k} \beta cl(U_n) = \phi$ This implies that

$U_k \cap \beta cl(U_{n \neq k} U_n) = \phi$ .Thus, $\beta cl(X \setminus U_k) \subset X \setminus U_k$. Since βcl(K) = K for each K ⊂Z , we have $\beta cl(f^{-1}(K)) = \beta cl(X \setminus U_k) \subset X \setminus U_k = f^{-1}(K) = f^{-1}(\beta cl(K))$.Hence we obtain that f is β- continuous. Since f is not constant, this is a contradiction with the Z- β connectedness of X. Hence, X is strongly β- connected.

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